Journal of Physical Sciences, Vol. 14, 2010, 133-143
ISSN: 0972-8791, www.vidyasagar.ac.in/journal
Published on December 24, 2010

A New Model of Chaotic Dynamics-the Complemented Shift Map

Indranil Bhaumik[1] and Binayak S. Choudhury[2]

Department of Mathematics
Bengal Engineering and Science University, Shibpur
West Bengal, Howrah -711103, India
[1] ranaindra2006@yahoo.co.in
[2] binayak12@yahoo.co.in

Received October 25, 2010; accepted November12, 2010

ABSTRACT

Recently, the present authors introduced the notion of the complemented shift map in the symbol space Σ_2. In this paper we have proved some further chaotic properties of the complemented shift map and pointed out some differences in the dynamics of the complemented shift map and the shift map.

Keywords: Complemented shift map, Chaotic dependence on initial conditions, Opposite points, Totally transitive, Chaos.

AMS Subject Classifications 2010: 37B10, 37D45.

1. Introduction

A general dynamical system is sometimes defined as a pair (X, f) consisting of a set X together with a continuous map f from X into itself. However dynamical systems are also studied in abstract metric spaces. In several of such studies a dynamical system is described by a metric space (X, ρ) along with a self map f. In most of the cases, X is assumed to be compact without being an isolated point. Chaotic dynamical systems constitute a special class of dynamical systems. In 1975, Li and Yorke [9] gave the first mathematical definition of chaos through the introduction of δ-scrambled set and in 1993, S. Li [8] introduced the notion of ω-chaos through the introduction of ω-scrambled set. Devaney [4] chaos is another popular type of chaos. In particular there are several works on symbolic dynamics which are dynamics represented by maps on symbol spaces. Some of these works are noted in references [1, 2, 5, 6, 7, 10]. Of particular interest is the space Σ_2 which has been considered in a large number of works. Devaney [4] and Robinson [11] both have given vivid description of the space Σ_2. Recently, the present authors have extended the idea of the shift map to the generalized shift map in [1] and proved that the generalized shift map is chaotic on Σ_2.

We now describe the space Σ_2 which we explicitly consider in our present work and the complemented shift map defined on it. The space $\Sigma_2 = \{\alpha : \alpha = (\alpha_0\alpha_1............), \alpha_i = 0 \text{ or } 1\}$ is the symbol space containing two symbols 0 and 1. Also we all know that the space Σ_2 is a compact metric space with the metric $d(s,t) = \sum_{i=0}^{\infty} \frac{|s_i - t_i|}{2^{i+1}}$, where $s = (s_0 s_1.......)$ and $t = (t_0 t_1........)$ are any two points of Σ_2. We note that maximum distance between any two points of Σ_2 with our chosen metric is 1, because $d(s,t) \le (\frac{1}{2} + \frac{1}{2^2} + ...) = 1$.

In [2], we introduced the concept of the complemented shift map σ' on Σ_2 and proved that the complemented shift map σ' is ω-chaotic with some additional features and is topologically conjugate to the shift map (the conjugacy is $h(a) = (a_0 a_1' a_2 a_3'..........)$, where $a = (a_0 a_1..........)$ is any point of Σ_2). In this paper we have proved some further strong chaotic properties of the complemented shift map and also introduced the concept of opposite points on Σ_2. In Theorem 3.1, we have proved that the dynamical system (Σ_2, σ') has chaotic dependence on initial conditions. In Theorem 3.2, we have proved that the complemented shift map $\sigma' : \Sigma_2 \to \Sigma_2$ is totally transitive on Σ_2. In the last section, we have given a counter example to prove that not all topologically transitive maps are totally transitive. A comparison of the behaviors of shift map and complemented shift map has been given in Section 4.

2. Mathematical Preliminaries

In this section we have presented some definitions and lemmas which are essential for establishing the theorems in the next section.

Definition 2.1 (Shift map [4]): The shift map $\sigma : \Sigma_2 \to \Sigma_2$ is defined by $\sigma(\alpha_0\alpha_1.................) = (\alpha_1\alpha_2............)$, where $\alpha = (\alpha_0\alpha_1..............)$ is any point of Σ_2.

Next we give the definition of the complemented shift map which we have introduced in [2].

Definition 2.2 (Complemented shift map [2]): Let $s = (s_0 s_1...........)$ be any point of Σ_2. Then the complemented shift map $\sigma' : \Sigma_2 \to \Sigma_2$ is defined by

$\sigma'(s) = (s_1' s_2' ....)$, where s_i' is the complement of s_i. So it is the map which shifts the first element of a point and then also changes all others into its complement.

We now introduce the concept of opposite points in the symbol space.

Definition 2.3 (Opposite points): Let x, y be two points of Σ_2. Then the points x, y are called opposite points of Σ_2, if $d(x, y) = 1$, that is, the distance of x, y is maximum in Σ_2.

Definition 2.4 (Sensitive dependence on initial conditions [4]): A continuous map $f : X \to X$ is said to have sensitive dependence on initial conditions if there exists $\delta > 0$ such that, for any $x \in X$ and any neighborhood $N(x)$ of x, there exist $y \in N(x)$ and $n \geq 0$ such that $\rho(f^n(x), f^n(y)) > \delta$, where (X, ρ) is a compact metric space.

Definition 2.5 (Topologically transitive [4]): A continuous map $f : S \to S$ is called topologically transitive if for any pair of non empty open sets $U, V \subset S$ there exists $k \geq 0$ such that $f^k(U) \cap V \neq \phi$, where (S, ρ) is a compact metric space.

Definition 2.6 (Totally transitive [12]): Let (X, ρ) be a compact metric space. A continuous map $f : X \to X$ is called totally transitive if f^n is topologically transitive for all $n \geq 1$.

Definition 2.7 (Li -Yorke pair [3]): A pair $(x, y) \in X^2$ is called Li –Yorke (with modulus $\delta > 0$) if and only if $\limsup_{n\to\infty} \rho(f^n(x), f^n(y)) \geq \delta$ and $\liminf_{n\to\infty} \rho(f^n(x), f^n(y)) = 0$, where X is a compact metric space with the metric ρ and f is a continuous mapping on X. The set of all Li -Yorke pairs of modulus δ is denoted by $LY(f, \delta)$.

Definition 2.8 (Chaotic dependence on initial conditions [3]): A dynamical system (X, f) has called chaotic dependence on initial conditions if for any $x \in X$ and every neighborhood $N(x)$ of x there is a $y \in N(x)$ such that the pair $(x, y) \in X^2$ is Li -Yorke.

Definition 2.9 (Distance between two sets): Let (X, ρ) be a compact metric space and A, B be two non empty subsets of X . Then the distance between A and B is denoted by $d(A, B)$ and defined by $d(A, B) = \inf\{d(x, y) : x \in A, y \in B\}$.

Definition 2.10 (Topological conjugacy [4]): Let $f : A \to A$ and $g : B \to B$ be two continuous mappings. Then f and g are said to be topologically conjugate if there exists a homeomorphism $h : A \to B$ such that $h \circ f = g \circ h$. The homeomorphism h is called a topological conjugacy between f and g.

It is well known that mappings which are topologically conjugate are completely equivalent in terms of their dynamics.

We also need the following lemmas.

Lemma 2.1 [4] Let $s,t \in \Sigma_2$ and $s_i = t_i$, for $i = 0,1,.....,m$. Then $d(s,t) < \frac{1}{2^m}$ and conversely if $d(s,t) < \frac{1}{2^m}$, $s_i = t_i$, for $i = 0,1,.....,m.$

It was proved in [2] that the complemented shift map is a continuous map in the symbol space Σ_2. We give the proof again here to make this paper self contained.

Lemma 2.2 The complemented shift map $\sigma' : \Sigma_2 \to \Sigma_2$ is a continuous map on the symbol space Σ_2.

Proof: Let $\varepsilon > 0$ be arbitrary. We choose n so large that $\frac{1}{2^n} < \varepsilon$. Let $s = (s_0 s_1 ....)$ and $t = (t_0 t_1 ....)$ be any two points of Σ_2. We now choose $\delta = \frac{1}{2^{n+1}}$ and denote the complement of any binary numeral α_i by α_i'.

Then $d(s,t) < \delta = \frac{1}{2^{n+1}} \Rightarrow d((s_0 s_1 ......s_{n+1}......),(t_0 t_1 ......t_{n+1}......)) < \frac{1}{2^{n+1}}$

$$\Rightarrow s_i = t_i, \text{ for } i = 0,1,2,........,n+1, \text{ (by Lemma 2.1)}$$
$$\Rightarrow s_i' = t_i', \text{ for } i = 0,1,2,........,n+1$$
$$\Rightarrow s_i' = t_i', \text{ for } i = 1,2,........,n+1$$
$$\Rightarrow d((s_1' ....s_{n+1}' ....),(t_1' ....t_{n+1}' ....)) < \frac{1}{2^n}, \text{ (by Lemma 2.1)}$$
$$\Rightarrow d(\sigma'(s), \sigma'(t)) < \frac{1}{2^n} < \varepsilon .$$

This proves that the complemented shift map $\sigma' : \Sigma_2 \to \Sigma_2$ is a continuous map on the symbol space Σ_2.

3. The Main Theorems

Theorem 3.1 The dynamical system (Σ_2, σ') has chaotic dependence on initial conditions.

Proof: Let $s = (s_0 s_1 ....)$ be any point of Σ_2. Also let U be any open neighborhood of s. Since U is open, we can take an open ball $V \subset U$, with radius

$\varepsilon > 0$. We choose n so large that $\frac{1}{2^n} < \varepsilon$. The following notations will assist us in proving Theorem 3.1.

1. Let $S = s_0 s_1 ....s_i$ and $P = p_0 p_1 ....p_m$ are two finite sequences of 0's and 1's, then $SP = s_0 s_1 ....s_i p_0 p_1 ....p_m$. Further, if we suppose that $T_1, T_2, ..., T_p$ are p finite sequences of 0's and 1's, then $T_1 T_2 ....T_p$ can be defined in a similar manner as above.

2. If α_p is any binary numeral, we denote the complement of α_p by α'_p, that is, if $\alpha_p = 0$ or 1, $\alpha'_p = 1$ or 0.

3. Let $A(s,2n+2) = (s_{n+1} s_{n+2} ....s_{2n+1} s'_{2n+2} s'_{2n+3} ....s'_{3n+2})$,

$A(s,2n+4) = (s_{3n+3} s_{3n+4} ....s_{4n+4} s'_{4n+5} s'_{4n+6} ....s'_{5n+6})$,

$A(s,2n+6) = (s_{5n+7} s_{5n+8} ....s_{6n+9} s'_{6n+10} s'_{6n+11} ....s'_{7n+12})$, and so on.

Note that for any even integer $k > 0, A(s,2n+k)$ is a finite string of length $2n+k$.

4. Lastly, we take $t \in \Sigma_2$ such that,

$t = (s_0 s_1 ....s_n A(s,2n+2) A(s,2n+4) A(s,2n+6) ....)$.

With those four notations and Lemma 2.1 as above we now prove the theorem. By construction s and t agree upto s_n. Hence $d(s,t) < \frac{1}{2^n} < \varepsilon$, by Lemma 2.1. So $t \in V \Rightarrow t \in U$.

Then two cases come up for consideration.

Case I: When n is an odd integer.

Here $\sigma'^{n+1}(s) = (s_{n+1} s_{n+2} ....s_{2n+1} ....)$ and

$\sigma'^{n+1}(t) = (s_{n+1} s_{n+2} ....s_{2n+1} ....)$.

Note that t consists of infinitely many finite sequences of the type $A(s,2n+k)$. So we get,

$$\liminf_{n\to\infty} d(\sigma'^n(s), \sigma'^n(t)) \le \lim_{n\to\infty} d((s_{n+1} s_{n+2} ....s_{2n+1} .....), (s_{n+1} s_{n+2} ....s_{2n+1} .....))$$
$$\le \lim_{n\to\infty} \left(\frac{0}{2} + \frac{0}{2^2} + ... + \frac{0}{2^{n+1}}\right)$$
$$= 0.$$

Hence, $\liminf_{n\to\infty} d(\sigma'^n(s), \sigma'^n(t)) = 0$. (3.1)

Similarly, $\sigma'^{2n+2}(s) = (s_{2n+2} s_{2n+3} ....s_{3n+2} ....)$ and

$$\sigma'^{2n+2}(t) = (s'_{2n+2}s'_{2n+3}..................s'_{3n+2}..................).$$

So we get,

$$\limsup_{n\to\infty} d(\sigma'^n(s), \sigma'^n(t)) \geq \lim_{n\to\infty} d((s_{2n+2}s_{2n+3}.....s_{3n+2}....), (s'_{2n+2}s'_{2n+3}.....s'_{3n+2}....))$$

$$\geq \lim_{n\to\infty}(\frac{1}{2} + \frac{1}{2^2} + ... + \frac{1}{2^{n+1}})$$

$$= 1.$$

Hence, $\limsup_{n\to\infty} d(\sigma'^n(s), \sigma'^n(t)) = 1$. (3.2)

Case II: When n is an even integer.

Then $\sigma'^{n+1}(s) = (s'_{n+1}s'_{n+2}...............s'_{2n+1}..........)$ and

$$\sigma'^{n+1}(t) = (s'_{n+1}s'_{n+2}..............s'_{2n+1}.........)$$

In this case also t consists of infinitely many finite sequences of the type $A(s, 2n+k)$.

So we get,

$$\liminf_{n\to\infty} d(\sigma'^n(s), \sigma'^n(t)) \leq \lim_{n\to\infty} d((s'_{n+1}s'_{n+2}........s'_{2n+1}.......), (s'_{n+1}s'_{n+2}.......s'_{2n+1}......))$$

$$\leq \lim_{n\to\infty}(\frac{0}{2} + \frac{0}{2^2} + ... + \frac{0}{2^{n+1}})$$

$$= 0.$$

Hence, $\liminf_{n\to\infty} d(\sigma'^n(s), \sigma'^n(t)) = 0$. (3.3)

Similarly, $\sigma'^{2n+2}(s) = (s_{2n+2}s_{2n+3}..........s_{3n+2}.......)$ and

$$\sigma'^{2n+2}(t) = (s'_{2n+2}s'_{2n+3}........s'_{3n+2}......).$$

So we get,

$$\limsup_{n\to\infty} d(\sigma'^n(s), \sigma'^n(t)) \geq \lim_{n\to\infty} d((s_{2n+2}s_{2n+3}.....s_{3n+2}....), (s'_{2n+2}s'_{2n+3}......s'_{3n+2}....))$$

$$\geq \lim_{n\to\infty}(\frac{1}{2} + \frac{1}{2^2} + ... + \frac{1}{2^{n+1}})$$

$$= 1.$$

Hence, $\limsup_{n\to\infty} d(\sigma'^n(s), \sigma'^n(t)) = 1$. (3.4)

By virtue of (3.1), (3.2), (3.3), (3.4) in the two cases above we get that

$\liminf_{n\to\infty} d(\sigma'^n(s), \sigma'^n(t)) = 0$ and $\limsup_{n\to\infty} d(\sigma'^n(s), \sigma'^n(t)) = 1$. (3.5)

By virtue of (3.5) it is proved that the pair (s,t) is Li-Yorke. Hence the dynamical system (Σ_2, σ') has chaotic dependence on initial conditions.

Theorem 3.2 The complemented shift map $\sigma' : \Sigma_2 \to \Sigma_2$ is totally transitive on the symbol space Σ_2.

Proof: We are to prove that σ'^n is topologically transitive for all $n \geq 1$. Let U and V be two non empty open subsets of Σ_2 and $\varepsilon_1, \varepsilon_2 > 0$. Also let $s = (s_0 s_1 ....) \in U$ be a point such that $\min\{d(s,\beta)\} \geq \varepsilon_1$, for any β belongs to the boundary of the set U. Similarly, let $t = (t_0 t_1 ....) \in V$ be any point such that $\min\{d(t,\gamma)\} \geq \varepsilon_2$ for any γ belongs to the boundary of the set V. Next we choose two odd integers k_1 and k_2 so large that $\frac{1}{2^{nk_1 - 1}} < \varepsilon_1$ and $\frac{1}{2^{nk_2}} < \varepsilon_2$. Then two cases come up for consideration.

Case I: When n is an even integer.

We now consider the point $\alpha = (s_0 s_1 ....s_{nk_1 - 1} t_0 t_1 ....t_{nk_2} ....)$. Then by Lemma 2.1, $d(s,\alpha) < \frac{1}{2^{nk_1 - 1}} < \varepsilon_1$.

Hence $\alpha \in U$, that is, $(\sigma'^n)^{k_1}(\alpha) \in (\sigma'^n)^{k_1}(U)$.

On the other hand, $(\sigma'^n)^{k_1}(\alpha) = (t_0 t_1 ....t_{nk_2} ....)$. Hence $d((\sigma'^n)^{k_1}(\alpha), t) < \frac{1}{2^{nk_2}} < \varepsilon_2$, by applying Lemma 2.1 again. This gives $(\sigma'^n)^{k_1}(\alpha) \in V$.

Hence we get that $(\sigma'^n)^{k_1}(U) \cap V \neq \phi$, where n is any even integer. So the complemented shift map $\sigma' : \Sigma_2 \to \Sigma_2$ is totally transitive on Σ_2 when n is an even integer.

Case II: When n is an odd integer.

In this case we consider the point $\beta = (s_0 s_1 ....s_{nk_1 - 1} t'_0 t'_1 ....t'_{nk_2} ....)$.

Then by Lemma 2.1, $d(s,\beta) < \frac{1}{2^{nk_1 - 1}} < \varepsilon_1$.

Hence $\beta \in U$, that is, $(\sigma'^n)^{k_1}(\beta) \in (\sigma'^n)^{k_1}(U)$.

On the other hand, $(\sigma'^n)^{k_1}(\beta) = (t_0 t_1 ....t_{nk_2} ....)$. Hence $d((\sigma'^n)^{k_1}(\beta), t) < \frac{1}{2^{nk_2}} < \varepsilon_2$, by applying Lemma 2.1 again. This gives $(\sigma'^n)^{k_1}(\beta) \in V$.

Hence we get that $(\sigma'^n)^{k_1}(U)\cap V \neq \phi$, where n is any odd integer. So the complemented shift map $\sigma':\Sigma_2 \to \Sigma_2$ is totally transitive on Σ_2 when n is an odd integer.

Combining those two cases as above we get that the complemented shift map is totally transitive on Σ_2.

4. Some Special Properties

In this section we discuss some basic differences of dynamics of the complemented shift map and the shift map. We also present a comparative study between the shift map and the complemented shift map. Here the period means prime period.

We start with the periodic points of those maps. If $\sigma:\Sigma_2 \to \Sigma_2$ is the shift map, we all know that any repeating sequence of 0's and 1's is always a periodic point of σ. As for example, $\alpha = (\alpha_0\alpha_1.........\alpha_m\alpha_0\alpha_1.........\alpha_m......)$ is a periodic point of period $m+1$ of σ, for all $m \geq 0$. But in the case of the complemented shift map σ', the situation is different. In this case not all repeating sequences of 0's and 1's are periodic in the same period that of σ. If x is a periodic point of σ such that it consists of repeating sequences of even number of terms, that is, $x = (x_0x_1...........x_{2n-1}x_0x_1........x_{2n-1}........)$ then obviously x is a periodic point of period $2n$ of σ'. On the other hand if we consider the point $y = (y_0y_1.........y_{2n}y_0y_1.........y_{2n}.........)$, it is a periodic point of σ' with period $2(2n+1)$, because $\sigma'^{2n+1}(y) = (y'_0y'_1..........y'_{2n}y'_0y'_1.........y'_{2n}.......) \neq y$, but $\sigma'^{2(2n+1)}(y) = (y_0y_1................y_{2n}y_0y_1............y_{2n}............) = y$. Hence for odd repeating sequences of 0's and 1's the case is different. So for an odd case we choose the point in a different manner. We now consider the point $z = (z_0z_1.........z_{2n}z'_0z'_1..........z'_{2n}z_0z_1..........z_{2n}z'_0z'_1..........z'_{2n}.......)$, then $\sigma'^{2n+1}(z) = (z_0z_1.............z_{2n}z'_0z'_1.............z'_{2n}..........) = z$. So z is a periodic point of period $2n+1$ of σ'. Similarly, we can get that z is a periodic point of σ with period $2(2n+1)$. Hence we conclude that the periodic points of σ and σ' are not same.

We know that one fixed point of any continuous map can not be mapped into another fixed point by the map itself. But in symbolic dynamics there is an interesting situation. We consider the points $\sigma_{f_1} = (0000.........)$ and $\sigma_{f_2} = (1111.........)$ of Σ_2. These are the only fixed points of σ. Also we can not jump from σ_{f_1} to σ_{f_2} (or from σ_{f_2} to σ_{f_1}) under any iteration of σ. But we can always jump from σ_{f_1} to σ_{f_2} (or from σ_{f_2} to σ_{f_1}) under iteration of σ'. Because $\sigma'(\sigma_{f_1}) = \sigma_{f_2}$ and $\sigma'(\sigma_{f_2}) = \sigma_{f_1}$. Similarly, the points $\sigma'_{f_1} = (0101.................)$

and $\sigma'_{f_2} = (1010................)$ of Σ_2 are the only fixed points of σ'. Hence we can not jump from σ'_{f_1} to σ'_{f_2} (or from σ'_{f_2} to σ'_{f_1}) under any iteration of σ'. But $\sigma(\sigma'_{f_1}) = \sigma'_{f_2}$ and $\sigma(\sigma'_{f_2}) = \sigma'_{f_1}$. So we can say that the fixed points of the complemented shift map are mapped into one from another by the shift map. Similarly, the fixed points of the shift map are mapped into one from another by the complemented shift map.

By the concept of opposite points we see that any two points of Σ_2 are not always opposite points. It is also true that for any point $x \in \Sigma_2$; $x, \sigma(x)$ are not always opposite points. Similarly, for any point $y \in \Sigma_2$; $y, \sigma'(y)$ are not always opposite points. But for any point $\alpha = (\alpha_0\alpha_1.................)$ of Σ_2, the distance between $\sigma(\alpha)$ and $\sigma'(\alpha)$ is always maximum on Σ_2.

Because, $d(\sigma(\alpha), \sigma'(\alpha)) = d((\alpha_1\alpha_2.............), (\alpha'_1\alpha'_2.............))$

$$= (\frac{1}{2} + \frac{1}{2^2} + ...)$$

$$= 1 \text{(which is maximum).}$$

Hence we conclude that for any point $\alpha \in \Sigma_2; \sigma(\alpha), \sigma'(\alpha)$ are always opposite points.

5. Conclusions

We all know that the shift map is the only map in the symbol space Σ_2 which can be considered as a model of chaotic maps. Recently, we have introduced another chaotic map, namely, the complemented shift map in the symbol space Σ_2 and have proved that it is a ω-chaotic map. Since ω-chaos is equivalent to chaos in the sense of Devaney, the complemented shift map is Devaney chaotic. Hence it is also Li-Yorke chaotic, because Devaney chaos is stronger than Li-Yorke chaos.

The property in Definition 2.8 is very important for any dynamical system, because this property is mainly based on Li-Yorke pair but has some common features with sensitive dependence on initial conditions. Hence we can say that the complemented shift map has property which is very similar to Li-Yorke pair but also has some common features with sensitivity.

In this paper we have proved that the complemented shift map is totally transitive on Σ_2. So a question arises, are all topologically transitive maps totally transitive? The following example gives a suitable answer to this question.

Example 5.1 Let $f(x)$ be a continuous map from $[0,1]$ onto itself defined by

$$f(x)=\begin{cases} x+\frac{2}{3}, & 0\le x\le\frac{1}{3} \\ -x+\frac{4}{3}, & \frac{1}{3}\le x\le\frac{2}{3} \\ -2x+2, & \frac{2}{3}\le x\le 1 \end{cases}$$

It can be easily proved that the map f is topologically transitive on [0,1]. On the other hand it is not totally transitive, since the subintervals $[0,\frac{2}{3}]$ and $[\frac{2}{3},1]$ are invariant under f^2, so f^2 is not topologically transitive on [0,1]. Hence not all topologically transitive maps are totally transitive.

The complemented shift map $\sigma':\Sigma_2\to\Sigma_2$ is chaotic in the sense of Li-Yorke, because it is chaotic in the sense of Devaney. It can also be directly proved by the scrambled set S consisting of points Γ_α, where Γ_α is defined by, $\Gamma_\alpha=\alpha_0 01\alpha_0\alpha_0\alpha_1\alpha_1 0011\alpha_0\alpha_0\alpha_0\alpha_1\alpha_1\alpha_1\alpha_2\alpha_2\alpha_2 000111.......................$, for all $\alpha=(\alpha_0\alpha_1.........)$ in Σ_2. So our constructed map is a strong chaotic map in the symbol space Σ_2. Since the shift map is often used to model the chaoticity of a dynamical system, we can now use the complemented shift map in place of the shift map to model the chaoticity of a dynamical system. So we conclude that the complemented shift map is a new model for chaotic dynamical systems.

Acknowledgements:
Indranil Bhaumik acknowledges his father Mr. Sadhan Chandra Bhaumik for his help in preparing the manuscript.

REFERENCES

1. I. Bhaumik and B. S. Choudhury, Dynamics of the generalized shift map, Bull. Cal. Math. Soc., 101(5), 2009, 463-470.
2. I. Bhaumik and B. S. Choudhury, Topologically conjugate maps and ω-chaos in symbol space, Int. J. Appl. Math., 23(2), 2010, 309-321.
3. F. Blanchard, E. Glasner, S. Kolyada and A. Maass, On Li – Yorke pairs, J. Rei. Ange. Math., 547, 2002, 51-68.
4. R. L. Devaney, An introduction to chaotic dynamical systems, 2nd edition, Addison –Wesley, Redwood City, CA, 1989.
5. B. S. Du, On the nature of chaos, arXiv:math.DS 0602585, February 2006.

6. X. C. Fu, W. Lu, P. Ashwin and J. Duan, Symbolic representations of iterated maps, Topo. Meth. Non. Anal., 18, 2001, 119-147.
7. B. P. Kitchens, Symbolic dynamics-one sided, two sided and countable state Markov shifts, Universitext, Springer-Verlag, Berlin, 1998.
8. S. Li, ω-chaos and topological entropy, Tran. Amer. Math. Soc., 339, 1993, 243-249.
9. T. Y. Li and J. A. Yorke, Period three implies chaos, Amer. Math. Mon., 82, 1975, 985-992.
10. W. Parry, Symbolic dynamics and transformation of the unit interval, Tran. Amer. Math. Soc., 122 (2), 1966, 368-378.
11. C. Robinson, Dynamical system: Stability, symbolic dynamics and chaos, 2nd edition, CRC press, Boca Raton, FL., 1999.
12. S. Ruette, Chaos for continuous interval map, www.math.u-psud.fr/ruette/, December 15, 2003.