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Reliability Analysis for Accelerated Life-Test with Progressive Hybrid Censored Data Using Geometric Process

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ABSTRACT

This paper considers the Geometric Process implementation of the constant stress accelerated life test model based on the progressive Type-I hybrid censored data. By assuming that the life variables under increasing stress levels form a Geometric Process, the likelihood functions are derived and then reduced to a single nonlinear equation to be solved numerically to obtain the maximum likelihood estimates (MLEs) of the parameters. Two bootstrap confidence intervals are proposed. The point and interval estimations for the component's reliability are also obtained. Finally, the Monte-Carlo simulation study is carried out to illustrate the proposed procedures.

Keywords: Geometric Process; Accelerated life test; Progressive Type-I hybrid censoring; Maximum likelihood estimations; Confidence intervals

AMS Mathematics Subject Classification (2010):

1. Introduction

Due to the rapid development of the product design and manufacture in life cycle, accelerated life testing (ALT) is adopted and widely used in manufacturing industries. Briefly, ALT is a method for making inference of the life character of devices at normal use conditions from failure data obtained at severe conditions regarding its relationship with the external stress variables. Three types of stress loadings are usually applied in ALT: constant stress, step stress and linearly increasing stress. The constant stress loading, which is a time-independent test setting, has several advantages over the time-dependent stress ones. It has been studied by several authors, see [1–3]. Commonly, all available test data obtained from ALT are used in statistical analysis. However, in life testing experiments, often the data are censored. The two most common censoring schemes are termed as Type-I and Type-II censoring schemes. A mixture of Type-I and Type-II censoring schemes is known as the hybrid censoring scheme. The three conventional censoring schemes all have the drawbacks that they do not allow for removal of units at points other than the terminal of the experiment. The progressively hybrid censoring scheme (PHCS), which has this advantage, has become very popular in the reliability and life-testing experiments in the last few years. For the details of the PHCS, one may refer to Ref. [4–8].All the works have focused on the parameter inference problem under PHCS for different distributions. There is little work on the application of the PHCS to the ALTs. Li *et al*. [9] first discussed a simple step-stress accelerated test model for progressive Type-I hybrid censored exponential life data.

The Geometric Process (GP) model is first applied to investigate a repair replacement model for a one-unit deteriorating system. Large numbers of studies in maintenance problems and system reliability have shown that the GP model is a good and simple model for analysis of data with a single trend or multiple trends, for example, the work of [10–12].So far, there is no study that utilizes the GP in the analysis of ALT with censored data, except for Shan's thesis [13], where the author introduced the GP model for the analysis of ALT with complete, Type-I and Type-II censored exponential samples under the constant stress.

In this paper, we consider the GP process implementation of the constant stress ALT model subject to progressive Type-I hybrid censoring scheme. The maximum likelihood and bootstrap confidence interval estimates of the model parameters and the item's reliability are considered and their results are compared in Monte-Carlo simulation.

2. Model description and notations

2.1. The Geometric Process and PHCS

Geometric Process was introduced by Lam [14, 15] when he studied the problem of repair replacement. It is defined below.

Definition 2.1. (Geometric Process). A counting process (CP) is a common method to model the total number of events that have occurred in the interval. The CP is said to be a Geometric Process (GP) with parameter a if there exists a real number $a > 0$ such that $Y_i = a^{i-1}X_i, i = 1,2,...,$ are independently and identically distributed (iid) random variables with distribution function *F*. Here, the parameter a is the ratio of the GP and the random variables $X_i (i = 1,2,...)$are the time intervals between the $(i-1)_{th}$ and i_{th} event of a CP$\{N(t), t \geq 0\}$ for i=1, 2…

Kundu[5] first proposed the progressively hybrid censoring scheme. Suppose *n* independent identical items are placed on test, at the time of the i_{th} failure, $x^{i:m:n}$, R_i of the remaining units are randomly removed. In the presence of progressively Type-I hybrid censoring schemes, we have one of the following types of observations:

1. Case I: $\{x^{1:m:n}, x^{2:m:n}, \cdots, x^{m:m:n}\}$ if $x^{m:m:n} < T_0$;
2. Case II: $\{x^{1:m:n}, x^{2:m:n}, \cdots, x^{J:m:n}\}$ if $x^{J:m:n} < T_0 < x^{J+1:m:n}$,

where *m* , T_0 ,$R_i$are fixed before and$R_1 + R_2 + \cdots + R_m + m = n$.

Let the ending time of the test be $T^* = \min\{X^{m:m:n}, T_0\}$ and the failure number before T^*(includingT^*) beD^*.Therefore,

$$D^* = \begin{cases} m & T^* = X^{m:m:n} \quad \text{caseI} \\ J & T^* = T_0 \quad \text{caseII} \end{cases}. \tag{1}$$

The likelihood function is

$$L \propto \prod_{i=1}^{D^*} f(x^{i:m:n})[1 - F(x^{i:m:n})]^{R_i}[1 - F(T^*)]^{R_T}, \tag{2}$$

where

$$R_T = n - D^* - \sum_{i=1}^{D^*} R_i.$$

2.2. The Accelerated Life Tests and Geometric Process

The geometric model for accelerated life test is based on the following assumptions:

Assumption 1. *s* stress levels $s_1, s_2, \cdots, s_s$ are used.

Assumption 2. For any level of stress, the life of test unit follows rayleigh distribution with failure rate λ_k(Ray (λ_k)). The probability density function (pdf) is given by

$$f_{x_k}(x) = \begin{cases} \lambda_k^2 x e^{-\frac{1}{2}\lambda_k^2 x^2} & x > 0 \\ 0 & otherwise \end{cases}, \tag{3}$$

Assumption 3. The mean life is assumed to be a log-linear function of the stress level *s*.

$$\ln\left(\sqrt{\frac{\pi}{2}}\frac{1}{\lambda_k}\right) = \alpha + \beta\phi(s_k). \tag{4}$$

where $\alpha, \beta(\beta < 0)$are unknown parameters depending on the nature of the product and the test method, $\phi(s)$ is a function of stress *s*. When $\phi(s)$ is an increasing function of *s*, the mean life at stress levels $s_1, s_2, \cdots, s_k$satisfies$\frac{1}{\lambda_1} > \frac{1}{\lambda_2} > \cdots > \frac{1}{\lambda_k}$.

Assumption 4. Let the sequence of random variables $X_0, X_1, \cdots X_s$denote the lifetimes under each stress level, where X_0 denotes the items' lifetime under the design stress s_0 at which items will operate ordinarily. We assume $\{X_{0,} X_1, X_2, \cdots, X_s\}$be a geometric process with ratio *a*>0.

Assumption1-3are most commonly used in ALT [2, 9].Assumption 4 may be stronger than the usual discussion of the ALT in literatures, but in this way we can take advantage of the PHCS and ALT without increasing the complexity of calculation. The next theorem discusses how the assumption of geometric process (Assumption 4) is satisfied when there is log-linear relationship between a life characteristic and the stress level (Assumption 3).

Theorem 2.1. If $\phi(s_{k+1})-\phi(s_k)$ is a constant for $k=0,1,2,\cdots,s-1$, then $\{X_0,X_1,X_2,\cdots,X_s\}$ forms a geometric process.

Proof. From (4) we can get:

$$\ln\frac{\lambda_k}{\lambda_{k+1}}=\beta(\phi(s_{k+1})-\phi(s_k)).$$

Therefore,

$$\frac{\lambda_{k+1}}{\lambda_k}=\mathrm{e}^{-\beta(\phi(s_{k+1})-\phi(s_k))}.$$

Because $\Delta\phi(s)=\phi(s_{k+1})-\phi(s_k)$ is a constant, note the right part of the above equation by *a*, thus,

$$\lambda_k=a\lambda_{k-1}=a^2\lambda_{k-2}=\cdots=a^k\lambda.$$

Let $Y_i=a^iX_i, i=0,1,2,\cdots,s$, then,

$$\begin{aligned}f_{y_i}(y)&=\frac{1}{a^i}f_{X_i}(y/a^i)\\&=\frac{1}{a^i}(a^i\lambda)^2\mathrm{e}^{-\frac{1}{2}(a^i\lambda)^2\frac{y}{a^{i2}}}\\&=\lambda^2y\mathrm{e}^{-\frac{1}{2}\lambda^2y^2}.\end{aligned}$$

We can see $Y_i\overset{\text{i.i.d.}}{\sim}\mathrm{Ray}(\lambda)$. From Definition 2.1, $\{X_0,X_1,X_2,\cdots,X_s\}$ forms a geometric process with ration *a*. □

From the properties of the GP, if the density function of X_0 is *f*, then the probability density function of X_k will be given by $a^kf(a^kx), k=0,1,2\cdots,s$. Therefore, if lifetime under a sequence of increasing stress levels form a geometric process with ratio *a*, and the items' life at the design stress level follows $\mathrm{Ray}(\lambda)$, then the life distribution at the k_{th} stress level is $\mathrm{Ray}(a^k\lambda)$.

3. The Maximum Likelihood Estimators

From (2) and (3) we can obtain the likelihood function of the observations tested under stress s_k:

$$L_k(a,\lambda)\propto a^{2kD_k^*}\lambda^{2D_k^*}\exp\{-\frac{1}{2}\mathrm{a}^{2\mathrm{k}}\lambda^2\left(\sum_{\mathrm{i}=1}^{\mathrm{D}_\mathrm{k}^*}(1+\mathrm{R_i})\mathrm{x_k^{i^2}}+\mathrm{T_k^{*^2}}\mathrm{R_{T,k}}\right)\},\quad(5)$$

where x_k^i denotes the i_{th} failure at the k_{th} stress level, $D_k^*$is the fail numbers under this stress level and $T_k^*=\min\{x_k^i,T_0\}$,$R_{T,k}=n-D_k^*-\sum_{i=1}^{D_k^*}R_i$.

The total likelihood function is

$$L(a,\lambda)\propto\prod_{k=1}^{s}a^{2kD_k^*}\lambda^{2D_k^*}\exp\{-\frac{1}{2}\mathrm{a}^{2\mathrm{k}}\lambda^2\left(\sum_{\mathrm{i}=1}^{\mathrm{D}_\mathrm{k}^*}(1+\mathrm{R_i})\mathrm{x_k^{i^2}}+\mathrm{T_k^{*^2}}\mathrm{R_{T,k}}\right)\}.\quad(6)$$

The log-likelihood function is (we still use the notation $L(a,\lambda)$)

$$L(a,\lambda)\propto\sum_{k=1}^{s}\left(2kD_k^*\ln a+2D_k^*\ln\lambda-\frac{1}{2}\mathrm{a}^{2\mathrm{k}}\lambda^2\left(\sum_{\mathrm{i}=1}^{\mathrm{D}_\mathrm{k}^*}(1+\mathrm{R_i})\mathrm{x_k^{i^2}}+\mathrm{T_k^{*^2}}\mathrm{R_{T,k}}\right)\right).\quad(7)$$

The first derivatives of $L(a, \lambda)$ with respect to *a* and λ are:

$$\frac{\partial L}{\partial a} = \sum_{k=1}^{s} \left(\frac{2kD_k^*}{a} - ka^{2k-1}\lambda^2 g_k \right) \tag{8}$$

$$\frac{\partial L}{\partial \lambda} = \sum_{k=1}^{s} \left(\frac{2D_k^*}{\lambda} - a^{2k}\lambda g_k \right), \tag{9}$$

where $g_k = \sum_{i=1}^{D_k^*}(1+R_i)x_k^{i^2} + T_k^{*^2} R_{T,k}$.

From $\frac{\partial L}{\partial \lambda} = 0$, the estimate of λ can be obtained as a function of *a*,

$$\hat{\lambda}(a) = \frac{2\sum_{k=1}^{s} D_k^*}{\sum_{k=1}^{s} a^{2k} g_k}, \tag{10}$$

Therefore, the estimate of *a* can be obtained easily by solving

$$\sum_{k=1}^{s} \left(-\frac{2}{a^2} kD_k^* - \hat{\lambda}^2(a)k(2k-1)a^{2k-2}g_k \right) = 0. \tag{11}$$

The Newton-Raphson method or the bisection method can be evoked to solve the above nonlinear equation of *a*. Once $\hat{a}$ is obtained, $\hat{\lambda}$ is obtained as $\hat{\lambda} = \hat{\lambda}(\hat{a})$. Based on the invariance of MLE, the MLE of the reliability of the component at time *t* can be easily obtained by $R(t) = \mathrm{e}^{-\frac{1}{2}\lambda^2 t^2}$.

4. Bootstrap confidence intervals

We construct the confidence intervals for a and λ based on the parametric bootstrap, using the percentile bootstrap (Boot-p) interval method [16] and the bootstrap-t (Boot-t) method [17].

Boot-p method:

1. Based on the progressive Type-I hybrid censored sample, obtain $\hat{a}$ and $\hat{\lambda}$, the MLEs of a and λ, by the method proposed in section 3.

2. For $b = 1,2,\cdots,B$, based on $\hat{a}$ and $\hat{\lambda}$, generate the bootstrap sample $X_k^i, k = 1,2,\cdots,s, i = 1,2,\cdots,D_k^*$, where $X_k^i \sim \mathrm{Ray}(\hat{a}^k \lambda)$.

3. Obtain $\hat{a}_b$ and $\hat{\lambda}_b$, the MLEs of a and λ, using the proposed method.

4. Repeat steps 2-3 for *B* times and obtain $\hat{a}_b$ and $\hat{\lambda}_b$, $b = 1,2,\cdots,B$.

5. The bootstrap percentile confidence interval endpoints for $\hat{a}$ and $\hat{\lambda}$ are the $\alpha/2$ and $1-\alpha/2$ quantiles of $\hat{a}_1, \hat{a}_2, \cdots, \hat{a}_B$ and $\hat{\lambda}_1, \hat{\lambda}_2, \cdots, \hat{\lambda}_B$, respectively.

Also, according to the invariance of MLE, the confidence interval endpoints of the unit's reliability are the $\alpha/2$ and $1-\alpha/2$ quantiles of

$$\mathrm{e}^{\frac{1}{2}(\hat{\lambda}_1)^2 t^2}, \mathrm{e}^{\frac{1}{2}(\hat{\lambda}_2)^2 t^2}, \cdots, \mathrm{e}^{\frac{1}{2}(\hat{\lambda}_B)^2 t^2}.$$

Boot-t method:

1. Estimate a and λ, say $\hat{a}$, $\hat{\lambda}$, by the method proposed in section 3 as before.

2. Based on $\hat{a}$ and $\hat{\lambda}$, generate the bootstrap sample

$$X_k^i, k=1,2,\cdots,s, i=1,2,\cdots,D_k^*, \text{ where } X_k^i \sim \text{Ray}(\hat{a}^k\lambda).$$

3. Obtain $\hat{a}_b$ and $\hat{\lambda}_b$, the MLEs of a and λ, using the proposed method, and compute $\hat{V}(\hat{a}_b)=\text{Var}(\hat{a}_b)$, and $\hat{V}(\hat{\lambda}_b)=\text{Var}(\hat{\lambda}_b)$, where $\text{Var}(\hat{a}_b)$ and $\text{Var}(\hat{\lambda}_b)$ can be calculated through the observed fisher information matrix, see the work of [5].

4. Determine the T_b statistic for a and λ respectively,

$$T_b^a=\frac{(\hat{a}_b-\hat{a})}{\sqrt{\hat{V}(\hat{a}_b)}},\quad T_b^\lambda=\frac{(\hat{\lambda}_b-\hat{\lambda})}{\sqrt{\hat{V}(\hat{\lambda}_b)}}.$$

5. Repeat steps 2-4 for *B* times and obtain $\hat{a}_b$ and $\hat{\lambda}_b$, $b=1,2,\cdots,B$.

6. Note the α quantiles of $T_1^a,T_2^a\cdots,T_B^a$ and $T_1^\lambda,T_2^\lambda\cdots,T_B^\lambda$ by $T_a^{-1}(\alpha)$ and $T_\lambda^{-1}(\alpha)$, respectively. Then the boot-t confidence interval endpoints for $\hat{a}$ and $\hat{\lambda}$ are give by

$$[\hat{a}+\sqrt{\hat{V}(\hat{a})}T_a^{-1}(\frac{\alpha}{2}),\hat{a}+\sqrt{\hat{V}(\hat{a})}T_a^{-1}(1-\frac{\alpha}{2})]$$

and

$$[\hat{\lambda}+\sqrt{\hat{V}(\hat{\lambda})}T_\lambda^{-1}(\frac{\alpha}{2}),\hat{\lambda}+\sqrt{\hat{V}(\hat{\lambda})}T_\lambda^{-1}(1-\frac{\alpha}{2})]$$

respectively.

Similarly, the boot-t confidence intervals of the reliability of the item can be obtained by

$$[\exp\{-\frac{1}{2}(\hat{\lambda}+\sqrt{\hat{V}(\hat{\lambda})}T_\lambda^{-1}(1-\frac{\alpha}{2}))^2t^2\},\exp\{-\frac{1}{2}(\hat{\lambda}+\sqrt{\hat{V}(\hat{\lambda})}T_\lambda^{-1}(\frac{\alpha}{2}))^2t^2\}].$$

5. Numerical study and conclusion

In this section, we use Monte-Carlo simulations to compare different methods with different parameter values. The Type-I progressively hybrid censored data for a given set *n*,*m*, $R=(R_1,R_2,\cdots,R_m)$ and T_0 are generated using the algorithm described in [5] and [18]. For simplify, without loss of generality, let $\phi(s)=s$ in the accelerated model (4).Thus, the ratio of the GP is $a=e^{-\beta\Delta s}$. We consider different n,m,T_0 and $\beta,\Delta s$ values .We use the sampling scheme:

$$R_1=\cdots=R_{m-1}=1, R_m=n-2m+1.$$

The absolute relative bias (RABias) and mean square error (MSE) have been considered, where

$$\text{RABias}(\hat{\theta})=|\frac{\hat{\theta}-\theta}{\theta}|,\text{MSE}(\hat{\theta})=\text{E}(\hat{\theta}-\theta)^2.$$

Simulation results are presented to compare the performance of the different parameters $\beta, \Delta s$ in the model for different sampling schemesn, m, T_0. We replicate the process 1000 times in each case and report the average estimators, the RABias, MSE and the lower and upper bounds of the 95% confidence intervals. The reliability of the item at time *t*=0.7 is considered. The results are presented in Table 1-6. Figure 2-3 display the reliability of the unit.

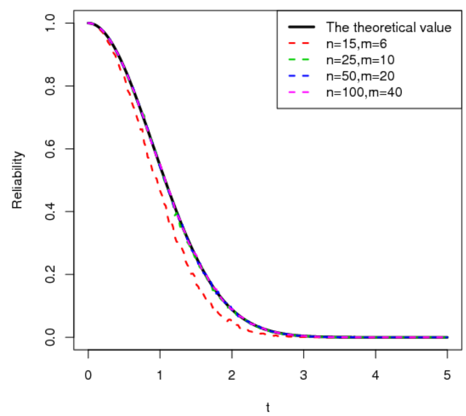

Figure 2: The component reliability with $\lambda = 1.1, \beta = -0.5, \Delta s = 0.52$ and $s = 4, T_0 = 0.5$

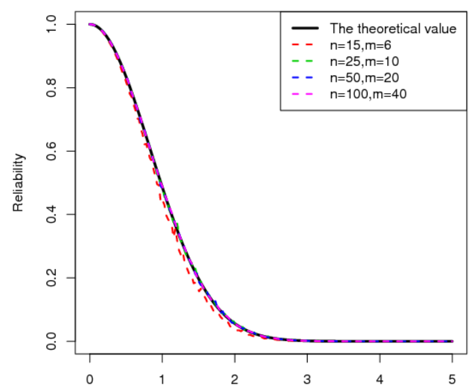

Figure 3: The component reliability with $\lambda = 1.2, \beta = -0.5, \Delta s = 0.52$ and $s = 6, T_0 = 0.5$

From the results of the simulation study, we observe the following.

1. For fixed fail ratio m/n as *n* increases, the RAbiases and the MSEs decrease for all cases, the reliability curve is more close to the theatrical one, as expected.

2. For the fixed $\beta, \Delta s$, the RABias and MSE of the estimators from samples tested under the six stages (*s*=6) ALT have good statistical properties than the four stages (*s*=4) one, which is quite intuitive.

3. For fixed values of the rationm/n, by increasing the tested numbers *n*, the lengths of both the two bootstrap CIs become less.

4. For small sample sizes, the boot-t CIs are not better than the boot-p CIs in the sense of having longer widths, which is a little counterintuitive.

In total, it can be obverted from the simulation results that the accuracy of estimations is closely related to the values of the parameters of the model and the choice of the progressive censored schemes, however, the method does improve for large sample size.

Table1: The MLE estimators with $\lambda = 1.1, \beta = -0.5, \Delta s = 0.52$ and $s = 4$

n	m	T_0	$\hat{a}$	$\hat{\lambda}$	$\hat{R}$	a_RABias	a_MSE	λ_RABias	λ_MSE
15	6	0.3	1.3067	1.5744	0.5448	0.0005	0.0039	0.4690	0.3349

15	6	0.4	1.3109	1.3436	0.6426	0.0092	0.0038	0.2477	0.1081
15	6	0.5	1.3057	1.2461	0.6836	0.0061	0.0033	0.1287	0.0509
25	10	0.3	1.3065	1.3737	0.6298	0.0069	0.0024	0.2486	0.1084
25	10	0.4	1.3074	1.1096	0.7396	0.0086	0.0022	0.0279	0.0162
25	10	0.5	1.3081	1.1109	0.7391	0.0043	0.0021	0.0084	0.0138
50	20	0.3	1.3016	1.1368	0.7286	0.0020	0.0009	0.0230	0.0073
50	20	0.4	1.3072	1.0947	0.7456	0.0033	0.0011	0.0034	0.0089
50	20	0.5	1.3022	1.0971	0.7446	0.0012	0.0011	0.0046	0.0092
100	40	0.3	1.3040	1.1254	0.7332	0.0033	0.0004	0.0222	0.0038
100	40	0.4	1.3016	1.1027	0.7424	0.0008	0.0005	0.0073	0.0044
100	40	0.5	1.3001	1.1024	0.7425	0.0001	0.0007	0.0005	0.0040

Table 2: The MLE estimators with $\lambda = 1.2, \beta = -0.5, \Delta s = 0.52$ and $s = 6$

n	m	T_0	$\hat{a}$	$\hat{\lambda}$	$\hat{R}$	a_RABias	a_MSE	λ_RABias	λ_MSE
15	6	0.3	1.3071	1.4232	0.6088	0.0076	0.0014	0.1811	0.0725
15	6	0.4	1.3089	1.3009	0.6606	0.0048	0.0014	0.1043	0.0454
15	6	0.5	1.3128	1.2526	0.6808	0.0065	0.0021	0.0584	0.0430
25	10	0.3	1.3086	1.2934	0.6637	0.0013	0.0010	0.1077	0.0361
25	10	0.4	1.3061	1.2021	0.7019	0.0087	0.0015	0.0061	0.0172
25	10	0.5	1.3042	1.2075	0.6996	0.0056	0.0012	0.0097	0.0184
50	20	0.3	1.3004	1.2224	0.6934	0.0024	0.0004	0.0075	0.0072
50	20	0.4	1.3005	1.2075	0.6996	0.0010	0.0005	0.0042	0.0085
50	20	0.5	1.3041	1.1927	0.7057	0.0055	0.0006	0.0167	0.0092
100	40	0.3	1.3009	1.2129	0.6974	0.0009	0.0002	0.0070	0.0038
100	40	0.4	1.3009	1.2023	0.7018	0.0005	0.0002	0.0004	0.0034
100	40	0.5	1.3024	1.1894	0.7071	0.0014	0.0002	0.0094	0.0045

Table 3: 0.95 boot-p confidence interval for $\lambda = 1.1, \beta = -0.5, \Delta s = 0.52$ and $s = 4$

n	m	T_0	a_low	a_up	λ_low	λ_up	R_low	R_up
15	6	0.3	1.2577	1.4989	1.4045	2.1796	0.3123	0.6168
15	6	0.4	1.1481	1.3974	1.2532	2.1546	0.3207	0.6806
15	6	0.5	1.2098	1.4252	0.9495	1.5399	0.5594	0.8018
25	10	0.3	1.2230	1.3798	1.2538	1.8801	0.4206	0.6803
25	10	0.4	1.2028	1.4066	1.0107	1.5486	0.5557	0.7786
25	10	0.5	1.3159	1.4771	0.7375	1.0947	0.7456	0.8752
50	20	0.3	1.2547	1.3908	0.9758	1.3298	0.6484	0.7919
50	20	0.4	1.2514	1.3588	0.9159	1.2825	0.6683	0.8142
50	20	0.5	1.2372	1.3328	0.9518	1.2684	0.6743	0.8010
100	40	0.3	1.2477	1.3396	1.1333	1.3980	0.6195	0.7300
100	40	0.4	1.3158	1.4072	0.8465	1.0545	0.7615	0.8390
100	40	0.5	1.2544	1.3338	1.0605	1.2787	0.6699	0.7592

Table 4: 0.95 boot-t confidence interval for $\lambda = 1.1, \beta = -0.5, \Delta s = 0.52$ and $s = 4$

n	m	T_0	a_low	a_up	λ_low	λ_up	R_low	R_up
15	6	0.3	1.2381	1.4982	1.0867	3.2686	0.0730	0.7488
15	6	0.4	1.2479	1.5004	1.1359	1.8219	0.4434	0.7290
15	6	0.5	1.1348	1.3538	1.0076	2.4714	0.2239	0.7798
25	10	0.3	1.2902	1.4727	1.1597	1.7635	0.4668	0.7193
25	10	0.4	1.1711	1.3411	0.9243	1.4570	0.5945	0.8111
25	10	0.5	1.2529	1.4268	0.8798	1.4574	0.5943	0.8272
50	20	0.3	1.2005	1.3034	1.0426	1.3355	0.6460	0.7662
50	20	0.4	1.2271	1.3554	1.0701	1.5732	0.5453	0.7554
50	20	0.5	1.3304	1.4633	0.8548	1.1637	0.7176	0.8361
100	40	0.3	1.2933	1.3757	1.0615	1.3230	0.6513	0.7588
100	40	0.4	1.2932	1.3746	0.9402	1.1541	0.7216	0.8053
100	40	0.5	1.2405	1.3226	1.0555	1.3078	0.6577	0.7611

Table 5: 0.95 boot-p confidence interval for $\lambda = 1.2, \beta = -0.5, \Delta s = 0.52$ and $s = 6$

n	m	T_0	a_low	a_up	λ_low	λ_up	R_low	R_up
15	6	0.3	1.1832	1.3340	1.4077	2.3290	0.2647	0.6154
15	6	0.4	1.2796	1.4728	0.9820	1.8883	0.4174	0.7896
15	6	0.5	1.3484	1.5207	0.8687	1.4418	0.6009	0.8312
25	10	0.3	1.2527	1.3616	1.1977	1.7703	0.4640	0.7036
25	10	0.4	1.2928	1.4104	0.8769	1.2547	0.6800	0.8283
25	10	0.5	1.2341	1.3410	1.0415	1.5580	0.5517	0.7666
50	20	0.3	1.2951	1.3788	0.9842	1.2858	0.6669	0.7887
50	20	0.4	1.2461	1.3365	1.0335	1.3866	0.6243	0.7697
50	20	0.5	1.2432	1.3184	1.1834	1.5700	0.5467	0.7095
100	40	0.3	1.2528	1.3118	1.1440	1.3833	0.6257	0.7257
100	40	0.4	1.2702	1.3224	1.1021	1.2905	0.6650	0.7426
100	40	0.5	1.2500	1.3081	1.2111	1.4875	0.5815	0.6981

Table 6: 0.95 boot-t confidence interval for $\lambda = 1.2, \beta = -0.5, \Delta s = 0.52$ and $s = 6$

n	m	T_0	a_low	a_up	λ_low	λ_up	R_low	R_up
15	6	0.3	1.1801	1.3960	1.0439	3.2452	0.0758	0.7657
15	6	0.4	1.2439	1.3962	0.9764	1.8434	0.4349	0.7917
15	6	0.5	1.2477	1.4069	0.8639	1.5841	0.5408	0.8329
25	10	0.3	1.2425	1.3628	1.1781	1.8621	0.4276	0.7117
25	10	0.4	1.2585	1.3741	0.9549	1.4293	0.6062	0.7998
25	10	0.5	1.2423	1.3672	0.9254	1.4108	0.6141	0.8107
50	20	0.3	1.2651	1.3476	1.0249	1.3514	0.6393	0.7731
50	20	0.4	1.2753	1.3598	1.0385	1.4324	0.6049	0.7678
50	20	0.5	1.2722	1.3574	1.0302	1.3486	0.6404	0.7710
100	40	0.3	1.2461	1.3115	1.1451	1.4302	0.6059	0.7252
100	40	0.4	1.2727	1.3367	1.1469	1.4218	0.6094	0.7245
100	40	0.5	1.2901	1.3474	1.0534	1.3108	0.6564	0.7619

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