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Fuzzy Normality and Some of its α-Forms

Md. Sahadat Hossain and Dewan Muslim Ali

Department of mathematics
Rajshahi University, Rajshahi-6205
Bangladesh
e-mail: sahadat@ru.ac.bd

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ABSTRACT

In this paper, we introduce and study three classes of FN - fuzzy topological spaces and we establish some relationships among them. We also study some other properties of these spaces and obtain their several features.

Keywords: Fuzzy topological space, Normal fuzzy topological space, good extension, continuous function, one-one, onto, open, closed map. lower semi continuous function.
AMS subject classification: 54A40

1. Introduction

The concepts of fuzzy sets introduced by American Mathematician Zadesh[1] first time in 1965. Chang[2] and Lowen[3] developed the theory of fuzzy topological spaces based on Zadesh's concept. From then a number of research papers have been published dealing with various aspects of such spaces. Fuzzy normal topological spaces have been introduced earlier by Hutton, Sinha , Srivastava[4] et al. In this paper we defined three definitions of normal fuzzy topological spaces while all these three definitions are 'good extensions' of their counter parts in topological spaces. Here we have proved that Normal fuzzy topological spaces are hereditary, productive and projective.

1.1. Definition[1] :- A function u from a set X into the unit interval I is called a fuzzy set in X. For every $x \in X$, $u(x) \in I$ is called the grade of membership (g.m.f) of x in u . Some authors say that u is a fuzzy subset of X instead of saying that u is a fuzzy set in X.

1.2. Definition[2] :- Let f be a mapping from a set X into a set Y and u be a fuzzy subset of X. Then f and u induced a fuzzy subset v of Y defined by

$$v(y) = \sup \{ u(x) \} \text{ if } f^{-1}[\{y\}] \neq \phi , \ x \in X$$

$= 0$ otherwise .

1.3. Definition[2] :- Let f be a mapping from a set X into Y and v be a fuzzy subset of Y. Then the inverse of v written as $f^{-1}(v)$ is a fuzzy subset of X and is defined by

$$f^{-1}(v)(x) = v(f(x)), \text{ for } x \in X.$$

1.4. Definition[2] :- Let $I = [0, 1]$, X be a non empty set , and I^X be the collection of all mappings from X into I , ie the class of all fuzzy sets in X . A fuzzy topology on X is defined as a family t of members of I^X, satisfying the following conditions.

(i) $1, 0 \in t$,
(ii) If $u_i \in t$ for each $i \in \Lambda$, then $\cup_{i \in \Lambda} u_i \in t$.
(iii) If $u_1, u_2 \in t$ then $u_1 \cap u_2 \in t$.

The pair (X , t) is called a fuzzy topological space (fts, in short) and members of t are called t- open (or simply open) fuzzy sets . A fuzzy set v is called a t- closed (or closed) fuzzy set if $1 - v \in t$.

1.5. Definition[3] :- (According to lowen) A fuzzy topology on a non empty set X is a collection t of fuzzy subsets of X such that
(i) all constant fuzzy subsets of X belong to t .
(ii) t is closed under formation of fuzzy union of arbitrary collection of members of t .
(iii) t is closed under formation of intersection of finite collection of members of t.

1.6. Definition[4] :- The function $f : (X, t) \longrightarrow (Y, s)$ is called fuzzy continuous if and only if for every $v \in s$, $f^{-1}(v) \in t$, the function f is called fuzzy homeomorphic if and only if f is bijective and both f and f^{-1} are fuzzy continuous.

1.7. Definition[4] :- The function $f : (X, t) \longrightarrow (Y, s)$ is called fuzzy open if and only if for each open fuzzy set u in (X , t) , f (u) is a open fuzzy set in (Y , s) .

1.8. Definition[4] :- The function $f : (X, t) \longrightarrow (Y, s)$ is called fuzzy closed if and only if for each closed fuzzy set u in (X , t) , f(u) is a closed fuzzy set in (Y , s).

1.9. Proposition[4] :- Let $f : (X, t) \longrightarrow (Y, s)$ be a fuzzy continuous function , then for every s – closed v , $f^{-1}(v)$ is t – closed.

1.10. Definition[5] :- If u_1 and u_2 are two fuzzy subsets of X and Y respectively then the Cartesian product $u_1 \times u_2$ of two fuzzy subsets u_1 and u_2 is a fuzzy subsets of $X \times Y$ defined by $(u_1 \times u_2)(x, y) = \min(u_1(x), u_2(y))$, for each pair $(x, y) \in X \times Y$.

1.11. Definition[7] :- Let f be a real valued function on a topological space. If $\{x : f(x) > \alpha\}$ is open for every real α, then f is called a lower semi continuous function.

1.12. Definition[8] :- Let $\{X_i, i \in \Lambda\}$, be any class of sets and let X denoted the Cartesian product of these sets, ie $X = \Pi_{i\in\Lambda} X_i$. Note that X consists of all points $p = \langle a_i, i \in \Lambda \rangle$, where $a_i \in X_i$. Recall that, for each $j_o \in \Lambda$, we define the projection π_{jo} from the product set X to the coordinate space X_{jo}. ie $\pi_{jo} : X \longrightarrow X_{jo}$ by $\pi_{jo}(\langle a_i : i \in \Lambda \rangle) = a_{jo}$,

These projections are used to define the product topology.

1.12. Definition[5] :- Let $\{X_\alpha\}_{\alpha\in\Lambda}$ be a family of nonempty sets. Let $X = \Pi_{\alpha\in\Lambda} X_\alpha$ be the usual product of X_α's and let π_α be the projection from X into X_α. Further assume that each X_α is an fts with fuzzy topology t_α. Now the fuzzy topology generated by $\{\pi_\alpha^{-1}(b_\alpha) : b_\alpha \in t_\alpha, \alpha \in \Lambda\}$ as a subbasis, is called the product fuzzy topology on X. Clearly if w is a basis element in the product, then there exist $\alpha_1, \alpha_2, \ldots \alpha_n \in \Lambda$ such that $w(x) = \min\{b_\alpha(x_\alpha) : \alpha = 1, 2, 3, \ldots n\}$, where $x = (x_\alpha)_{\alpha\in\Lambda} \in X$.

1.14. Definition[3] :- Let X be a nonempty set and T be a topology on X. Let $t = \omega(T)$ be the set of all lower semi continuous (lsc) functions from(X , T) to I (with usual topology). Thus $\omega(T) = \{u \in I^X : u^{-1}(\alpha, 1] \in T\}$ for each $\alpha \in I_1$. It can be shown that $\omega(T)$ is a fuzzy topology on X.

Let P be the property of a topological space (X , T) and FP be its fuzzy topological analogue. Then FP is called a 'good extension' of P " iff the statement (X , T) has P iff $(X, \omega(T))$ has FP " holds good for every topological space (X , T).

2. Normal fuzzy topological spaces.

2.1. Definition :- Let (X , t) be a fuzzy topological space and $\alpha \in I_1 = [0, 1)$.

(a) (X , t) is α - FN(i) space $\Leftrightarrow$ $\forall\, w, w^* \in t^c$, with $w \cap w^* \le \alpha$, $\exists\, u, u^* \in t$ such that

$$u(x) = 1,\ \forall x \in w^{-1}\{1\},\ u^*(y) = 1,\ \forall y \in w^{*-1}\{1\} \text{ and } u \cap u^* = 0.$$

(b) (X , t) is α - FN(ii) space $\Leftrightarrow$ $\forall\, w, w^* \in t^c$ with $w \cap w^* = 0$, $\exists\, u, u^* \in t$ such that

$u(x) = 1$, $\forall\, x \in w^{-1}\{1\}$, $u^*(y) = 1$, $\forall\, y \in w^{*\,-1}\{1\}$ and $u \cap u^* \leq \alpha$.

(c) (X , t) is a α - FN(iii) space $\Leftrightarrow \forall\, w , w^* \in t^c$ with $w \cap w^* \leq \alpha$, $\exists\, u , u^*$, $\in t$ such that $u(x) = 1$, $\forall\, x \in w^{-1}\{1\}$, $u^*(y) = 1$, $\forall\, y \in w^{*-1}\{1\}$ and $u \cap u^* \leq \alpha$.

2.2. Theorem :- The following implications are true:
(X , t) is 0 - FN(i) $\Leftrightarrow$ (X , t) is 0 - FN(ii) $\Leftrightarrow$ (X , t) is 0 -FN(iii) .

Proof:- Suppose (X , t) be 0 -FN(i). We shall prove that (X , t) is 0 -FN(ii). Let $w , w^* \in t^c$ with $w \cap w^* = 0$. This implies that $w \cap w^* \leq 0$. Since (X , t) is 0 - FN(i), for $\alpha \in I_1$, $\exists\, u , u^* \in t$ such that $u(x) = 1$, $\forall\, x \in w^{-1}\{1\}$, $u^*(y) = 1$, $\forall\, y \in w^{*\,-1}\{1\}$ and $u \cap u^* = 0$. It can be written as $u \cap u^* \leq 0$. Hence it is clear that (X , t) is α - FN(ii).

Conversely, suppose that (X , t) is 0 - FN(ii). We shall prove that (X , t) is 0- FN(i). Let $w , w^* \in t^c$ with $w \cap w^* \leq 0$, ie $w \cap w^* = 0$. Since (X , t) is 0 –FN(ii) , $\exists\, u , u^* \in t$ such that $u(x) = 1$, $\forall\, x \in w^{-1}\{1\}$, $u^*(y) = 1$, $\forall\, y \in w^{*\,-1}$, and $u \cap u^* \leq 0$, ie $u \cap u^* = 0$. Hence it is clear that (X , t) is 0 – FN(i) .

Next, suppose that (X , t) is 0 – FN(ii) . We shall prove that (X , t) is 0 – FN(iii). Let $w , w^* \in t^c$ with $w \cap w^* \leq 0$, ie $w \cap w^* = 0$. Since (X , t) is 0 – FN(ii), $\exists\, u , u^* \in t$ such that $u(x) = 1$, $\forall\, x \in w^{-1}\{1\}$, $u^*(y) = 1$,$\forall\, y \in w^{*\,-1}\{1\}$ and $u \cap u^* \leq 0$. Hence it is clear that (X , t) is α-FN(iii).

Conversely, suppose that (X , t) is 0 – FN(iii) . We shall prove that (X , t) is 0 – FN(ii). Let $w , w^* \in t^c$ with $w \cap w^* = 0$, ie $w \cap w^* \leq 0$. Since (X , t) is 0 – FN(iii) , $\exists\, u , u^* \in t$ such that $u(x) = 1$, $\forall\, x \in w^{-1}\{1\}$, $u^*(y) = 1$, $\forall\, y \in w^{*\,-1}\{1\}$ and $u \cap u^* \leq 0$. Hence it is clear that (X , t) is α-FN(ii).

2.3. Theorem :- If $0 \leq \alpha \leq \beta < 1$, then

(a) (X , t) is β - FN(i) $\Rightarrow$ (X , t) is α - FN(i) .

(b) (X , t) is α - FN(ii) $\Rightarrow$ (X , t) is β - FN(ii).

Proof :- First , suppose that (X , t) is β - FN(i) . We shall prove that (X , t) is α - FN(i) . Let $w , w^* \in t^c$ with $w \cap w^* \leq \alpha$. Since $0 \leq \alpha \leq \beta < 1$, then $w \cap w^* \leq \beta$. Since (X , t) is β - FN(i), for $\beta \in I_1$, $\exists\, u , u^* \in t$ such that $u(x) = 1$, $\forall\, x \in w^{-1}\{1\}$, $u^*(y) = 1$, $\forall\, y \in w^{*\,-1}\{1\}$ and $u \cap u^* = 0$. Hence it is clear that (X , t) is α - FN(i).

Next, suppose that (X , t) is α - FN(ii) . We shall prove that (X , t) is β - FN(ii). Let $w , w^* \in t^c$ with $w \cap w^* = 0$. Since (X , t) is α -FN(ii) , $\exists\, u , u^* \in t$ such that $u(x) = 1$, $x \in w^{-1}\{1\}$, $u^*(y) = 1$, $y \in w^{*\,-1}\{1\}$ and $u \cap u^* \leq \alpha$. Since $0 \leq \alpha \leq \beta <1$, then $u \cap u^* \leq \beta$. Hence it is clear that (X , t) is β - FN(ii) .

2.4. Theorem :- Let (X , t) be a fuzzy topological space , and $I_\alpha(t) = \{ u^{-1}(\alpha, 1] : u \in t \}$, then

(X , t) is 0 – FN(iii) space $\Rightarrow$ (X , $I_0(t)$) is Normal space .

Proof :- Suppose that (X , t) be a 0 - FN(iii) space . We shall prove that (X , $I_0(t)$) is Normal space . Let V , V^* be closed set in $I_0(t)$, and $V \cap V^* = \phi$. Then V^c , $V^{*c} \in I_0(t)$ and $(V \cap V^*)^c = V^c \cup V^{*c} = X$. Since V^c , $V^{*c} \in I_0(t)$, then , $\exists$ u , $u^* \in t$ such that $V^c = u^{-1}(0, 1]$ and $V^{*c} = u^{*-1}(0, 1]$ and $u^{-1}(0, 1] \cup u^{*-1}(0, 1] = V^c \cup V^{*c} = X$. Hence $(u \cup u^*)^{-1}(0, 1] = X$. Now we find u^c , $u^{*c} \in t^c$ such that $((u \cup u^*)^{-1}(0, 1])^c = \phi$. This implies that $(u \cup u^*)^c = u^c \cap u^{*c} = 0$. Since (X , t) is 0 - FN(iii) , $\exists$ v , $v^* \in t$ such that $v \geq 1_{(u^c)^{-1}\{1\}}$, $v^* \geq 1_{((u^*)^c)^{-1}\{1\}}$, $v \cap v^* = 0$. But from the definition of $I_0(t)$, $v^{-1}(0, 1]$, $v^{*-1}(0, 1] \in I_0(t)$, and we get $v^{-1}(0, 1] \supseteq 1_{(u^c)^{-1}\{1\}}(0, 1]$, $v^{*-1}(0, 1] \supseteq 1_{((u^*)^c)^{-1}\{1\}}(0, 1]$, $(v \cap v^*)^{-1}(0, 1] = \phi$. Put $W = v^{-1}(0, 1]$, $W^* = v^{*-1}(0, 1]$ in $I_0(t)$. Finally we find that , $W \supseteq V$, $W^* \supseteq V^*$ and $W \cap W^* = \phi$. Hence (X , $I_0(t)$) is a normal space.

2.5. Theorem :- Let (X , T) be a topological space consider the following statements:

(a) (X , T) be a Normal space

(b) (X , $\omega(T)$) be α - FN(i) space .

(c) (X , $\omega(T)$) be α - FN(ii) space .

(d) (X , $\omega(T)$) be α - FN(iii) space .

Then (a) $\Rightarrow$ (b) $\Rightarrow$ (c) $\Rightarrow$ (a) .

Proof :- First suppose that (X , T) is a Normal space . We shall prove that (X , $\omega(T)$) be α - FN(i) space . Let w , w^* be closed in $\omega(T)$ and $w \cap w^* \leq \alpha$. Then it is clear that w^c, $w^{*c} \in \omega(T)$ and $(w \cap w^*)^c \geq 1 - \alpha > 0$. But from the definition of $\omega(T)$, $(w^c)^{-1}(0, 1]$, $(w^{*c})^{-1}(0, 1] \in T$. Now we see that $((w \cap w^*)^c)^{-1}(0, 1] = X$, and we also see that $((w^c)^{-1}(0, 1])^c = w^{-1}\{1\}$ and $((w^{*c})^{-1}(0, 1])^c = (w^*)^{-1}\{1\}$ be closed in T. Now $(((w \cap w^*)^c)^{-1}(0, 1])^c = (w \cap w^*)^{-1}\{1\} = w^{-1}\{1\} \cap w^{*-1}\{1\} = \phi$. Since (X , t) is fuzzy Normal , then , $\exists$ V , $V^* \in T$ such that $V \supseteq w^{-1}\{1\}$, $V^* \supseteq w^{*-1}\{1\}$ and $V \cap V^* = \phi$. But from the definition of $\omega(T)$, 1_V , $1_{V^*} \in w(T)$, $1_V \geq 1_{w^{-1}\{1\}}$, $1_{V^*} \geq 1_{(w^*)^{-1}\{1\}}$ and $1_{V \cap V^*} = 0$.

Hence (X , $\omega(T)$) is α - FN(i) space .

It can easily be shown that (b) $\Rightarrow$ (c) $\Rightarrow$ (d) .

We , therefore prove that (d) $\Rightarrow$ (a).

Suppose, that (X , ω(T)) is α - FN(iii) . We shall prove that (X , T) is a normal space. Let $V , V^* \in T^c$ and $V \cap V^c = \phi$. Then it is clear that $1_V , 1_{V^*}$ be closed in ω(T) and $1_{V \cap V^*} = 0$. Since (X , ω(T)) is α - FN(iii) , $\exists$ u , $u^* \in$ ω(T) such that $u \geq 1_V$, $u^* \geq 1_{V^*}$ and $u \cap u^* \leq \alpha$. But from the definition of ω(T) , $u^{-1}(\alpha , 1]$, $u^{*-1}(\alpha , 1] \in T$ and $u^{-1}(\alpha , 1] \supseteq (1_V)^{-1}(\alpha , 1] = V$, $u^{*-1}(\alpha , 1] \supseteq (1_{V^*})^{-1}(\alpha , 1] = V^*$ and $u^{-1}(\alpha , 1] \cap u^{*-1}(\alpha , 1] = (u \cap u^*)^{-1}(\alpha , 1] = \phi$. Hence it is clear that (X , T) is a normal space .

Thus it is seen that α - FN(p) is a good extension of its topological counter part . (p = i , ii , iii)

2.6. Theorem :- Let (X , t) and (Y , s) be two fuzzy topological spaces and $f : X \longrightarrow Y$ be a continuous , one-one , onto and open map then ,

- **(a)** (X , t) is α -FN(i) $\Rightarrow$ (Y , s) is α - FN(i) .
- **(b)** (X , t) is α -FN(ii) $\Rightarrow$ (Y , s) is α - FN(ii) .
- **(c)** (X , t) is α -FN(iii) $\Rightarrow$ (Y , s) is α - FN(iii) .

Proof :- Suppose (X , t) be α - FN(i). We shall prove that (Y , s) is α -FN(i). Let $w , w^* \in s^c$ with $w \cap w^* \leq \alpha$ then $f^{-1}(w)$, $f^{-1}(w^*) \in t^c$ as f is continuous . Now $f^{-1}(w \cap w^*) \leq \alpha$ as $w \cap w^* \leq \alpha \Rightarrow f^{-1}(w) \cap f^{-1}(w^*) \leq \alpha$. Since (X , t) is α - FN(i) , for $\alpha \in I_1$, then $\exists$ u , $u^* \in$ t such that u(x) = 1 , $x \in (f^{-1}(w))^{-1}\{1\}$, $u^*(y) = 1$, $y \in (f^{-1}(w^*))^{-1}\{1\}$ and $u \cap u^* = 0$. This implies that f(u) , f(v) $\in$ s as f is open .
Now $f(u) (p) = \{ \text{Sup } u(x) \quad ; f(x) = p \}$, $f^{-1}(p) \in (f^{-1}(w))^{-1}\{1\}$
ie f(u) (p) = 1 , $p \in w^{-1}\{1\}$
and $f(u^*) (q) = \{ \text{Sup } u^*(y) \quad ; f(y) = q \}$, $f^{-1}(q) \in (f^{-1}(w))^{-1}\{1\}$
ie $f(u^*) (q) = 1$, $q \in w^{*-1}\{1\}$ and $f(u) \cap f(u^*) = f (u \cap u^*) = 0$ as $u \cap u^* = 0$.
Now it is clear that $\exists$ f(u) , $f(u^*) \in$ s such that f(u) (p) = 1 , $p \in w^{-1}\{1\}$, $f(u^*) (q) = 1$, $q \in w^{*-1}\{1\}$ and $f(u) \cap f(u^*) = 0$. Hence (Y , s) is α - FN(i) .
Similarly (b) and (c) can be proved.

2.7. Theorem :- Let (X , t) and (Y , s) be two fuzzy topological spaces and $f : X \longrightarrow Y$ be continuous , one-one , onto and closed map then ,

- **(a)** (Y , s) is α - FN(i) $\Rightarrow$ (X , t) is α - FN(i) .
- **(b)** (Y , s) is α - FN(ii) $\Rightarrow$ (X , t) is α - FN(ii) .
- **(c)** (Y , s) is α - FN(iii) $\Rightarrow$ (X , t) is α - FN(iii) .

Proof:- Suppose (Y , s) be α - FN(i) . We shall prove that (X , t) is α - FN(i) . Let $w , w^* \in t^c$ with $w \cap w^* \leq \alpha$, then f(w) , $f(w^*) \in s^c$ as f is closed and $f(w) \cap f(w^*) = f (w \cap w^*) \leq \alpha$, as $w \cap w^* \leq \alpha$. Since (Y , s) is α -FN(i) , for $\alpha \in I_1$, then $\exists$ u , $u^* \in$ s such that u(x) = 1 , $x \in (f(w))^{-1}\{1\}$, $u^*(y) = 1$, $y \in$

$(f(w^*))^{-1}\{1\}$ and $u \cap u^* = 0$. This implies that $f^{-1}(u)$, $f^{-1}(u^*) \in t$ as f is continuous and $u , u^* \in s$.
Now $f^{-1}(u)(p) = u(f(p)) = u(x) = 1$ as $f(p) = x \in (f(w))^{-1}\{1\}$
ie $f^{-1}(u)(p) = 1$, $p \in w^{-1}\{1\}$
$f^{-1}(u^*)(q) = u^*(f(q)) = u^*(y) = 1$ as $f(q) = y \in (f(w^*))^{-1}\{1\}$
ie $f^{-1}(u^*)(q) = 1$, $q \in w^{*-1}\{1\}$
and $f^{-1}(u) \cap f^{-1}(u^*) = f^{-1}(u \cap u^*) = 0$.
Now it is clear that $\exists f^{-1}(u) , f^{-1}(u^*) \in t$ such that $f^{-1}(u)(p) = 1, p \in w^{-1}\{1\}$, $f^{-1}(u^*)(q) = 1, q \in w^{*-1}\{1\}$ and $f^{-1}(u) \cap f^{-1}(u^*) = 0$. Hence (X , t) is α - FN(i).
Similarly (b) and (c) can be proved.

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