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A New Fuzzy Reliability Model to Avoid Resonance and its Numerical Solutions

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ABSTRACT

There is a lot of fuzziness in resonance problem. Translating the fuzzy membership function into probability density function by using the normalized method, the fuzzy reliability model of blade to avoid resonance was established. Two methods were proposed to solve the model, which were called direct sampling method and typical distribution sampling method respectively, and the calculation formulas of their corresponding coefficients of variation were given. Some examples were carried out to compare the two methods and study the influence of fuzzy degrees on reliability.

Keywords: reliability model; numerical method; coefficient of variation; sample distribution; fuzzy degree

1. Introduction

There are always a lot of fuzzy uncertainties in engineering. Therefore, the fuzzy theory was introduced into reliability engineering, and the fuzzy reliability theory was founded and developed gradually [1-4]. The resonance reliability assessment is a typical fuzzy problem. Resonance failure occurs when the excitation frequency equals or closes to the inherent frequency. However, "close" is a fuzzy concept. In practice, it is difficult to describe the concept "close" by using a definite zone. So the resonance zone has no clear boundary and is fuzzy. In other words, whether the resonance failure occurs is a fuzzy event. Meanwhile, for some components, such as aero-engine blade (especially with damping structure), apart from the fuzziness of resonance zone, its inherent frequency also has a lot of fuzziness due to the multiple influences of the various known and unknown factors such as machining accuracy, installation tightness, damping factors and so on. Some researchers have studied the fuzzy reliability to avoid resonance [5-8]. However, as mentioned in the literature [5], the studies just considered the fuzziness of resonance zone but ignored the fuzziness of basic variables.

The fuzzy reliability model of blade to avoid resonance, which considered the fuzziness of both resonance zone and inherent frequency, was presented in this paper. The expression and measurement of the fuzziness were discussed in section 2. Translating the fuzzy membership function into probability density function by using the normalized method, the reliability model was proposed in section 3, and its numerical methods, which were called Direct Sampling Method and Typical Distribution Sampling Method respectively, were given to solve the model. In section 4, the coefficients of variation of numerical solutions were calculated. In section 5, simulations were carried out in order to: (i) explain how one can utilize the model and its numerical methods, (ii) compare the two numerical solution methods and (iii) study the influence of fuzzy degree on reliability assessment. Some conclusions were made in section 6.

2. Expression and measurement of the fuzziness

2.1. Expression of the fuzziness

The resonance zone is a fuzzy sub-set of the frequency domain. The event "resonance will not occur" is a fuzzy event, which is noted as $\tilde{w}_R$. The membership function $\mu_{\tilde{w}_R}(w)$ is used to describe the degree of w belongs to the resonance zone. It can be expressed by normal bathtub curve [5]:

$$\mu_{\tilde{w}_R}(w) = \begin{cases} 1-\exp\{-(w+\alpha)^2/\beta^2\} & w<-\alpha \\ 0 & -\alpha \le w \le \alpha \\ 1-\exp\{-(w-\alpha)^2/\beta^2\} & w>\alpha \end{cases} \tag{1}$$

where $w = f_e - \tilde{f}_n$, f_e is the random excitation frequency, $\tilde{f}_n$ is the fuzzy inherent frequency, α and β are the position and shape parameters respectively of the fuzzy parts.

The fuzziness of inherent frequency means that its range $\tilde{F}_n$ is a fuzzy subset of the frequency domain. The membership function $\mu_{\tilde{f}_n}(f_n)$ is used to describe the degree of f_n belongs to $\tilde{F}_n$. In engineering $\mu_{\tilde{f}_n}(f_n)$ can be considered as trapezoidal, which should satisfy some conditions (reference [9]) and its expression is shown as follows [10]:

$$\mu_{\tilde{f}_n}(f_n) = \begin{cases} 1-(a-f_n)/M_1 & a-M_1 \le f_n < a \\ 1 & a \le f_n \le b \\ 1-(f_n-b)/M_2 & b < f_n \le b+M_2 \\ 0 & else \end{cases} \tag{2}$$

where $[a,b]$ is definite and called flat zone, $[a-M_1,a]$ is left fuzzy zone and M_1 is the length of the zone, $[b,b+M_2]$ is right fuzzy zone and M_2 denotes its length.

2.2. Measurement of the fuzziness

Fuzzy degree denotes the degree of the fuzziness. In the continuous domain, fuzzy degree of event or variable can be calculated by the following formula:

$$D(\mu) = \int_{-\infty}^{+\infty} |\mu(x) - \mu_{0.5}(x)| \mathrm{d}x \tag{3}$$

where $\mu_{0.5}(x)=\begin{cases}1 & \mu(x)\geq 0.5\\ 0 & \mu(x)<0.5\end{cases}$.

Substituting (1) into (3), the fuzzy degree of resonance zone is obtained:

$$D(\mu_{\tilde{w}_R})=\beta[2\sqrt{\ln 2}-4\sqrt{\pi}\Phi(\sqrt{2\ln 2})+3\sqrt{\pi}]\approx 0.7400\beta \tag{4}$$

In formula (4), it can be seen that the fuzzy degree of resonance zone is related to the shape parameter β, and not related to the position parameter. Substituting (2) into (3), the fuzzy degree of inherent frequency is obtained:

$$\begin{aligned}D(\mu)&=\int_{-\infty}^{a-M_1/2}\mu(x)\mathrm{d}x+\int_{a-M_1/2}^{b+M_2/2}(1-\mu(x))\mathrm{d}x+\int_{b+M_2/2}^{+\infty}\mu(x)\mathrm{d}x\\&=\frac{1}{4}(M_1+M_2)\end{aligned} \tag{5}$$

In formula (5), it can be seen that the fuzzy degree of trapezoidal inherent frequency is related to the lengths M_1 and M_2 of fuzzy zones, and not related to the position parameter a and b.

3. Reliability model and its numerical solutions

Let the probability density function (PDF) of random excitation frequency f_e is $g_{f_e}(f_e)$, and the membership function of fuzzy inherent frequency $\tilde{f}_n$ is $\mu_{\tilde{f}_n}(f_n)$. The resonance zone is fuzzy, and the membership function of fuzzy event $\tilde{w}_R$ “blade resonance will not occur” is $\mu_{\tilde{w}_R}(w)$.

The reliability of blade to avoid resonance is the probability of the event $\tilde{w}_R$ occurs, i.e. $R=P(\tilde{w}_R)$. The probability of $\tilde{w}_R$ can be calculated by the following formula [11]:

$$P(\tilde{w}_R)=\int_{-\infty}^{+\infty}\mu_{\tilde{w}_R}(w)f(w)\mathrm{d}w \tag{6}$$

where $f(w)$ is the PDF of w. Because f_e and $\tilde{f}_n$ are independent and $w=f_e-\tilde{f}_n$, (6) can be expressed as:

$$P(\tilde{w}_R)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\mu_{\tilde{w}_R}(f_e,f_n)g_{f_e}(f_e)g_{f_n}(f_n)\mathrm{d}f_e\mathrm{d}f_n \tag{7}$$

where $g_{f_e}(f_e)$ and $g_{f_n}(f_n)$ are respectively the PDF of f_e and $\tilde{f}_n$. Because $\tilde{f}_n$ is a fuzzy variable and has no PDF, it is needed to translate the membership function

$\mu_{\tilde{f}_n}(f_n)$ into a PDF $g_{f_n}(f_n)$ first. According to the probability theory, $g_{f_n}(f_n)$ must satisfy two properties: (i) $g_{f_n}(f_n) \geq 0; f_n \in R$; (ii) $\int_{-\infty}^{+\infty} g_{f_n}(f_n)\mathrm{d}f_n = 1$. Obviously, $\mu_{\tilde{f}_n}(f_n)$ satisfies the former one, but not satisfies the latter one.

Let $Q = \int_{-\infty}^{+\infty} \mu_{\tilde{f}_n}(f_n)\mathrm{d}f_n$, and translate $\mu_{\tilde{f}_n}(f_n)$ according to the following formula:

$$g_{f_n}(f_n) = \frac{\mu_{\tilde{f}_n}(f_n)}{\int_{-\infty}^{+\infty} \mu_{\tilde{f}_n}(f_n)\mathrm{d}f_n} = \frac{\mu_{\tilde{f}_n}(f_n)}{Q} \tag{8}$$

The obtained $g_{f_n}(f_n)$ satisfies the properties (i) and (ii) of PDF. Substituting (8) into (7), the reliability R can be calculated as:

$$R = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \mu_{\tilde{w}_R}(f_e, f_n) g_{f_e}(f_e) \frac{\mu_{\tilde{f}_n}(f_n)}{Q} \mathrm{d}f_e \mathrm{d}f_n \tag{9}$$

Formula (9) is just the reliability model of blade to avoid resonance.

3.1. Solution of the model

The closed solution of (9) is difficult to obtain, so two numerical methods are given.

Direct sampling method (DSM): Sample the inherent frequency from $g_{f_n}(f_n)$ directly. Specifically, sample from $g_{f_e}(f_e)$ and $g_{f_n}(f_n)$ for N times respectively, and get the samples $f_{ei}(i = 1, 2, \cdots, N)$ and $f_{ni}(i = 1, 2, \cdots, N)$. Because (9) has the equivalent form $R = E[\mu_{\tilde{w}_R}(f_e, f_n)]$, R can be estimated approximately according the following formula:

$$R \approx \hat{R} = \frac{1}{N}\sum_{i=1}^{N} \mu_{\tilde{w}_R}(f_{ei}, f_{ni}) \tag{10}$$

Typical distribution sampling method (TDSM): $g_{f_n}(f_n)$ is not a typical PDF and there is no existing command to sample it in common mathematics software, so when DSM is adopted, it needs to programme using Monte-Carlo sampling method. Thus, it will take more time to sample. In order to raise the efficiency, a numerical method which can sample from typical PDF is proposed.

For arbitrary PDF $g(\cdot)$ in real number field, (9) can be rewritten as:

$$R=\frac{1}{Q}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\mu_{\tilde{w}_R}(f_e,f_n)g_{f_e}(f_e)g(f_n)\frac{\mu_{\tilde{f}_n}(f_n)}{g(f_n)}\mathrm{d}f_e\mathrm{d}f_n$$
$$=\frac{1}{Q}E[\mu_{\tilde{w}_R}(f_e,f_n)\frac{\mu_{\tilde{f}_n}(f_n)}{g(f_n)}]\triangleq\frac{1}{Q}E[H(f_e,f_n)]$$

So sample from $g_{f_e}(f_e)$ and $g(f_n)$ for N times, R can be estimated as:

$$R\approx\hat{R}=\frac{1}{QN}\sum_{i=1}^{N}H(f_{ei},f_{ni}) \tag{11}$$

$g(f_n)$ is an arbitrary PDF, so it can be any common typical PDF such as uniform distribution , normal distribution and so on. In this way, time can be saved in sampling and the computational efficiency will be raised.

4. Coefficient of variation

The coefficient of variation **(**CV) reflects the relative dispersity of numerical solution, and its calculation formula is shown as follows:

$$CV(\hat{R})=\sqrt{\mathrm{var}(\hat{R})}\,/\,|E[\hat{R}]| \tag{12}$$

4.1. CV of DSM

Substituting (10) into (12), the following formulas can be obtained:

$$E[\hat{R}]=E[\mu_{\tilde{w}_R}(f_e,f_n)]\approx\frac{1}{N}\sum_{i=1}^{N}\mu_{\tilde{w}_R}(f_{ei},f_{ni}) \tag{13}$$

$$\mathrm{var}(\hat{R})=\frac{\mathrm{var}(\mu_{\tilde{w}_R}(f_e,f_n))}{N}\approx\frac{1}{N(N-1)}\sum_{i=1}^{N}[\mu_{\tilde{w}_R}(f_{ei},f_{ni})-\sum_{i=1}^{N}\mu_{\tilde{w}_R}(f_{ei},f_{ni})]^2 \tag{14}$$

Substituting (13) and (14) into (12), the CV of solution (10) can be calculated.

4.1. CV of TDSM

Similarly, for the TDSM solution (11), there are:

$$E[\hat{R}]=\frac{1}{Q}E[H(f_e,f_n)]\approx\frac{1}{QN}\sum_{i=1}^{N}H(f_{ei},f_{ni}) \tag{15}$$

$$\mathrm{var}[\hat{R}]=\frac{\mathrm{var}[H(f_e,f_n)]}{Q^2N}\approx\frac{1}{Q^2N(N-1)}\sum_{i=1}^{N}H(f_{ei},f_{ni})-\frac{1}{N}[\sum_{i=1}^{N}H(f_{ei},f_{ni})]^2 \tag{16}$$

Substituting (15) and (16) into (12), the CV of solution (11) can be calculated.

5. Numerical simulation study

The first bend frequency of some shoulder blade is about 180Hz. The frequency manufacture tolerance is 5%, while the ultra-tolerance phenomenon often occurs in practice and the acceptable range is ±8% in engineering. The resonance maybe appears when the blade is subjected to the excitation frequency which obeys random distribution N (230,10). The reliability of blade to avoid first bending resonance could be assessed. Generally, when the engine works, the resonance margin of the lower order mode of vibration must be bigger than 10%. Thus the membership function parameters of resonance zone can be obtained as α =18.8865 and β =3.281 by using the fuzzy comprehensive evaluation method. Meanwhile, according to other known conditions, the median of flat zone of inherent frequency is 180Hz, and the length is 180×10%=18, i.e. the flat zone is [171,189]. The lengths of bilateral fuzzy zones are both 180×(8%-5%)=5.4, i.e. $M_1 = M_2 = M$ =5.4. According to (10) ~ (11), the reliability to avoid first bending resonance can be assessed using DSM and TDSM.

5.1. Reliabilities with different methods

In this example, DSM and TDSM are adopted respectively to assess the reliability of the blade above to avoid first bending resonance, and the spent time is recorded. When TDSM is adopted, the chosen sample distributions are respectively typical uniform distribution and normal distribution, and untypical linear distribution.

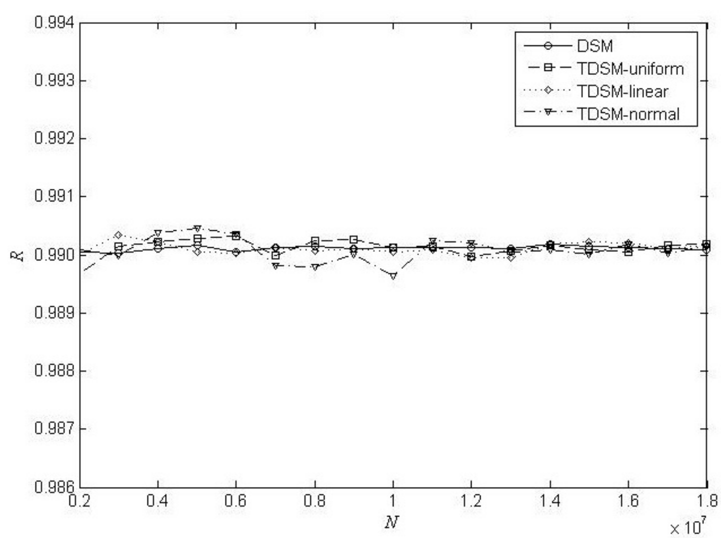

Figure 1. Reliability curves

Reliability curves with different methods are shown in Fig 1. The abscissa is sampling times. It can be seen that the reliability curve obtained by DSM is flatter, which means the reliability assessment is stable with lesser samples. Meanwhile, all the three curves obtained by TDSM have bigger fluctuation with lesser samples, and when the sampling times reach 1.1×10^7, they tend to flat and are consistent with the curve obtained by DSM. This means that more sampling times are required to improve the stability of the reliability obtained by TDSM.

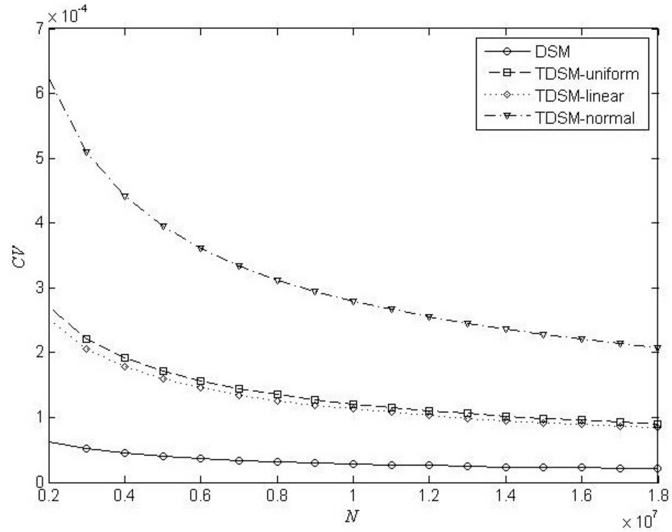

Figure 2. Coefficient of Variation curves

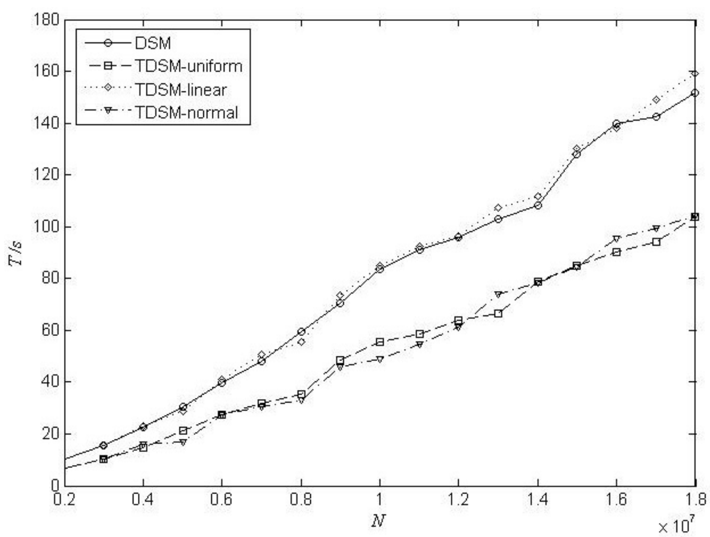

Figure 3. Calculation time curves

CV curves and calculation time curves with different methods are shown in Fig 2 and Fig 3 respectively. In Fig 2, it can be seen that the CV of DSM is smallest and the CV of TDSM is bigger. Meanwhile, with increasing of sample times, all the four curves decrease, i.e. the relative dispersity of reliability assessments decreases, which are consistent with the statistical law. From Fig 3, it can be seen that under the same sample times, DSM and TDSM with linear sample distribution need more time; while TDSM with typical uniform and normal sample distributions need less time and the computational efficiencies are higher. With increasing of sample times, the time differences in different methods are bigger and bigger.

In a word, compared with DSM, TDSM loses some stability, but it improves computational efficiency when the sample distribution is typical. Thus, the advantage of TDSM is that it can improve computational efficiency by sampling from typical PDF instead of the complicated sample distribution in DSM.

5.2. The influence of fuzzy degree on reliability

It is shown in (4) and (5) that the fuzzy degrees of resonance zone and inherent frequency are determined by the parameters β, M_1 and M_2. Therefore, these parameters are changed instead of the variations of fuzzy degrees in this example.

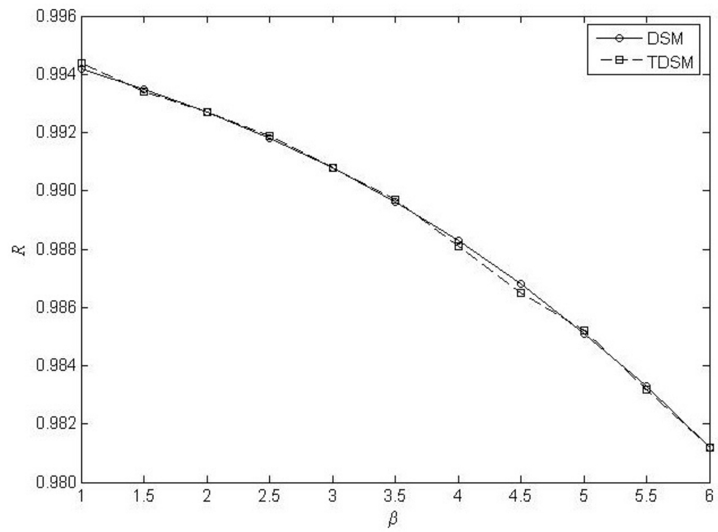

Figure 4. Reliability curves with the change of parameter β

Reliability curves with the change of parameter β are shown in Fig.4. It can be seen that reliabilities decrease with increasing of the fuzzy degree of resonance zone (i.e. β increases). In other words, the fuzzier the resonance zone is, the lower the reliabilities will be.

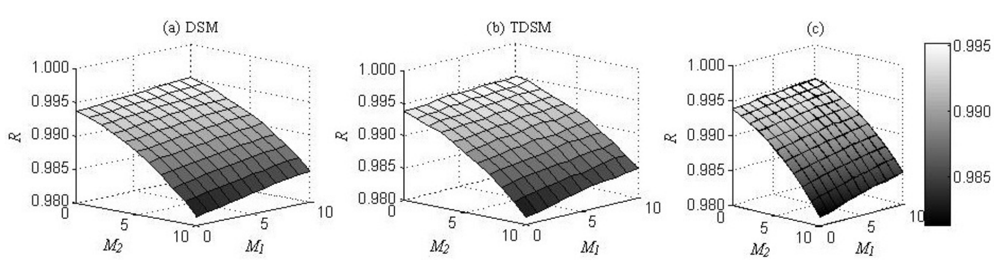

Figure 5. Reliability surfaces with the change of parameters M_1 and M_2

Reliability surfaces with the change of parameters M_1 and M_2 are shown in Fig 5, where (a) is the reliability surface obtained by DSM, (b) is the reliability surface obtained by TDSM, and (c) is the color bar figure of the two surfaces. It can be seen that reliabilities increase with increasing of the left side fuzzy degree of inherent frequency (i.e. M_1 increases), and decrease with increasing of the right side fuzzy degree (i.e. M_2 increases). This is because the mean 230Hz of excitation frequency is in the right side of flat zone $[a,b]$ of the first bending frequency, when the left side fuzzy degree of inherent frequency increases (M_1 increases), the possibility "the inherent frequency is far away from the excitation frequency" increases, thus the reliabilities increase. And when the right side fuzzy degree of inherent frequency increases (M_2 increases), the possibility "the inherent frequency is close to the excitation frequency" increases, and thus the reliabilities decrease. In engineering, if the mean of excitation frequency is in the left side of flat zone of inherent frequency, the reliabilities will have opposite variations regularity with each parameter.

In Fig 5 (c), it can be seen that the reliability assessment surfaces obtained by DSM and TDSM are very close. In practice, engineers can choose DSM or TDSM according to actual demands: if it requires higher stability, DSM should be chosen; and if it requires higher computational efficiency, TDSM with typical sample distribution should be chosen.

6. Conclusions

The assessment problem of the reliability to avoid resonance was studied in this paper. The fuzzy reliability model was established by translating the membership function into the corresponding probability density function by using the normalized method. Meanwhile, direct sampling method and typical distribution sampling method were proposed to solve the model, and the calculation formulas of the coefficient of variation of the two solutions were given. The simulation examples showed that both DSM and TDSM can assess the reliability to avoid resonance. DSM is suitable for the reliability assessment problems which require a more steady result, and TDSM is suitable for the reliability assessment problems which require a higher computational efficiency. Furthermore, the fuzzier the resonance zone is, the smaller the reliability is. And if the mean of excitation frequency is in the right side of the flat zone of inherent frequency, the reliabilities increase with increasing of the left side fuzzy degree, and decrease with increasing of the right side fuzzy degree, and vice versa.

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