

A Quality Control Approach Based on CPM Scheduling

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ABSTRACT

Any complex large scale project consist a lot of activities, so it is very difficult to maintain schedule with standard quality. This paper presents a straight forward method to maintain quality without violating the minimum required time. Firstly, exemplify Critical Path Management (CPM) schedule from the multifaceted project by selecting parallel activities. Secondly, it represents the technique of control project standard by sampling attributes, and finally, compares the testing results by sampling attributes and those of from Normal distribution method.

1. Introduction

Complete successfully any large scale project is very difficult task and to maintain standard quality is also a complex job for the project manager. This paper will originate a technique which allows completing the task within minimum time holding appropriate quality. To do this sequence of work, illustrate the expected time for individual activity by using the formula of estimated time. This formula requires Optimistic time, pessimistic time and Normal time for each activity. After selecting the activity, draw a network diagram [1] and finally calculate the critical activity for the critical path of the project and the critical activity for the project.

2. Formulation

The estimation of minimum required time for the completion of each activity is important in the node labeling and its analysis. This can be done using three assumptions which is based on the whole project stated above. These three terms are very much important in the proper estimation of time for each activity.

Optimistic time (X_o), Most pessimistic time (Y_p), Most likely or normal time (N_m) [2]. This is the time that would most often occur should this effort be repeated over again. The estimated time of the completion of an activity is given by,

$$T_e = (X_o + 4N_m + Y_p) / 6$$

where, T_e : estimated time, X_o : most optimistic time

Y_p : most pessimistic time

N_m : normal time

And the dispersion or variance of activity completion time is given by

$$\text{Variance} = \left(\frac{Y_p - X_o}{6} \right)^2$$

Activity	Optimistic Time (x_o)	Most pessimistic time (Y_p)	Most likely time (N_m)	Estimated time	Variance
Foundation Up to PL	45	60	40	44.17	6.25
Making ground floor	25	27	22	23.33	0.111
Internal & external plinth wall (column)	10	15	12	12.16	0.694
Internal and external brick wall	8	10	6	7.00	0.111
Lintel work	4	8	6	6.00	0.444
Electrical other purpose	6	10	8	8.00	0.444
Railing and starting work	20	25	22	22.16	0.694
Doors and windows work	6	8	7	7.00	0.111
1st floor Making	24	26	22	23.00	0.111
2nd Floor Making	24	26	22	23.00	0.111
3rd Floor Making	24	26	22	23.00	0.111
Sanitary and Water Supply Works	10	14	8	9.33	0.444
Internal Electrical Works	7	10	5	6.16	0.250

Table: Estimation of shortest possible time

The total time for successful completion of a project is according to the estimate time for every activity depending on the node labeling of the project network. [1].

3. Determination of Critical Path

The determination of the critical path of the project is extremely depends on the earliest Starting Time (EST), Latest Starting Time (LST), Earliest finished time and slack time of the project [2]. Thus to evaluate the critical activity of the project we must have to way out these times for our project. The method of the estimating of these times is present here:

To find out EST, LST, EFT and Slack time is very intricate for the complex project of large activities. Therefore, an algorithm is to develop that can evaluate the EST, LST, EFT and slack within a moment for any complex project. If any project has $(n + 1)$ node then we can easily defined these entire node by a simple algorithm and it will be more flexible to handle the project of $(n + 1)$ node. We can also easily formulate our project node by this algorithm.

Suppose a project network has $(n + 1)$ node, the initial event being 0 and the last event being n . The start of the project at x_0 be defined at time 0. Nodal event times must be consistent with activity duration. So that an activity's successor node event time must be larger than an activity predecessor node event time plus it duration. In defining an activity starting from event i and ending at event j , the relationship can be expressed as the inequality constraints, $x_j \geq x_i + D_{ij}$, where D_{ij} is the duration of activity (i, j) .

To obtain Earliest Event Time (EET) and Latest Event Time (LET), present all the relative activity (table 1) by using Earliest Starting Time, latest starting time and slack time. In order to find EET and LET algorithms have shown in this section.

Calculation of Earliest Event Time	Calculation of Latest Event Time
Step 1: Set $E(0)$ is equal to 0 Step 2: Input each Event $E(i)$ until last event Step 4: Input the duration for each corresponding Event of the project Step 5: Compute the Last Maximum value For each Node using $E(j) = \text{Max} \{ E(i) + D(ij) \}$ Step 6: Stop process.	Step 1: Let $L(n)$ equal the required completion time of the project. [$L(n)$ must equal or exceed $E(n)$.] Step 2: For $i = n-1, n-2, \dots, 0$, let Step 3: Calculate $L(i) = \text{minimum} \{ L(j) - D_{ij} \}$ for each event Step 3: Stop the Process [where the minimum is computed over all activities (i,j) that have i as the starting event.

Earliest event times are computed as the maximum of the earliest start times plus activity durations for each of the activities immediately preceding an event. The earliest start time for each activity (i,j) is equal to the earliest possible time for the preceding event $E(i)$:

$$ES(i, j) = E(i)$$

The earliest finish time of each activity (i,j) can be calculated by:

$$EF(i, j) = E(i) + D_{ij}$$

The latest possible time $L(j)$ can be determined by above algorithm at which each event j in the network can occur. An activity (i, j) is a critical activity if it satisfies all of the following conditions:

$$E(i) = L(i) \quad (i)$$

$$E(j) = L(j) \quad (ii)$$

$$E(i) + D(ij) = L(j) \quad (iii)$$

Hence activities between critical events are also on a critical path as long as the activity's earliest starting time equals its latest starting time. $ES(i,j) = LS(i,j)$.

4. Critical Path on the project activity

The critical path is one having longest-time span through the total system of events. The Critical path is the path where the slack time is zero. The following table 4.1 shows the critical path of our selected project:

Activity	Duration D_{ij}	Earliest start time $E(i) = ES(i,j)$	Latest finish time $L(j) = LF(i,j)$	Latest start time $LS(i,j)$
A (0,1)	44.16	0*	44.16	0*
B (0,2)	23.33	44.16	67.49	32.99
C (1,2)	12.16	44.16*	67.49	44.16*
D (1,3)	7.00	56.32	85.65	55.32
E (2,3)	6.00	56.32*	85.65	56.32*
F (2,4)	8.00	62.32	107.81	76.48
G (3,4)	22.16	62.32*	107.81	62.32*
H (3,5)	7.00	84.48	176.81	146.48
I (4,5)	69.00	84.48*	176.81	84.48*
J(4,6)	9.33	153.48	183.14	144.15
K(5,6)	6.16	153.48*	183.14	159.64*

Table 4.1: Identification of activities on the critical path

(*) Indicate the Critical activity of the project.

5. Method

Sampling by attributes is a widely applied quality control method. The procedure is intended to determine whether or not a particular group of work acceptable. A group work items to be tested is called a *batch*. Each batch is tested to determine if it satisfies a minimum acceptable quality level (AQL) expressed as the maximum percentage of defective items in a batch or process.

In its basic form, sampling by attributes is applied by testing a pre-defined number of sample items from a batch. If, the number of defective items is greater than a trigger level, then the batch is rejected as being likely to be of unacceptable quality. Otherwise, the batch is accepted. . More formally, a batch is defined as acceptable if it contains a fraction r_1 or less defective items. Similarly, a batch is defined as unacceptable if it contains a fraction r_2 or more defective units. Generally, the acceptance fraction is less than or equal to the rejection fraction $r_1 \leq r_2$, and the two fractions are often equal so that there is no ambiguous range of batch acceptability between r_1 and r_2 . Given a sample size and a trigger level for lot rejection or acceptance, and also like to determine the probabilities that acceptable batch might be incorrectly rejected (termed *producer's risk*) or that deficient batch might be incorrectly accepted (termed *consumer's risk*).

Consider a batch of finite number N , in which m items are defective (bad) and the remaining $(N - m)$ items are non-defective (good). If a random sample of n items is taken from this batch, then it is possible to determine the probability of having different numbers of defective items in the sample. With a pre-defined acceptable number of defective items, this study develop the probability of accepting a batch as a function of the sample size, the allowable number of defective items, and the actual fraction of defective items.

The number of possible samples with exactly x defectives is the combination associated with obtaining x defectives from m possible defective items and $(n - x)$ good items from $(N - m)$ good items: $\binom{m}{x} \binom{N-m}{n-x}$. Given these possible numbers of samples, the probability of having exactly x defective items in the sample is given by the ratio as the hyper geometric series:

$$P(X = x) = \frac{\binom{m}{x} \binom{N-m}{n-x}}{\binom{N}{n}} \quad (1)$$

With this function, calculate the probability of obtaining different numbers of defectives in a sample of a given size. Suppose that the actual fraction of defectives in the batch is p and the actual fraction of non defectives is q , then p plus q is one, resulting in $m = Np$, and $N - m = Nq$. Then, a function $g(p)$ representing the probability of having r or less defective items in a sample of size n is obtained by substituting m and N into Eq. (1) and summing over the acceptable defective number of items:

$$g(p) = \sum_{x=0}^r P(X = x) = \sum_{x=0}^r \frac{\binom{Np}{x} \binom{Nq}{n-x}}{\binom{N}{n}} \quad (2)$$

If the number of items in the batch N is large in comparison with the sample size n , then the function $g(p)$ can be approximated by the binomial distribution:

$$g(p) = \sum_{x=0}^r \binom{n}{x} p^x q^{n-x} \quad (3)$$

$$\text{Or } g(p) = 1 - \sum_{x=r+1}^n \binom{n}{x} p^x q^{n-x}$$

The function $g(p)$ indicates the probability of accepting a batch, given the sample size n and the number of allowable defective items in the sample r . The function $g(p)$ can be represented graphical for each combination of sample size n and number of allowable defective items r , as shown in Figure 5-1. Each curve is referred to as the operating characteristic curve (OCC) in this graph. For the special case of a single sample ($n = 1$), the function $g(p)$ can be simplified:

$$g(p) = \binom{1}{0} p^0 q^1 = q \quad (4)$$

so that the probability of accepting a batch is equal to the fraction of acceptable items in the batch. For example, there is a probability of 0.5 that the lot may be

accepted from a single sample test even if fifty percent of the batch is defective.

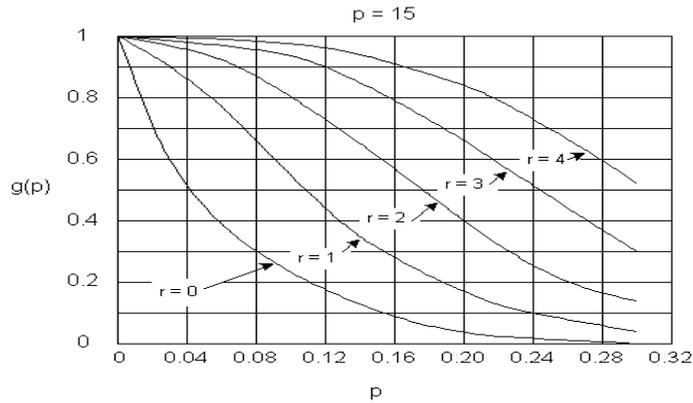


Figure 5-1 OCC Indicating Probability of batch Acceptance

For any combination of n and r , we can read off the value of $g(p)$ for a given p from the corresponding OCC. For example, $n = 11$ is specified in Figure 5-1. Then, for various values of r has shown in the table:

$r = 0$	$p = 24\%$	$g(p) = 2\%$
$r = 0$	$p = 4\%$	$g(p) = 54\%$
$r = 1$	$p = 24\%$	$g(p) = 10\%$
$r = 1$	$p = 4\%$	$g(p) = 88\%$

The operating characteristic curve indicates that a sample size of 11 is chosen with a trigger level for rejection of one item. With a four percent acceptable level and a greater than four percent defective fraction, the consumer's risk is at most eighty-eight percent. In contrast, with a four percent acceptable level and a four percent defective fraction, the producer's risk is at most $1 - 0.88 = 0.12$ or twelve percent.

In specifying the sampling plan implicit in the operating characteristic curve, the supplier and consumer of materials or work must agree on the levels of risk acceptable to themselves. If the batch is of acceptable quality, the supplier would like to minimize the chance or risk that a lot is rejected solely on the basis of a lower than average quality sample.

6. Analysis

Suppose that the sample size is six ($n = 6$) from the critical path activity table of eleven items ($N = 11$). The batch of materials is to be rejected if any of the six samples is defective ($r = 0$). In this case, the probability of acceptance as a function of the actual number of defective items can be computed by noting that for $r = 0$, only one term ($x = 6$) need be considered in Eq. (2). Thus, for $N = 11$ and $n = 6$:

$$g(p) = \frac{\binom{11}{p} C_0 \binom{11}{q} C_6}{\binom{11}{C_6}} \text{ [Taking } N = (11 \times 10) \text{ for small 11 activity project]}$$

For a two percent defective fraction ($p = 2\%$), the resulting acceptance value is: $g(p) = \frac{\binom{11}{p} C_0 \binom{11}{q} C_6}{\binom{11}{C_6}} = \frac{1913554188}{2141851635} = .8934$

Using the binomial approximation in Eq. (4), the comparable calculation would be: $g(p) \approx \binom{6}{C_0} p^0 q^6 = .8858$; which is a difference of 0.0076, or 0.07 percent from the actual value of 0.8934 found above. If the acceptable defective proportion was two percent (so $p_1 = 0.02$), then the chance of an incorrect rejection (or producer's risk) is $1 - g(0.02) = 1 - 0.9 = 0.1$ or ten percent. Note that a prudent producer should ensure better than minimum quality products to reduce the probability or chance of rejection under this sampling plan. If the actual proportion of defectives was one percent, then the producer's risk would be only five percent with this sampling plan.

7. Results and Discussions

Project Variance = sum of the variance of the activities in the critical path. We know the standard deviation is given by the square root of the variance. So, project standard deviation $\sigma_T = \sqrt{9.886} = 3.144$.

The probability of finishing a project on time is calculated based on two assumptions. These are as follows:

- i) Project completion time follow a normal probability distribution
- ii) Activity times are statistically independent

With these assumptions the bell shaped curve shown in the following figure used to represent the projects completion dates. 183.14. Days

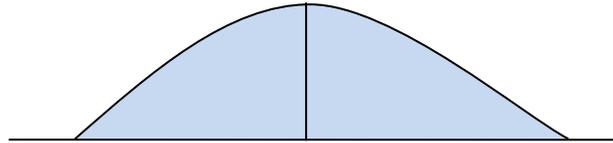


Figure 6.1: Probability distribution for project completion time.

It also means that there is a 50% chance the project will be completed in less than the expected 183.14 days and 50% chance that the project will be completed in more than the expected time. Using normal distribution we have

$$Z = \frac{\text{Due date} - \text{Expected date of completion}}{\sigma_T} = \frac{188.30 - 183.14}{3.144} = 1.64$$

Where, Z= standard deviations from the due date or the expected date.

From normal table we find a probability of 0.4495 corresponding to the Z value. Thus, there are 44.95% chances that the project will be able to finish the project in 188.30 days or less. This is shown in the following figure:

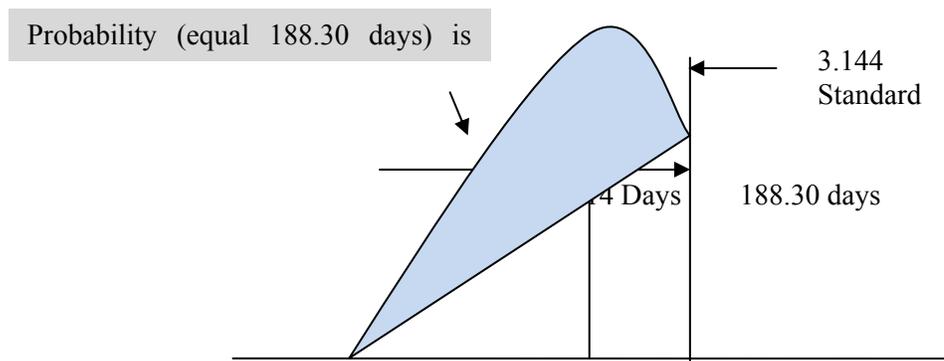


Figure 6.2: Probability of meeting 188.30 days deadline.

Thus the discussion provides us the following information for the successful completion of the project:

- i) The expected completion time of the project is 183.14
- ii) Probability of finishing the project within the deadline is 45% (approx.)
- iii) Identify the critical activities delay of any of which will delay the project completion time has been presented in table 4.1.

From the calculation we observe that statistically the project maintain standard quality. On the contrary, we discuss about the dead line of work and found that it also successfully completed within the projected time.

8. Conclusion

Though it is important to complete any work within the estimated time, with this it is also crying need to maintain standard quality. This paper presents a sequential step to formulate a technique to control time limit maintaining standard quality. Finally, Experimental result is effective to control quality of any large scale project.

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