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Fuzzy N Policy Queues with Infinite Capacity

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ABSTRACT

One of the important issues in Queuing model is to control the queue with different situation. In this paper, we have studied a N policy queues with infinite capacity under uncertain arrival and service information. The purpose of this paper is to construct the membership function of the system characteristic of a N-policy queue with infinite capacity under imprecise data. Fuzzy set theory is applied to estimate the uncertainty associated with the input parameters and triangular membership function has been used to analyse the model.

Keywords : Infinite capacity, N-policy, Fuzzy sets, Triangular membership function.

1. Introduction

Queueing models with a control operating policy are effective approaches for performance analysis of Computer and Telecommunication systems, manufacturing, production systems and inventory control. one of the well known control operating policies of queueing model is the N-policy. In N-policy the server is turned on when $N \geq 1$ or more units are in presence, and off when the system is empty, queues with N-policy has been studied by many Researchers like Kella 1989, Lee et al 1994a, 1994b, 1995 ; Lee & Park 1997 ; Pearn et al 2004 ; Wang et al 2004 ; Arumuganathan and Jeyakumar 2005 Choudhury and Madan, 2005. Rich and Vast literature survey found for N-policy queues.

Existing research works, including those mentioned above, have been developed to search for the optimal operating policy of queueing model when the arrival and service patterns are known exactly. However, in many real-world applications, these parameters may not be estimated precisely. For example arrival and service pattern of customer is more suitably described in linguistic terms such as “frequently arrivals” or “fast slow or moderate services rather than” by probability distributions. Both interarrival times and service times are more possibilistic than probabilistic in many practical situations. Thus the N-policy queueing models with fuzzy parameters is potentially much more useful and realistic than the commonly used crisp queues.

2. Literature Review for Fuzzy Queues

Efficient methods have been developed for analyzing the queuing system when its parameters such as arrival rate and service rate are known exactly. However, there are cases that these parameters may not be presented precisely due to uncontrollable factors. Specifically in many practical applications, the statistical data may be obtained subjectively ie. arrival rate and service rate are more suitably described by linguistic terms such as fast, moderate, or slow rather by probability distribution based on statistical theory. Imprecise information of this kind will determine the system performance measure accurately. To deal with imprecise information, Zadeh introduced the concept of fuzziness. Fuzzy set theory is a well known concept for modeling imprecision or uncertainty arising from mental phenomenon. Specifically fuzzy queues have been discussed by several researchers.

Buckley investigated multiple channel queueing system with finite or infinite waiting capacity and calling population. Negi and Lee formulated the α cut and two variable simulation approach for analyzing fuzzy queues on the basis of Zadeh extension principle. Li and Lee investigated the analytical results for $M/F/1/\infty$ and $FM/FM/1/\infty$ (where F represents fuzzy time and FM represents fuzzified exponential distribution) using Markov Chain.

Unfortunately their approach provided only crisp solutions. In other words the membership functions of the performance measures are not completely described. Kao et al applied parametric programming to construct the membership functions of the performance measures for four simple fuzzy queues with one or two fuzzy variable namely M/F/1, F/M/1, F/F/1 and FM/FM/1 where F denotes fuzzy time and FM denotes fuzzified exponential time.

N-policy queue problem with fuzzy environment are considered in this paper. The aim of this paper is to provide the membership function of the system performance for the fuzzy N-policy queue where arrival rate and service rate are fuzzy triangular numbers. Triangular membership function has been used to analyse the performance measures of N-policy queue.

3. Fuzzy Set Theory and Fuzzy Arithmetic Operations

Fuzzy set theory: We include a brief introduction on fuzzy set theory. More details are available in H.J. Zimmermann, Fuzzy Set Theory and Its Applications.

Definition 1. A fuzzy set is a set where the members are allowed to have partial membership and hence the degree of membership varies from 0 to 1. It is expressed as, $A = \left\{\left(x,\ \mu_{\tilde{A}}(x)\right) | \, x \in V\right\}$ where X is the universe of discourse and $\mu_{\tilde{A}}(x)$ is the universe of discourse and $\mu_{\tilde{A}}(x)$ = 0 or 1, i.e., x is a non-member in A if $\mu_{\tilde{A}}(x)$ = 0, and x is a member in A if $\mu_{\tilde{A}}(x)$ = 1.

Definition 2. If a fuzzy set A is defined on X, for any $\alpha \in [0, 1]$, the α –cuts ${}^{\alpha}A$ is represented by the following crisp set,

Strong α -cuts : ${}^{\alpha+}A = \{x \in X \mid \mu_A(x) > \alpha\}; \alpha \in [0,1].$

Weak α -cuts : ${}^{\alpha}A = \{x \in X \mid \mu_A(x) \geq \alpha\}; \alpha \in [0,1].$

Therefore, it is inferred that fuzzy set A can be treated as crisp set ${}^{\alpha}A$ in which all the members have their membership values greater than or at least equal to α. The concept of α -cut' is one of the most important concepts in fuzzy set theory. And here, we define support and height of a fuzzy set in terms of α -cut.

Definition 3. The support of a fuzzy set A is a crisp set represented as Supp A(x) such that, $\forall\{x \in X \mid \mu(x) > 0\}$. Thus, support of a fuzzy set is the set of all members with a strong α -cut where $\alpha = 0$.

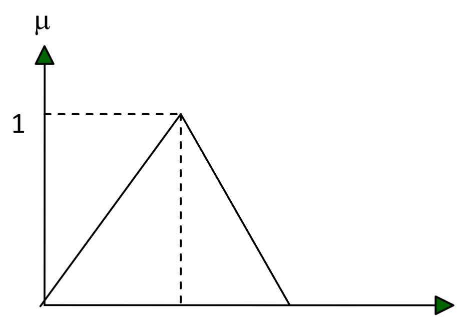
Definition 4. The height of a fuzzy set $h\{A(x) \mid x \in X\}$ is the maximum value of its membership function $\mu(x)$ such that ${}^{\alpha}A = \{x \in X \mid \mu_A(x) \geq \alpha\}$ and $0 \notin \alpha$. .

A fuzzy set where Max $\{\mu(x)\} = 1$ is called as a normal fuzzy set, otherwise, it is referred as sub-normal fuzzy set.

Definition 5. Triangular Fuzzy Number

A cover and normalized fuzzy set defined on R whose membership function is piecewise continuous is called fuzzy number. A triangular fuzzy number $\tilde{A}$ with membership function $\mu_{\tilde{A}(x)}$ is defined on R by

$$A = \begin{cases} \dfrac{x - a_1}{a_M - a_1} & \text{for } a_1 \leq x \leq a_M \\ \dfrac{x - a_2}{a_M - a_2} & \text{for } a_M \leq x \leq a_2 \\ 0 & \text{Otherwise} \end{cases}$$



Where $[a_1, a_2]$ is the supporting interval and the point $(a_M,1)$ is the peak. The third line is below figure can be dropped.

Often in application the point $a_M \in (a_1, a_2)$ is located at the middle of the supporting interval that is $a_M = (a_2 + a_1) / 2$

Fuzzy Arithmetic Operations : We define fuzzy arithmetic operations on fuzzy numbers in terms of the α -cuts. Let, A and B are two fuzzy sets and if '$*$' denotes any of the four basic arithmetic operations ($+$, $-$, $*$ and $/$) then a fuzzy set

Z = (A ∗ B) and Z ∈ R, can be defined as, ${}^{\alpha}(A * B) = {}^{\alpha}A * {}^{\alpha}B$, such that ∀ α ∈ (0, 1]. However, if '∗' is a division operator, then ${}^{\alpha}(A * B) = {}^{\alpha}A * {}^{\alpha}B$, such that ∀ α ∈ (0, 1] and $0 \notin {}^{\alpha}B$.

Theorem : (First decomposition theorem) for every A ∈ X,

$$A = \bigcup_{\alpha \in (0,1]} {}_{\alpha}A, \text{ where } \alpha\, {}_{\alpha}A(x)$$

From first decomposition theorem, if Z = (A ∗ B) and Z ∈ R,

$$(A * B) = \bigcup_{\alpha \in (0,1]} {}_{\alpha}(A * B)$$

Since ${}^{\alpha}(A * B)$ is a closed interval for each α ∈ (0, 1] with both A and B fuzzy, (A ∗ B) is also a fuzzy number.

4. N-Policy M/M/1/ Queue

We consider an N-policy M/M/1 queue with infinite capacity. It is assumed that arrivals of customers follow a poisson process with parameters λ. Service times are according to exponential distribution with service rate μ. The server can serve only one customer at a time. Arrived customers form a single waiting line at a server based on the order of their arrivals that is in a first-come, first-served discipline. If the server is busy, the arriving customers must wait until the server is available. Let W_q and N_q denote the expected waiting time in the queue and the expected number of customers in the system respectively. By Markov Chain approach W_q and N_s are derived as follows.

$$W_q = \frac{N-1}{2\lambda} + \frac{\lambda}{\mu(\mu-\lambda)} \; ; \qquad N_S = \frac{N-1}{2} + \frac{\lambda}{(\mu-\lambda)}$$

Under steady state condition $0 < \frac{\lambda}{\mu} < 1$.

5. Fuzzy N-Policy Queue

We consider an N-policy queueing model with infinite capacity in which arriving customers follow a poisson process with a fuzzy arrival rate $\tilde{\lambda}$ and service times are exponential with a fuzzy service rate $\tilde{\mu}$. $\tilde{\lambda}$ and $\tilde{\mu}$ are imprecise and uncertain. ∴ $\tilde{\lambda}$ and $\tilde{\mu}$ are defined by triangular fuzzy numbers such that

$$\tilde{\lambda} = [\lambda_1, \lambda_2, \lambda_3] \qquad ; \qquad \tilde{\mu} = [\mu_1, \mu_2, \mu_3]$$

where $\lambda_1 < \lambda_2 < \lambda_3$ and $\lambda_1 < \lambda_2 < \lambda_3$.

We apply arithmetic operators on fuzzy quantities and then defuzzify the same to convert them to crisp output.

The membership function of $\eta_{\tilde{\lambda}}(\tilde{\lambda})$ and $\eta_{\tilde{\mu}}(\tilde{\mu})$ are defined as follows.

$$\eta_{\tilde{\lambda}}\left(\tilde{\lambda}\right)=\begin{cases}0 & \text{if } \lambda<\lambda_1\\ \dfrac{\lambda-\lambda_1}{\lambda_2-\lambda_1} & \text{if } \lambda_1\le\lambda\le\lambda_2\\ \dfrac{\lambda_3-\lambda}{\lambda_3-\lambda_2} & \text{if } \lambda_1\le\lambda\le\lambda_2\\ 0 & \text{if } \lambda<\lambda_3\end{cases}\quad;\quad \eta_{\tilde{\mu}}\left(\tilde{\mu}\right)=\begin{cases}0 & \text{if } \mu<\mu_1\\ \dfrac{\mu-\mu_1}{\mu_2-\mu_1} & \text{if } \mu_1\le\mu\le\mu_2\\ \dfrac{\mu_3-\mu}{\mu_3-\mu_2} & \text{if } \mu_1\le\mu\le\mu_2\\ 0 & \text{if } \mu<\mu_3\end{cases}$$

Using the concept of α cut method.

$$\alpha_\lambda=\left[\alpha\left(\lambda_2-\lambda_1\right)+\lambda_1,\lambda_3-\alpha\left(\lambda_3-\lambda_2\right)\right]\ ;\ \alpha_\mu=\left[\alpha\left(\mu_2-\mu_1\right)+\mu_1,\mu_3-\alpha\left(\mu_3-\mu_2\right)\right]$$

6. Solution Methodology

Let the system performance measures $\tilde{W}_q$ and $\tilde{N}_s$ are given by

$$\tilde{W}_q=\frac{N-1}{2\tilde{\lambda}}+\frac{\tilde{\lambda}}{\tilde{\mu}\left(\tilde{\mu}-\tilde{\lambda}\right)}\qquad;\quad \tilde{N}_s=\frac{N-1}{2}+\frac{\tilde{\lambda}}{\left(\tilde{\mu}-\tilde{\lambda}\right)}$$

Using a cuts of $\tilde{\lambda}$ and $\tilde{\mu}$

$$^{\alpha}\left[\frac{N-1}{2}\square\frac{1}{\lambda}\right]=\left[\left(\frac{N-1}{2}\right)\frac{1}{\lambda_3\text{-}\left(\lambda_3-\lambda_2\right)},\left(\frac{N-1}{2}\right)\frac{1}{\left[\alpha\left(\lambda_2-\lambda_1\right)+\lambda_1\right]}\right]$$

$$\frac{1}{\alpha\left(\lambda_2-\lambda_1\right)+\lambda_1}\ge\frac{1}{\lambda_3\text{-}\alpha\left(\lambda_3-\lambda_2\right)}\quad \text{for } \alpha\in[0,1]$$

$$^{\alpha}\left[\tilde{\mu}-\tilde{\lambda}\right]=\left[\alpha(\mu_2\text{-}\mu_1)+\mu_1-\lambda_1-\alpha(\mu_2\text{-}\mu_1),\ \mu_3-\alpha(\mu_3\text{-}\mu_2)-\lambda_3+\alpha(\lambda_3\text{-}\lambda_2)\right]$$

$$^{\alpha}\left(\frac{\tilde{\lambda}}{\tilde{\mu}}\right)=\left[\frac{\alpha(\lambda_2\text{-}\lambda_1)+\lambda_1}{\mu_3-\alpha(\mu_3\text{-}\mu_2)},\frac{\lambda_3-\alpha(\lambda_3\text{-}\lambda_2)}{\alpha(\mu_2\text{-}\mu_1)+\mu_1}\right]$$

$$\frac{\lambda_3-\alpha(\lambda_3\text{-}\lambda_2)}{\alpha(\mu_2\text{-}\mu_1)+\mu_1}\ge\frac{\alpha(\lambda_2\text{-}\lambda_1)+\lambda_1}{\mu_3-\alpha(\mu_3\text{-}\mu_2)}\quad \text{for } \alpha\in[0,1]$$

$$^{\alpha}\left[\frac{\tilde{\lambda}}{\tilde{\mu}}\square\frac{1}{\tilde{\mu}-\tilde{\lambda}}\right]=\left[\frac{\alpha(\lambda_2\text{-}\lambda_1)+\lambda_1}{\left[\mu_3-\alpha(\mu_3\text{-}\mu_2)\right]\left[\mu_3-\alpha(\mu_3\text{-}\mu_2)\text{-}\lambda_3-\alpha(\lambda_3\text{-}\lambda_2)\right]},\right.$$

$$\left.\frac{\lambda_3-\alpha(\lambda_3\text{-}\lambda_2)\alpha(\lambda_2\text{-}\lambda_1)+\lambda_1}{\left[\alpha(\mu_2\text{-}\mu_1)+\mu_1\right]\left[\alpha(\mu_2\text{-}\mu_1)+\mu_1-\lambda_1-(\lambda_2\text{-}\lambda_1)\right]}\right]$$

$$^{\alpha}\left(W_q\right) = \left[\left(\frac{N-1}{2}\right)\frac{1}{\lambda_3-\alpha(\lambda_3-\lambda_2)}+\frac{\alpha(\lambda_2-\lambda_1)+\lambda_1}{\left[\mu_3-\alpha(\mu_3-\mu_2)\right]\left[\begin{matrix}\mu_3-\alpha(\mu_3-\mu_2)\\ -\lambda_3-\alpha(\lambda_3-\lambda_2)\end{matrix}\right]}\right.,$$

$$\left.\left(\frac{N-1}{2}\right)\frac{1}{\alpha(\lambda_2-\lambda_1)+\lambda_1}+\frac{-\lambda_3-\alpha(\lambda_3-\lambda_2)}{\left[\alpha(\mu_2-\mu_1)+\mu_1\right]\left[\begin{matrix}\alpha(\mu_2-\mu_1)+\mu_1\lambda_1\\ -\alpha(\lambda_2-\lambda_1)\end{matrix}\right]}\right] \quad \ldots (1)$$

Similarly,

$$^{\alpha}\left(N_s\right) = \left[\left(\frac{N-1}{2}\right)+\frac{\alpha(\lambda_2-\lambda_1)+\lambda_1}{\left[\mu_3-\alpha(\mu_3-\mu_2)\right]-\lambda_3+\alpha(\lambda_3-\lambda_2)},\right.$$

$$\left.\left(\frac{N-1}{2}\right)+\frac{\lambda_3-\alpha(\lambda_3-\lambda_2)}{\alpha(\mu_2-\mu_1)+\mu_1-\lambda_1-\alpha(\lambda_2-\lambda_1)}\right] \quad \ldots (2)$$

We put $\alpha = 0$ and $\alpha = 1$ in (1) and (2) and obtain an approximate triangular fuzzy number for W_q and N_s as below.

$$W_q = \left[\left(\frac{N-1}{2}\right)\frac{1}{\lambda_3}+\frac{\lambda_1}{\mu_3(\mu_3-\lambda_3)},\left(\frac{N-1}{2}\right)\frac{1}{\lambda_2}+\frac{\lambda_2}{\mu_2(\mu_2-\lambda_2)},\right.$$

$$\left.\left(\frac{N-1}{2}\right)\frac{1}{\lambda_1}+\frac{\lambda_3}{\mu_1(\mu_1-\lambda_1)}\right]$$

where $W_{q_1} = \left(\frac{N-1}{2}\right)\frac{1}{\lambda_3}+\frac{\lambda_1}{\mu_3(\mu_3-\lambda_3)}$; $W_{q_2} = \left(\frac{N-1}{2}\right)\frac{1}{\lambda_2}+\frac{\lambda_2}{\mu_2(\mu_2-\lambda_2)}$

$$W_{q_3} = \left(\frac{N-1}{2}\right)\frac{1}{\lambda_1}+\frac{\lambda_3}{\mu_1(\mu_1-\lambda_1)}$$

$$\eta_{\tilde{W}_q}\left(\tilde{W}_q\right) = \begin{cases} 0 & \text{if } W_q < W_{q_1} \\ \dfrac{W_q - W_{q_1}}{W_{q_2} - W_{q_1}} & \text{if } W_{q_1} \le W_q \le W_{q_2} \\ \dfrac{W_{q_3} - W_q}{W_{q_3} - W_{q_2}} & \text{if } W_{q_2} \le W_q \le W_{q_3} \\ 0 & \text{if } W_q < W_{q_3} \end{cases}$$

$$N_s = \left[\frac{N-1}{2}+\frac{\lambda_1}{\mu_3-\lambda_3}, \frac{N-1}{2}+\frac{\lambda_2}{\mu_2-\lambda_2}, \frac{N-1}{2}+\frac{\lambda_3}{\mu_1-\lambda_1}\right]$$

where $N_{s_1} = \frac{N-1}{2}+\frac{\lambda_1}{\mu_3-\lambda_3}$; $N_{s_2} = \frac{N-1}{2}+\frac{\lambda_2}{\mu_2-\lambda_2}$

$$N_{s_2} = \frac{N-1}{2}+\frac{\lambda_3}{\mu_1-\lambda_1}$$

The membership function for $\eta_{\tilde{N}_s}\left(\tilde{N}_s\right)$ is given as

$$\eta_{\tilde{N}_s}\left(\tilde{N}_s\right) = \begin{cases} 0 & \text{if } N_s < W_{q_1} \\ \dfrac{N_s - N_{s_1}}{N_{s_2} - N_{s_1}} & \text{if } N_{s_1} \le N_s \le N_{s_1} \\ \dfrac{N_{s_3} - N_s}{N_{s_3} - N_{s_2}} & \text{if } N_{s_1} \le N_s \le N_{s_1} \\ 0 & \text{if } N_s < N_{s_1} \end{cases}$$

7. Numerical Illustration

To illustrate the validity of the proposed method, an example inspired by Tsung-Yin Wang, Dong Yub Yang, Meng-Julie is solved.

Let us study on Printed Circuit Board Assembly (PCBA). For cost saving purpose, the reflow machine begins operating whenever the number of PCB reaches a critical value N. PCB arrive at a machine in accordance with Poisson process and the service time of the reflow machine is exponential. The practitioner would like to know the system performance including the expected waiting time in the queue W_q and the expected number of customers in the System $\tilde{N}_s$ clearly. We can model this system as N-policy FM/FM/1 queue, and construct the triangular membership function of the system performances based on the α -cut approach.

Incomplete Information

Suppose that both the interarrival rate and service rate are triangular fuzzy numbers described by $\tilde{\lambda}$ = [1, 2, 3] and $\tilde{\mu}$ = [11, 12, 13] per hour respectively for the threshold value, we choose N = 3 and thereby the reflow machine operates when three PC B are accumulated. It is simple to find.

$\alpha_\lambda = [1 + \alpha, 3 - \alpha]$; $\alpha_\mu = [11 + \alpha, 13 - \alpha]$

$$^{\alpha}\left[W_q\right] = \left[\frac{1}{(3-\alpha)}+\frac{(\alpha+1)}{(13-\alpha)(13-\alpha-3+\alpha)}, \frac{1}{(\alpha+1)}+\frac{(3-\alpha)}{(\alpha+11)(\alpha+11-1-\alpha)}\right]$$

Sub α = 0 and α = 1 in $^{\alpha}\left[W_q\right]$ we get

W_q = [.3409, .51666, 1.02727]

$$^{\alpha}[N_s] = \left[1+\frac{(\alpha+1)}{10}, 1+\frac{(3-\alpha)}{10}\right]$$

Sub α = 0 and α = 1 in $^{\alpha}[N_s]$ we get N_s = [1.1, 1.2, 1.3]

∴ Membership functions of $\tilde{W}_q$ and $\tilde{N}_s$ are given by

$$\eta_{\tilde{W}_q}\left(\tilde{W}_q\right) = \begin{cases} 0 & , \text{ if } W_q < 0.3409 \\ \dfrac{W_q - 0.3409}{0.17576} & , \text{ if } 0.3409 \le W_q \le 0.51666 \\ \dfrac{1.02727 - W_q}{0.51061} & , \text{ if } 0.51666 \le W_q \le 1.02727 \\ 0 & , \text{ if } W_q < 1.02727 \end{cases}$$

$$\eta_{\tilde{N}_s}\left(\tilde{N}_s\right) = \begin{cases} 0 & , \text{ if } N_s < 1.1 \\ \dfrac{N_s - 1.1}{0.1} & , \text{ if } 1.1 \le N_s \le 1.2 \\ \dfrac{1.3 - N_s}{0.1} & , \text{ if } 1.2 \le N_s \le 1.3 \\ 0 & , \text{ if } N_s < 1.3 \end{cases}$$

Complete Information :

λ = 2, μ = 12, N = 3

$$W_q = \left(\frac{N - 1}{2}\right)\frac{1}{\lambda}+\frac{\lambda}{\mu(\mu - \lambda)} \quad ; \quad N_s = \left(\frac{N - 1}{2}\right)+\frac{\lambda}{(\mu - \lambda)}$$

$$W_q = 1.0166667 \quad ; \quad N_s = 1.2$$

8. Conclusion

In this paper, a system performance measures of fuzzy N-policy queue under imprecise information has been studied. Fuzzy triangular membership function has been used to model fuzzy N-policy queue. Considering the arrival rate and service rate are fuzzy triangular Number and performance measures are computed using fuzzy arithmetic operators. It is a well established fact in the literature that practical situation, the arrival rate and service rate are not known exactly. Fuzzy queues are more realistic than the crisp queues for decision makers.

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