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On the M-Projective Curvature Tensor of P-Sasakian Manifolds

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ABSTRACT

The present paper deals with Para Sasakian manifolds with m-projective curvature tensor.

Keywords: P-Sasakian manifold, m-projective curvature tensor, concircular curvature tensor, η-Einstein manifold.

AMS Mathematics Subject Classification (2010):

1. Introduction

Sat$\bar{o}$ [6, 7] introduced the notion of an almost para contactstructure, either P-Sasakian or SP-Sasakian, and gave a lot of very interesting results about such manifolds. In [3] Bhagwat Prasad define and studied a tensor field on Riemannian manifold of dimension n, called the pseudo projective curvature tensor which in a particular case becomes a projective curvature tensor.

In this paper, we investigate the properties of the P-Sasakian manifold equipped with m-projective curvature tensor. An n-dimensional P-Sasakian manifold is a said to be m-projectively flat if $\overline{P} = 0$, where $\overline{P}$ is the m-projective curvature tensor.

We show that m-projectively flat Para-Sasakian manifold is an Einstein manifold. Also we prove that an n-dimensional m-projectively flat P-Sasakian manifold is locally isometric with the Hyperbolic $H^n(-1)$.

Next, we investigate φ-m-projectively flat P-Sasakian manifold. A. Yildiz and M. Turan [2] studied the same condition on α-Sasakian manifold.We prove that φ-m-projectively flat P-Sasalian manifold is an η-Einstein manifold. Then we study P-Sasakian manifold in with $\tilde{C}(\xi, X).\overline{P} = 0$, were $\tilde{C}$ is a concircular curvature tensor.

In this case, we show that either manifold has scaler curvature $r = n(n-1)$ or manifold is locally isometric with the Hyperbolic $H^n(-1)$.
Finally, we study an n-dimensional P-Sasakian manifold satisfying $R(X,Y).\overline{P} = 0$ and we prove that such manifold is locally isometric with the Hyperbolic $H^n(-1)$.

2. Preliminaries

Let M be an n-dimensional contact manifold with contact form η, i.e, $\eta \wedge (d\lambda)^n \neq 0$. It is well known that a contact manifold admits a vector field ξ, called the characteristic vector field, such that $\eta(\xi) = 1$ and $\eta(\xi) = 1$ for every $X \in \chi(M)$. Moreover, M admits a Riemannian metric g and a tensor field φ of type (1,1) such that [6, 9]

$$\varphi^2 = I - \eta \otimes \xi, \tag{2.1}$$

$$g(X,\xi) = \eta(X), \tag{2.2}$$

$$g(X,\varphi Y) = d\eta(X,Y). \tag{2.3}$$

We then say that (φ,η,ξ,g) is a contact metric structure. A contact metric manifold is said to be a Sasakian if

$$(\nabla_X \varphi)Y = g(X,Y)\xi - \eta(Y)X, \tag{2.4}$$

In which case

$$\nabla_X \xi = -\varphi X, \qquad R(X,Y)\xi = \eta(Y)X - \eta(X)Y, \tag{2.5}$$

for all vector fields X, Y on M. Now, we give a structure similar to Sasakian but not having contact.
An n-dimensional differentiable manifold M is said to admit an almost para contact Riemannian structure (φ,η,ξ,g) such that [5, 6, 9]

$$\varphi\xi = 0, \quad \eta(\varphi) = 0, \quad \eta(\xi) = 1, \tag{2.6}$$

$$g(\xi,X) = \eta(X), \quad \varphi^2 X = X = X - \eta(X)\xi, \tag{2.7}$$

$$g(\varphi X,\varphi Y) = g(X,Y) - \eta(X)\eta(Y), \tag{2.8}$$

for all vector fields X, Y on M. The equation $\eta(\xi) = 1$ equivalent to $|\eta| \equiv 1$, then ξ is just the metric dual of η. If (φ,η,ξ,g) satisfy the equations

$$d\eta = 0, \quad \nabla_X \xi = \varphi X, \tag{2.9}$$

$$(\nabla_X \varphi)Y = -g(X,Y)\xi - \eta(Y)X + 2\eta(X)\eta(Y)\xi, \tag{2.10}$$

then M is called Para-Sasakian manifold or briefly, P-Sasakian manifold.
Especially a P-Sasakian manifold is called a special para-Sasakian manifold or briefly, a SP-Sasakian manifold if

$$(\nabla_X \eta)Y = -g(X,Y) + \eta(X)\eta(Y). \tag{2.11}$$

Also, a P-Sasakian manifold M is said to be $\eta-$Einstein manifold if its Ricci tensor is of the form

$$S(X,Y)=ag(X,Y)+b\eta(X)\eta(Y), \tag{2.12}$$

For any vector fields X, Y, where a and b are function on M.

If $b=0$, then $\eta-$Einstein manifold to becomes an Einstein manifold.

Further, on such an n-dimensional P-Sasakian manifold the following relations hold [1,4, 6, 9]

$$g(R(X,Y)Z,\xi)=\eta(R(X,Y)Z)=g(X,Z)\eta(Y)-g(Y,Z)\eta(X), \tag{2.13}$$

$$Q\xi=-(n-1)\xi, \tag{2.14}$$

$$R(X,Y)\xi=\eta(X)Y-\eta(Y)X, \tag{2.15}$$

$$R(\xi,X)Y=\eta(Y)X-g(X,Y)\xi, \tag{2.16}$$

$$R(\xi,X)\xi=X-\eta(X)\xi, \tag{2.17}$$

$$S(X,\xi)=-(n-1)\eta(X), \tag{2.18}$$

$$S(\varphi X,\varphi Y)=S(X,Y)+(n-1)\eta(X)\eta(Y), \tag{2.19}$$

for any vector fields X, Y, Z, where R(X,Y)Z is the curvature tensor and S is the Ricci tensor.

Definition 2.1. The M-projective curvature tensor $\overline{P}$ is defined as

$$\overline{P}(X,Y)Z=R(X,Y)Z-\frac{1}{2(n-1)}[S(Y,Z)X-S(X,Z)Y$$
$$+g(Y,Z)QX-g(X,Z)QY], \tag{2.20}$$

for all vector fields X, Y, Z on M [3]. Where Q is the Ricci operator defined by $S(X,Y)=g(QX,Y)$. The manifold is said to be m-projectively flat if $\overline{P}$ vanishes identically on M.

Definition 2.2. The concircular curvature tensor $\tilde{C}$ on P-Sasakian manifold M of dimensional n is defined by

$$\tilde{C}(X,Y)Z=R(X,Y)Z-\frac{r}{n(n-1)}[g(Y,Z)X-g(X,Z)Y] \tag{2.21}$$

for all vector fields X, Y , Z on M.

Definition 2.3. An n-dimensional, (n > 3), P-Sasakian manifold satisfying the condition

$$\varphi^2\overline{P}(\varphi X,\varphi Y)\varphi Z=0 \tag{2.22}$$

is called φ-m-projectively flat manifold.

3. Main Results

In this section, we prove the following theorems:

Theorem 3.1. An n-dimensional m-projectively flat P-Sasakian manifold is locally isometric to the Hyperbolic $H^n(-1)$.

Proof. If $\overline{P}=0$ then we get from (2.20) that

$$R(X,Y)Z=\frac{1}{2(n-1)}[S(Y,Z)X-S(X,Z)Y+g(Y,Z)QX-g(X,Z)QY]. \tag{3.1}$$

Putting $Z=\xi$ in (2.21) and using (2.7), (2.15) and (2.18) we obtain

$$\eta(X)Y-\eta(Y)X=\frac{1}{2(n-1)}[-(n-1)\eta(Y)X+(n-1)\eta(X)Y+\eta(Y)QX-\eta(X)QY]. \tag{3.2}$$

Taking $Y=\xi$ in (3.2) and using (2.6) we have

$$\eta(X)\xi-X=\frac{1}{2(n-1)}[-(n-1)X+(n-1)\eta(X)\xi+QX+(n-1)\eta(X)\xi].$$

Therefore with simplify of the above equation we get

$$QX=-(n-1)X. \tag{3.3}$$

Now, putting (3.3) in (3.1) we obtain

$$R(X,Y)Z=\frac{1}{2(n-1)}[S(Y,Z)X-S(X,Z)Y-(n-1)g(Y,Z)X+(n-1)g(X,Z)Y],$$

putting $X=\xi$ and using (2.16) and (2.18) we get

$$\eta(Z)Y-g(Y,Z)\xi=\frac{1}{2(n-1)}[S(Y,Z)\xi+(n-1)\eta(Z)Y-(n-1)g(Y,Z)\xi+(n-1)\eta(Z)Y],$$

with simplify of the above equation we obtain

$$S(Y,Z)=-(n-1)g(Y,Z). \tag{3.4}$$

thus the manifold is an Einstein manifold.
Now, putting (3.3) and (3.4) in (3.1) we have

$$R(X,Y)Z=\frac{1}{2(n-1)}[-(n-1)g(Y,Z)X+(n-1)g(X,Z)Y.-(n-1)g(Y,Z)X+(n-1)g(X,Z)Y].$$

Finally we get

$$R(X,Y)Z=-[g(Y,Z)X-g(X,Z)Y].$$

The above equation implies that M is of constant curvature -1 and consequently it is locally isometric with the Hyperbolic $H^n(-1)$.

This the completes the proof of the theorem. □

Now, we construct an example of m-projectively flat P-Sasakian manifold which support Theorem (3.1).

Example 1. We consider 3-dimensional manifold $M = \{(x,y,z) : (x,y,z) \in R^3\}$, where (x,y,z) are standard coordinates in R^3. We choose the vector fields

$$E_1 = e^z(-\frac{\partial}{\partial x} - \frac{\partial}{\partial y}) \ , \ \ E_2 = -e^z \frac{\partial}{\partial y} \quad , \quad E_3 = -\frac{\partial}{\partial z} \ ,$$

which are linearly independent at each point of M. Let g be the Riemannian metric defined by

$$g(E_1, e_2) = g(E_2, E_3) = g(E_3, E_1) = 0$$
$$g(E_1, E_1) = g(E_2, E_2) = g(E_3, E_3) = 1.$$

Let η be a 1-form defined by $\eta(Z) = g(Z, E_3)$ for any vector field Z on M.
We define the (1,1) tensor field φ as

$$\varphi(E_1) = E_1 \ , \ \ \varphi(E_2) = E_2 \quad , \quad \varphi(E_3) = 0.$$

The linearity property of φ and g yields that

$$\eta(e_3) = 1 \ , \ \varphi^2 U = -U + \eta(U)e_3$$
$$g(\varphi U, \varphi W) = g(U, W) - \eta(U)\eta(W),$$

for any vector fields U, W, Z on M.
Thus for $E_3 = \xi$, (φ, η, ξ, g) defines an almost para contact structure on M.
Let ∇ be the Levi-Civita connection with respect to g, then for any $f \in C^\infty(R^3)$ we have

$$\begin{aligned}[E_1, E_2] &= E_1(E_2 f) - E_2(E_1 f) \\ &= e^z(-\frac{\partial}{\partial x} - \frac{\partial}{\partial y})(-e^z \frac{\partial f}{\partial y}) - (-e^z \frac{\partial}{\partial y})(-e^z \frac{\partial f}{\partial x} - e^z \frac{\partial f}{\partial y}) \\ &= 0.\end{aligned}$$

Similarly we obtain $[E_1, E_3] = E_1$, $[E_2, E_3] = E_2$.
Using the Koszuls formula

$$\begin{aligned}2g(\nabla_X Y, Z) &= \nabla_X g(Y, Z) + \nabla_Y g(Z, X) - \nabla_Z g(X, Y) \\ &\quad + g([X, Y], Z) - g([Y, Z], X) + g([Z, X], Y),\end{aligned}$$

we have

$$\begin{aligned}2g(\nabla_{E_1} E_1, E_3) &= -g(E_1, E_1) + g(-E_1, E_1) \\ &= -2g(E_1, E_1),\end{aligned}$$

therefore $\nabla_{E_1} E_1 = -E_3$. Similarly, it follows that

$$\nabla_{E_1} E_2 = 0 \ , \ \nabla_{E_1} E_3 = E_1 \ , \ \nabla_{E_2} E_1 = 0 \, , \ \nabla_{E_2} E_2 = -E_3$$
$$\nabla_{E_2} E_3 = E_2 \ , \ \nabla_{E_3} E_1 = 0 \, , \ \nabla_{E_3} E_2 = 0 \ , \ \nabla_{E_3} E_3 = 0.$$

From the above, it can be easily seen that (φ,η,ξ,g) is a P-Sasakian structure on M. Hence M is a 3-dimensional P-Sasakian manifold.
Now, using the formula

$$R(X,Y)Z=\nabla_X\nabla_Y Z-\nabla_Y\nabla_X Z-\nabla_{[X,Y]}Z,$$

we can easily calculate the non-vanishing components of the curvature tensor as follows

$$R(E_1,E_2)E_2=\nabla_{E_1}\nabla_{E_2}E_2-\nabla_{E_2}\nabla_{E_1}E_2-\nabla_{[E_1,E_2]}E_2$$
$$=-E_1$$

Similarly,

$$R(E_1,E_3)E_3=-E_1\ ,\quad R(E_2,E_1)E_1=E_2\ ,\quad R(E_2,E_3)E_3=-E_2$$
$$R(E_3,E_1)E_1=-E_3,\quad R(E_3,E_2)E_2=-E_3\ ,\ R(E_3,E_2)E_3=-E_2.$$

The above relations implies that M is of constant curvature -1.
The definition of Ricci tensor in 3-dimensional manifold implies that

$$S(X,Y)=\sum_{i=1}^{3}\ g(R(E_i,X)Y,E_i).$$

From the above relation we can calculate the non-vanishing components of Ricci tensor S as follows

$$\begin{aligned}S(E_1,E_1)&=\sum_{i=1}^{3}\ g(R(E_i,E_1)E_1,E_i)\\&=g(R(E_1,E_1)E_1,E_1)+g(R(E_2,E_1)E_1,E_2)\\&\quad+g(R(E3,E_1)E_1,E_3)\\&=-2,\end{aligned}$$

therefore $S(E_1,E_1)=-2.$ Similarly we get

$$S(E_2,E_2)=-2\ ,\ S(E_3,E_3)=-2$$

We know that the scaler curvature of the 3-dimensional manifold is given by

$$r=\sum_{i=1}^{3}\ S(E_i,E_i).$$

In view of above relations, it follows that for all vector fields $X,Y\in\chi(M)$ the scaler curvature of the manifold is equal to -6 and the Ricci tensor

$$S(X,Y)=-2g(X,Y).$$

Also, $QX=-2X$. Now, in view of (2.20) we have

$$\begin{aligned}\overline{P}(E_1,E_3)E_3&=R(E_1,E_3)E_3-\frac{1}{2}[S(E_3,E_3)E_1-S(E_1,E_3)E_3\\&\quad+g(E_3,E_3)QE_1-g(E_1,E_3)QE_3]\\&=0.\end{aligned}$$

Similarly, for all $i, j, k = 1,2,3$ we obtain

$$\overline{P}(E_i, E_j)E_k = 0.$$

Therefore M is a 3-dimensional m-projectively flat P-Sasakian manifold.
Also, M is an 3-dimensional Einstein manifold whit the constant curvature -1. □

Theorem 3.2. Let M be an n-dimensional, (n > 3), φ-m-projectively flat P-Sasakian manifold. Then M is an η-Einstein manifold.

Proof. If M is φ-m-projectively flat P-Sasakian manifold then we get from (2.22) that

$$\varphi^2 \overline{P}(\varphi X, \varphi Y)\varphi Z = 0$$

this implies that

$$g(\varphi^2 \overline{P}(\varphi X, \varphi Y)\varphi Z, \varphi W) = 0$$

for any vector fields X, Y , Z and W on M . Using (2.20) we obtain

$$\begin{aligned} g(R(\varphi X, \varphi Y)\varphi Z, \varphi W) = \frac{1}{2(n-1)}[&S(\varphi Y, \varphi Z) g(\varphi X, \varphi W) \\ &- S(\varphi X, \varphi Z) g(\varphi Y, \varphi W) \\ &+ g(\varphi Y, \varphi Z) S(\varphi X, \varphi W) \\ &- g(\varphi X, \varphi Z) S(\varphi Y, \varphi W)]. \end{aligned}$$

Let $\{e_1, e_2, ..., e_n, \xi\}$ be a local orthonormal basis of vector fields in M.
Using that $\{\varphi e_1, \varphi e_2, ..., \varphi e_n, \xi\}$ is also a local orthonormal basis, if we put $X = W = e_i$ in above equation and sum up with respect to i, then

$$\begin{aligned} \sum_{i=1}^{n-1} g(R(\varphi e_i, \varphi Y)\varphi Z, \varphi e_i) = \frac{1}{2(n-1)} \sum_{i=1}^{n-1}[&S(\varphi Y, \varphi Z) g(\varphi e_i, \varphi e_i) \\ &- S(\varphi e_i, \varphi Z) g(\varphi Y, \varphi e_i) \\ &+ g(\varphi Y, \varphi Z) S(\varphi e_i, \varphi e_i) \\ &- g(\varphi e_i, \varphi Z) S(\varphi Y, \varphi e_i)]. \end{aligned} \tag{3.5}$$

It can be easily verify that

$$\sum_{i=1}^{n-1} g(R(\varphi e_i, \varphi Y)\varphi Z, \varphi e_i) = S(\varphi Y, \varphi Z) + g(\varphi Y, \varphi Z), \tag{3.6}$$

$$\sum_{i=1}^{n-1} S(\varphi e_i, \varphi e_i) = r - (n-1), \tag{3.7}$$

$$\sum_{i=1}^{n-1} g(\varphi Y, \varphi e_i) S(\varphi e_i, \varphi Z) = S(\varphi Y, \varphi Z), \tag{3.8}$$

$$\sum_{i=1}^{n-1} g(\varphi e_i, \varphi e_i) = n-1. \tag{3.9}$$

So by virtue of (3.6)-(3.9) the equation (3.5) can be written as

$$S(\varphi Y,\varphi Z)+g(\varphi Y,\varphi Z)=\frac{1}{2(n-1)}[(n-1)S(\varphi Y,\varphi Z)-S(\varphi Y,\varphi Z)$$
$$+(r-(n-1))S(\varphi Y,\varphi Z)-S(\varphi Y,\varphi Z)],$$

this implies that

$$S(\varphi Y,\varphi Z)=\frac{r-3(n-1)}{n+1}g(\varphi Y,\varphi Z),$$

in view of (2.6) and (2.19) we get

$$S(Y,Z)+(n-1)\eta(Y)\eta(Z)=\frac{r-3(n-1)}{n+1}[g(Y,Z)-\eta(Y)\eta(Z)].$$

Finally we obtain

$$S(Y,Z)=\frac{r-3(n-1)}{n+1}g(Y,Z)-[\frac{r-3(n-1)}{n+1}+(n+1)]\eta(Y)\eta(Z).$$

Therefore M is an η-Einstein manifold. □

Theorem 3.3. Let M be an n-dimensional P-Sasakian manifold. Then M satisfies in condition

$$\tilde{C}(\xi,U).\overline{P}=0$$

if and only if either M has scaler curvature $r=n(1-n)$ or M is locally isometric with the Hyperbolic $H^n(-1)$.

Proof. Since $\tilde{C}(\xi,U).\overline{P}=0$ we have

$$\tilde{C}(\xi,U).\overline{P}(X,Y)Z=0,$$

this implies that

$$[\tilde{C}(\xi,U),\overline{P}(X,Y)]Z-\overline{P}(\tilde{C}(\xi,U)X,Y)Z-\overline{P}(X,\tilde{C}(\xi,U)Y)Z=0,$$

in view of (2.21) we get

$$0=(-1-\frac{r}{n(n-1)})[-\eta(\overline{P}(X,Y)Z)U+\overline{P}(X,Y,Z,U)\xi$$
$$+\eta(X)\overline{P}(U,Y)Z-g(U,X)\overline{P}(\xi,Y)Z$$
$$+\eta(Y)\overline{P}(X,U)Z-g(U,Y)\overline{P}(X,\xi)Z$$
$$+\eta(Z)\overline{P}(X,Y)U-g(U,Z)\overline{P}(X,Y)\xi].$$

Therefore M has scalar curvature $r=n(1-n)$ or

$$0=-\eta(\overline{P}(X,Y)Z)U+\overline{P}(X,Y,Z,U)\xi+\eta(X)\overline{P}(U,Y)Z$$
$$-g(U,X)\overline{P}(\xi,Y)Z+\eta(Y)\overline{P}(X,U)Z-g(U,Y)\overline{P}(X,\xi)Z$$

$$+\eta(Z)\overline{P}(X,Y)U - g(U,Z)\overline{P}(X,Y)\xi.$$

Taking the inner product of the last equation with ξ we get

$$\begin{aligned}0 = &-\eta(\overline{P}(X,Y)Z)\eta(U) + \overline{P}(X,Y,Z,U)\\ &+\eta(X)\eta(\overline{P}(U,Y)Z) - g(U,X)\eta(\overline{P}(\xi,Y)Z)\\ &+\eta(Y)\eta(\overline{P}(X,U)Z) - g(U,Y)\eta(\overline{P}(X,\xi)Z)\\ &+\eta(Z)\eta(\overline{P}(X,Y)U) - g(U,Z)\eta(\overline{P}(X,Y)\xi).\end{aligned}$$

Finally, with simplify we get

$$\overline{P}(X,Y,Z,U) = 0,$$

which implies that M is m-projectively flat. Thus in view of Theorem(3.1), M is locally isometric with the Hyperbolic $H^n(-1)$. The converseis trivial. This the completes the proof of the theorem. □

Theorem 3.4. If an n-dimensional P-Sasakian manifold M satisfies

$$R(X,Y).\overline{P} = 0$$

then M is locally isometric with the Hyperbolic $H^n(-1)$.

Proof. If $R(X,Y).\overline{P} = 0$ then we have

$$R(X,Y).\overline{P}(U,V)W = 0,$$

for all vector fields X, Y , U, V and W on M , this implies that

$$\begin{aligned}0 = &R(X,Y)\overline{P}(U,V)W - \overline{P}(R(X,Y)U,V)W\\ &-\overline{P}(U,R(X,Y)V)W - \overline{P}(U,V)R(X,Y)W.\end{aligned}$$

Putting $X = \xi$ and taking the inner product of the last equation with ξ, we obtain

$$\begin{aligned}0 = &g[R(\xi,Y)\overline{P}(U,V)W,\xi] - g[\overline{P}(R(\xi,Y)U,V)W,\xi]\\ &-g[\overline{P}(U,R(\xi,Y)V)W,\xi] - g[\overline{P}(U,V)R(\xi,Y)W,\xi].\end{aligned}$$

In view of (2.7) and (2.16) we have

$$\begin{aligned}0 = &\overline{P}(U,V,W,Y) + \eta(Y)\eta(\overline{P}(U,V)W)\\ &-g(Y,U)\eta(\overline{P}(\xi,V)W) - \eta(U)\eta(\overline{P}(Y,V)W)\\ &+g(Y,V)\eta(\overline{P}(U,\xi)W) - \eta(V)\eta(\overline{P}(U,Y)W)\\ &+g(Y,W)\eta(\overline{P}(U,V)\xi) - \eta(W)\eta(\overline{P}(U,V)Y).\end{aligned}$$

With simplify of the above equation we obtain

$$\overline{P}(U,V,W,Y) = 0$$

Therefore M is m-projectively flat. In view of Theorem (3.1) manifold is locally isometric with the Hyperbolic $H^n(-1)$. □

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