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Estimations of components' life-parameters under multiple type-II censoring using masked data

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ABSTRACT

Based on masked data and multiple type-II censoring model, the likelihood function of the samples is investigated using probit analysis method. The maximum likelihood method is used to obtain the estimations. Meanwhile, two numerical simulation examples are given to illustrate how one can utilize the theoretical results to tackle the practical problem and study the influence of the observed number and masking level on the accuracy of the estimations.

Keywords masked data; multiple type-II censoring; probit analysis method; Maximum likelihood estimation; numerical simulation

1. Introduction.

Life data from multi-components systems are often analyzed to estimate the reliability of each component. These estimations are extremely useful since they can reflect components' reliability after the components are assembled into an operational system. Under appropriate conditions, these estimations can be used to predict the reliability of new configuration of components.

Consider a system of components connected in series. Due to some certain environmental conditions, the exact component which causes the system failure might be unknown. Instead, it is assumed that it belongs to some subset of the components which is considered potentially responsible for the failure. In this case, the cause of failure is masked.

Various studies have used masked data to estimate the unknown life-parameters and reliabilities of the system components. For example, [1] and [2] considered the maximum likelihood estimations (MLE) of exponential components, and [3] deduced the Bayes estimations. [4] presented MLE of unknown parameters of Weibull components for the cases of two-component and three-component series systems. Lynn Kuo and Tae Young Yang[5] obtained the Bayes estimations of Weibull components. For geometric components and pareto components, A.M. Sarhan considered the estimations of parameters in [6] and [7] respectively. [8] discussed the case of parallel systems of complementary exponential components.

In those studies, most of authors made the assumption that the test to be terminated when all the systems failed and the available sample is not censored. However, in some experiments and data collecting process, the failure of some systems may not be observed due to restrictions on data collection, experimental difficulties or some other extraordinary reasons. Using type-II censoring sample, the reliability estimations of geometric components[9] and Weibull components[10] have been presented. Here we consider a kind of censoring type named multiple type-II censoring, which may arise in practice, such as, in life tests when the failure times of some systems were not observed due to mechanical difficulties and so on. Another situation which multiple type-II censoring arise naturally is that some systems failed between two points of observation with exact failure times of these systems unobserved.

In this paper, masked life data from multiple type-II censoring test was used to derive the maximum likelihood estimations of components' life-parameters. In section 2, a general likelihood function of samples for J -components series system is developed. In section 3, we explore its use for the special case when components' lifetimes are exponentially distributed. In section 4, several numerical examples were given to illustrate the use of the method.

2. Likelihood Function.

Suppose n systems placed on the life test and the test terminates when all systems fail. Each of these systems is assumed to be a series system of J independent components. Let $T_i (i = 1, 2, \cdots, n)$ denote the lifetime of system i and $T_{ij} (j = 1, 2, \cdots, J)$ denote the lifetime of component j in system i. That is, $T_i = \min\{T_{i1}, \cdots, T_{iJ}\}$. For a fixed $j (j = 1, 2, \cdots, J)$, the random variables $T_{1j}, T_{2j}, \cdots, T_{nj}$ are independent and identically distributed. Let $f_j(t)$ and $\overline{F}_j(t)$ denote the probability density function (PDF) and survival function (SF) of the lifetime of component j with parameter vector θ_j.

Assume that we only observed the $r_1\text{th}, r_2\text{th}, \cdots, r_k\text{th}$ systems' failure times $y_1 = T_{r_1}, y_2 = T_{r_2}, \cdots, y_k = T_{r_k}$. Suppose these data satisfy $y_1 < y_2 < \cdots < y_k$. In addition, for each of these systems, we observed set of labels of the components that include the exact component which causes the system failure. Let $S_{r_i} (i = 1, \cdots, k)$

denote the set of components that may cause the r_ith system failure. Thus, the observable quantity for the r_ith system is (T_{r_i}, S_{r_i}). Thus, the multiple type-II censored samples are $(y_1, S_{r_1}), (y_2, S_{r_2}), \cdots, (y_k, S_{r_k})$. It is also assumed that masking occurs independently of the cause of failure.

Next we use probit analysis method to derive the likelihood function. Note that

$$(-\infty,+\infty)=(-\infty,y_1)\cup[y_1,y_1+\mathrm{d}y_1)\cup[y_1+\mathrm{d}y_1,y_2)\cup\cdots\cup[y_k,y_k+\mathrm{d}y_k)\cup[y_k+\mathrm{d}y_k,+\infty)$$

We divide the number axis into four parts using the multiple type-II censored data:

$$\text{(I)}\ (-\infty,y_1)\ ;\ \text{(II)}\bigcup_{j=1}^{k}[y_j,y_j+\mathrm{d}y_j)\ ;\ \text{(III)}\bigcup_{j=1}^{k-1}[y_j+\mathrm{d}y_j,y_{j+1})\ ;\ \text{(IV)}\ [y_k+\mathrm{d}y_k,+\infty)$$

In the following, we will analyze each part to derive the likelihood function.

(I)There are r_1-1 systems failing in this part. Suppose that the ith system failed in this part, that is, $T_i < y_1 = T_{r_1}$. The probability of this event is:

$$\begin{aligned}P(T_i<y_1)&=1-P(T_i\ge y_1)=1-P(\min(T_{i1},T_{i2},\cdots,T_{iJ})\ge y_1)\\&=1-P(T_{i1}\ge y_1,T_{i2}\ge y_1,\cdots,T_{iJ}\ge y_1)=1-\prod_{j=1}^{J}\bar{F}_j(y_1;\theta_j)\end{aligned}$$

So the total probability in this part is

$$I_1=[1-\prod_{j=1}^{J}\bar{F}_j(y_1;\theta_j)]^{r_1-1} \tag{1}$$

(II) There are k systems failing in this part, and the r_jth$(j=1,\cdots,k)$ system failed in $[y_j, y_j+\mathrm{d}y_j)$, the probability of this event is:

$$\begin{aligned}P(y_j\le T_{r_j}<y_j+\mathrm{d}y_j)&=\sum_{m\in S_{r_j}}P(y_j\le T_{r_jm}<y_j+\mathrm{d}y_j,T_{r_j1}>y_j,\cdots,T_{r_j(m-1)}>y_j,\\&\qquad T_{r_j(m+1)}>y_j,\cdots,T_{r_jJ}>y_j)\\&\approx\sum_{m\in S_{r_j}}f_m(y_j;\theta_m)\prod_{l\in J_m}\bar{F}_l(y_j;\theta_l)\end{aligned}$$

Where $J_m=1,2,\cdots,m-1,m+1,\cdots,J.$

Thus, the total probability in part (II) is

$$I_2=\prod_{j=1}^{k}[\sum_{m\in S_{r_j}}f_m(y_j;\theta_m)\prod_{l\in J_m}\bar{F}_l(y_j;\theta_l)] \tag{2}$$

(III) Consider the interval $[y_j+\mathrm{d}y_j, y_{j+1})(j=1,\cdots,k-1)$, and the number of systems which failed in this interval is $r_{j+1}-r_j-1 \triangleq m_j$. Suppose that the ith system failed in this part, and then we get the following equations:

$$\begin{aligned}P(y_j+\mathrm{d}y_j\le T_i<y_{j+1})&=P(T_i<y_{j+1})-P(T_i<y_j+\mathrm{d}y_j)=P(T_i\ge y_j+\mathrm{d}y_j)-P(T_i\ge y_{j+1})\\&=P(\min(T_{i1},T_{i2},\cdots,T_{iJ})\ge y_j+\mathrm{d}y_j)-P(\min(T_{i1},T_{i2},\cdots,T_{iJ})\ge y_{j+1})\\&=\prod_{l=1}^{J}\bar{F}_l(y_j+\mathrm{d}y_j;\theta_l)-\prod_{l=1}^{J}\bar{F}_l(y_{j+1};\theta_l)\approx\prod_{l=1}^{J}\bar{F}_l(y_j;\theta_l)-\prod_{l=1}^{J}\bar{F}_l(y_{j+1};\theta_l)\end{aligned}$$

So, the total probability in the interval $[y_j+\mathrm{d}y_j, y_{j+1})(j=1,\cdots,k-1)$ is

$$[\prod_{l=1}^{J}\overline{F}_l(y_j;\theta_l)-\prod_{l=1}^{J}\overline{F}_l(y_{j+1};\theta_l)]^{m_j}$$

Thus, the total probability in part (III) is

$$I_3=\prod_{j=1}^{k-1}[\prod_{l=1}^{J}\overline{F}_l(y_j;\theta_l)-\prod_{l=1}^{J}\overline{F}_l(y_{j+1};\theta_l)]^{m_j} \tag{3}$$

(IV) There are $n-r_k$ systems failing in this part. We also suppose that the ith system failed in this part, and then get the following probability:

$$P(T_i>y_k+\mathrm{d}y_k)=P(\min(T_{i1},T_{i2},\cdots,T_{iJ})>y_k+\mathrm{d}y_k)\approx\prod_{l=1}^{J}\overline{F}_l(y_k;\theta_l)$$

So the total probability in part (IV) is

$$I_4=[\prod_{l=1}^{J}\overline{F}_l(y_k;\theta_l)]^{n-r_k} \tag{4}$$

According to (1)-(4), we can easily get the likelihood function. It takes the following form:

$$\begin{aligned}L=C[1-\prod_{j=1}^{J}\overline{F}_j(y_1;\theta_j)]^{r_1-1}\prod_{j=1}^{k}[\sum_{m\in S_{r_j}}f_m(y_j;\theta_m)\prod_{l\in J_m}\overline{F}_l(y_j;\theta_l)]\bullet\\ \prod_{j=1}^{k-1}[\prod_{l=1}^{J}\overline{F}_l(y_j;\theta_l)-\prod_{l=1}^{J}\overline{F}_l(y_{j+1};\theta_l)]^{m_j}[\prod_{l=1}^{J}\overline{F}_l(y_k;\theta_l)]^{n-r_k}\end{aligned} \tag{5}$$

Where C is a constant.

Then the maximum likelihood estimation (MLE) of $\theta_i(i=1,2,\cdots,J)$ can be obtained by solving the likelihood equation $\dfrac{\partial\log L}{\partial\theta_i}=0$.

3. The exponential case.

When the lifetime of each component is exponential distribution with parameter $\lambda_j(j=1,2,\cdots,J)$, the likelihood function from (5) takes the following form:

$$\begin{aligned}L&=C[1-\prod_{j=1}^{J}\mathrm{e}^{-\lambda_j y_1}]^{r_1-1}\prod_{j=1}^{k}[\sum_{m\in S_{r_j}}\lambda_m\mathrm{e}^{-\lambda_m y_j}\prod_{l\in J_m}\mathrm{e}^{-\lambda_l y_j}]\prod_{j=1}^{k-1}[\prod_{l=1}^{J}\mathrm{e}^{-\lambda_l y_j}-\prod_{l=1}^{J}\mathrm{e}^{-\lambda_l y_{j+1}}]^{m_j}[\prod_{l=1}^{J}\mathrm{e}^{-\lambda_l y_k}]^{n-r_k}\\&=C[1-\mathrm{e}^{-ay_1}]^{r_1-1}\prod_{j=1}^{k}[\mathrm{e}^{-ay_j}\sum_{m\in S_{r_j}}\lambda_m]\prod_{j=1}^{k-1}[\mathrm{e}^{-ay_j}-\mathrm{e}^{-ay_{j+1}}]^{m_j}[\mathrm{e}^{-ay_k}]^{n-r_k}\end{aligned}$$

Where $a=\sum_{i=1}^{J}\lambda_i$.

Then the log-likelihood function is:

$$\begin{aligned}\log L=\log C+(r_1-1)\log(1-\mathrm{e}^{-ay_1})-a\sum_{i=1}^{k}y_i-a(n-r_k)y_k+\\ \sum_{j=1}^{k}\log(\sum_{m\in S_{r_j}}\lambda_m)+\sum_{j=1}^{k-1}m_j\log(\mathrm{e}^{-ay_j}-\mathrm{e}^{-ay_{j+1}})\end{aligned} \tag{6}$$

Further, the likelihood equations are:

$$\frac{\partial\log L}{\partial\lambda_i}=\frac{(r_1-1)y_1\mathrm{e}^{-ay_1}}{1-\mathrm{e}^{-ay_1}}-\sum_{i=1}^{k}y_i-(n-r_k)y_k+\sum_{j=1}^{k}\frac{\varepsilon_{ij}}{\sum_{m\in S_{r_j}}\lambda_m}+\sum_{j=1}^{k-1}m_j\frac{y_{j+1}\mathrm{e}^{-ay_{j+1}}-y_j\mathrm{e}^{-ay_j}}{\mathrm{e}^{-ay_j}-\mathrm{e}^{-ay_{j+1}}}=0 \tag{7}$$

Where $\varepsilon_{ij} = \begin{cases} 1 & i \in S_{r_j} \\ 0 & else \end{cases}$.

Consider the simplest case of $J = 2$. Let n_1 and n_2 be the number of the observed system failures when the cause of failure is known to be component 1 and 2 respectively, i.e. n_1 and n_2 are the number of observations when $S_{r_i} = \{1\}$ and $S_{r_i} = \{2\}$. Let n_{12} denote the number of masked observations when the failure is obtained.

Let $y_0 = 0, r_0 = 0$, then $m_0 = r_1 - r_0 - 1 = r_1 - 1$. From equations (6), the following equations were obtained:

$$\begin{cases} \dfrac{\partial \log L}{\partial \lambda_1} = -\sum\limits_{i=1}^{k} y_i - (n - r_k) y_k + \dfrac{n_1}{\lambda_1} + \dfrac{n_{12}}{\lambda_1 + \lambda_2} + \sum\limits_{j=0}^{k-1} m_j \dfrac{y_{j+1} e^{-ay_{j+1}} - y_j e^{-ay_j}}{e^{-ay_j} - e^{-ay_{j+1}}} = 0 \\ \dfrac{\partial \log L}{\partial \lambda_2} = -\sum\limits_{i=1}^{k} y_i - (n - r_k) y_k + \dfrac{n_2}{\lambda_2} + \dfrac{n_{12}}{\lambda_1 + \lambda_2} + \sum\limits_{j=0}^{k-1} m_j \dfrac{y_{j+1} e^{-ay_{j+1}} - y_j e^{-ay_j}}{e^{-ay_j} - e^{-ay_{j+1}}} = 0 \end{cases}$$

Two methods were used to deduce the approximate maximum likelihood estimation of $\lambda_j (j = 1, 2)$.

Method 1: Note that $n_1 + n_2 + n_{12} = k$ and $\dfrac{n_1}{\lambda_1} = \dfrac{n_2}{\lambda_2} \triangleq b$, we can obtain that b satisfies the following equation:

$$-\sum_{i=1}^{k} y_i - (n - r_k) y_k + \frac{kb}{n_1 + n_2} + \sum_{j=0}^{k-1} m_j \frac{y_{j+1} e^{-(n_1+n_2) y_{j+1}/b} - y_j e^{-(n_1+n_2) y_j/b}}{e^{-(n_1+n_2) y_j/b} - e^{-(n_1+n_2) y_{j+1}/b}} = 0 \tag{8}$$

The solution of the equation above can be got by numerical methods, then the MLE of $\lambda_j (j = 1, 2)$ can be easily obtained by $\lambda_j = \dfrac{n_j}{b} (j = 1, 2)$.

Method 2: We can use Taylor expansion to get the approximate MLE. From [11], we have

$$\frac{y_{j+1} e^{-ay_{j+1}} - y_j e^{-ay_j}}{e^{-ay_j} - e^{-ay_{j+1}}} \approx -\frac{1}{a} [\gamma_j + a\delta_j y_j + a(1 - \delta_j) y_{j+1}]$$

Where $p_j = \dfrac{r_j}{n+1}$, $q_j = 1 - p_j$, $\gamma_j = \dfrac{q_{j+1} \ln q_{j+1} - q_j \ln q_j}{q_j - q_{j+1}} + \delta_j \ln q_j - (1 - \delta_j) \ln q_{j+1}$,

$\delta_j = \dfrac{q_j}{q_j - q_{j+1}} - \dfrac{q_j q_{j+1}}{(q_j - q_{j+1})^2} \ln(\dfrac{q_j}{q_{j+1}})$.

Note that $a = \lambda_1 + \lambda_2$, the maximum likelihood equations can be written as:

$$\begin{cases} \dfrac{\partial \log L}{\partial \lambda_1} = -Y + \dfrac{n_1}{\lambda_1} + (n_{12} - \sum\limits_{j=0}^{k-1} m_j \gamma_j) \dfrac{1}{\lambda_1 + \lambda_2} = 0 \\ \dfrac{\partial \log L}{\partial \lambda_2} = -Y + \dfrac{n_2}{\lambda_2} + (n_{12} - \sum\limits_{j=0}^{k-1} m_j \gamma_j) \dfrac{1}{\lambda_1 + \lambda_2} = 0 \end{cases}$$

Where $Y = \sum_{j=0}^{k-1} m_j[\delta_j y_j + (1-\delta_j) y_{j+1}] + \sum_{j=1}^{k} y_j + (n - r_k) y_k$.

Solve the equations above and the approximate MLE of $\lambda_j (j = 1,2)$ can be obtained:

$$\hat{\lambda}_j = [n_j + (n_{12} - \sum_{j=0}^{k-1} m_j\gamma_j)\frac{n_j}{n_1 + n_2}] / Y = n_j (k - \sum_{j=0}^{k-1} m_j\gamma_j) / [Y(n_1 + n_2)] (j = 1,2)$$

4. Simulation study.

In this section, two numerical examples will be presented to show how one can apply the previous theoretical results obtained. It is assumed that n systems are put on the life test, and every system is a series system of two independent components. The lifetimes of the components are exponential distributions with parameters $\lambda_1 = 1$ and $\lambda_2 = 0.9$ respectively. In these examples, there are only k systems' lifetimes and the corresponding sets of components which may cause the system failure are observed.

Example 4.1. In this example, we generated a random sample with size $n = 30$ from the model above. The masking level is $l = 30\%$. The simulated data was presented in table 1. Then the data was used to calculate: (i) the approximate MLE of λ_1 and λ_2; (ii) the percentage errors associated with the estimations obtained. The percentage error associated with the estimation of $\lambda_j (j = 1,2)$, say $PE_{\hat{\lambda}_j}$, is given by the following formula:

$$PE_{\hat{\lambda}_j} = \frac{|\text{exact value of } \lambda_j - \text{estimated value of } \lambda_j|}{\text{exact value of } \lambda_j} \times 100\%$$

Note in table 1 that i denotes to the system number, t_i denotes the failure time of the system i, and S_i denotes the set of components which may cause the system i failure. $t_i =$ '$-$' and $S_i =$ '$-$' means the failure time and the corresponding possible failure reasons of the system i are not observed.

Table 1. The simulated data for example 1

i	t_i	S_i	i	t_i	S_i	i	t_i	S_i
1	0.0390	{1,2}	11	0.2426	{2}	21	0.6700	{2}
2	0.0572	{1}	12	—	—	22	—	—
3	—	—	13	0.3029	{1}	23	0.8035	{2}
4	0.1305	{2}	14	0.3084	{1}	24	—	—
5	0.1367	{1}	15	0.3098	{1,2}	25	1.0809	{1,2}
6	0.1477	{1}	16	0.3217	{2}	26	1.1033	{1}
7	—	—	17	0.4301	{1,2}	27	1.1619	{1}
8	0.1792	{1}	18	—	—	28	1.2308	{1,2}
9	0.2182	{1,2}	19	—	—	29	—	—
10	—	—	20	0.5083	{2}	30	—	—

Based on the simulated data, we got that $k = 20$, $n_1 = 8$, $n_2 = 6$ and $n_{12} = 6$. Then the MLE of the parameters and the percentage errors associated with the estimations were computed. After taking the data to the theoretical results in section 3, we got the point estimations $\hat{\lambda}_{Ij}$ by using method 1 and $\hat{\lambda}_{Ej}$ by using method 2. The results were $\hat{\lambda}_{I1} = 1.0523$, $\hat{\lambda}_{I2} = 0.7892$, $\hat{\lambda}_{E1} = 0.8257$, $\hat{\lambda}_{E2} = 0.6193$, and the corresponding percentage errors were $PE_{\hat{\lambda}_{I1}} = 0.0523$, $PE_{\hat{\lambda}_{I2}} = 0.1231$, $PE_{\hat{\lambda}_{T1}} = 0.1743$, $PE_{\hat{\lambda}_{T2}} = 0.3119$.

Example 4.2. In a practical experiment, the value of k may either be a fixed number or a random number. In order to illustrate the influence of k on the accuracy of the estimations, k is assumed to be a fixed number.

In this example, there were three test schemes:(I) $n = 50, k = 30$;(II) $n = 50, k = 35$; (III) $n = 50, k = 40$. For every scheme, the censored sample was simulated using masked data under $l = 20\%$, $l = 40\%$ and $l = 60\%$ respectively. For each particular choice of n , k and l , 1000 replicas of a random sample were generated. The MLE $\hat{\lambda}_j^{(i)} (j = 1,2; i = 1,\cdots,1000)$ was found for replica i . And then the mean squared errors (MSE) of these estimations were computed from the sample of 1000 replicas. The MSE associated with the estimation $\hat{\lambda}_j$, say $MSE_{\hat{\lambda}_j}$, is given by $MSE_{\hat{\lambda}_j} = \sum_{i=1}^{1000} (\lambda_j - \hat{\lambda}_j^{(i)})^2 / 1000$. The results were presented in table 2.

Table 2. The mean squared error of the estimations

MSE	$l = 20\%$			$l = 40\%$			$l = 60\%$		
	(I)	(II)	(III)	(I)	(II)	(III)	(I)	(II)	(III)
$MSE_{\hat{\lambda}_{I1}}$	0.0619	0.0579	0.0533	0.0784	0.0671	0.0617	0.0861	0.0826	0.0828
$MSE_{\hat{\lambda}_{I2}}$	0.0569	0.0504	0.0468	0.0771	0.0661	0.0570	0.1076	0.0807	0.0756
$MSE_{\hat{\lambda}_{E1}}$	0.1320	0.0944	0.0595	0.1434	0.0997	0.0684	0.1605	0.1181	0.0899
$MSE_{\hat{\lambda}_{E2}}$	0.1159	0.0738	0.0543	0.1185	0.0831	0.0597	0.1188	0.0970	0.0874

According to the results shown in table 2, one can conclude that:

(I) For the same scheme, in other words, for a given sample size n and a given k , the MSE associated with the estimations increase with increasing the masking level l .

(II) For a given sample size n and the masking level l , the MSE associated with the estimations decrease with increasing k .

(III) The MSE associated with $\hat{\lambda}_{Ij} (j = 1,2)$ is always smaller than that associated with $\hat{\lambda}_{Ej}$. Thus, $\hat{\lambda}_{Ij}$ has a higher accuracy than $\hat{\lambda}_{Ej}$.

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