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Multi-objective Two Stage Fuzzy Transportation Problem

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ABSTRACT

This paper presents a two stage cost minimizing fuzzy transportation problem with multi-objective constraints. Fuzzy geometric programming approach is used to determine the optimal solution of a multi-objective two stage fuzzy transportation problem in which supplies, demands are trapezoidal fuzzy numbers and fuzzy membership of the objective function is defined. A numerical illustration is given to check the validity of the proposed method.

***Keywords**:* Transportation problem, Trapezoidal fuzzy numbers, Two stage fuzzy transportation problem ,Multi-objective.

1. Introduction

Transportation models provide a powerful framework to meet this challenge. They ensure the efficient movement and timely availability of raw materials and finished goods. Transportation problem is a linear programming problem stemmed from a network structure consisting of a finite number of nodes and arcs attached to them. In a typical problem a production is to be transported from m sources to n destinations and their capacities are $a_1, a_2, ... a_m$ and $b_1, b_{2,} ... b_n$ respectively. In addition there is a penalty C_{ij} associated with transporting unit of production from source i to destination j. This penalty may be cost or delivery time or safety of delivery etc. A variable X_{ij} represents the unknown quantity to be shipped from source i to destination j. In general the real life problems are modeled with multi-objectives, which are measured in different scales and at the same time in conflict. In some circumstances due to storage constraints designations are unable to receive the quantity in excess of their minimum demand. After consuming parts of whole of this initial shipment they are prepared to receive the excess quantity in the second stage. According to Sonia and Rita Malhotra [19] in such situations the product transported to the destination has two stages. Just enough of the product is shipped in stage I so that the minimum requirements of the destinations are satisfied and having done this the surplus quantities (if any) at the sources are shipped to the destinations according to cost consideration. In both the stages the transportation of the product from sources to the destination is done in parallel. Efficient algorithms have been developed for solving the transportation problem when the cost coefficients and the supply and demand quantities are known exactly. However, there are cases that these parameters may not be presented in a precise manner. For example, the unit shipping cost may vary in a time frame. The supplies and demands may be uncertain due to some uncontrollable factors.

To deal quantitatively with imprecise information in making decisions, Bellman and Zadeh [1] and Zadeh [24] introduce the notion of fuzziness. Since the transportation problem is essentially a linear program, one straightforward idea is to apply the existing fuzzy linear programming techniques [3, 4, 8, 9, 13, 16, 18, 20] to the fuzzy transportation problem. Unfortunately, most of the existing techniques [3, 4, 8, 9, 18, 20] only provide crisp solutions. The method of Julien [9] and Parra et al. [16] is able to find the possibility distribution of the objective value provided all the inequality constraints are of ''≤'' type or ''≥'' type. However, due to the structure of the transportation problem, in some cases their method requires the refinement of the problem parameters to be able to derive the bounds of the objective value. There are also studies discussing the fuzzy transportation problem. Chanas et al. [6] investigate the transportation problem with fuzzy supplies and demands and solve them via the parametric programming technique in terms of the Bellman–Zadeh criterion. Their method is to derive the solution which simultaneously satisfies the constraints and the goal to a maximal degree. Chanas and Kuchta [5] discuss the type of transportation problems with fuzzy cost coefficients and transform the problem to a bicriterial transportation problem with crisp objective function. Their method is able to determine the efficient solutions of the transformed problem; nevertheless, only crisp solutions are provided. Verma et al. [21] apply the fuzzy programming technique with hyperbolic and exponential membership functions to solve a multi-objective transportation problem, the solution derived is a compromise solution. Similar to the method of Chanas and Kuchta [5], only crisp solutions are provided.Obviously, when the cost coefficients or the supply and demand quantities are fuzzy numbers, the total transportation cost will be fuzzy as well.

In this paper two stage fuzzy transportation problem is discussed with multi objective constraints where the supply and demand are trapezoidal fuzzy numbers. This paper aims to find out the best compromise solution among the set of feasible solutions for the multi-objective two stage transportation problem. To illustrate the proposed method , example is used . Finally, some conclusions are drawn from the discussions.

2. Definitions

2.1. Fuzzy number

A real fuzzy number $\bar{a}$ is a fuzzy subset of the real number R with membership function $\mu_{\bar{a}}$ satisfying the following conditions.

1. $\mu_{\bar{a}}$ is continuous from R to the closed interval [0, 1]
2. $\mu_{\bar{a}}$ is strictly increasing and continuous on [a_1, a_2]
3. $\mu_{\bar{a}}$ is strictly decreasing and continuous on [a_3, a_4]

where a_1, a_2, a_3 and a_4 are real numbers, and the fuzzy number denoted by $\bar{a} = [a_1, a_2, a_3, a_4]$ is called fuzzy trapezoidal number.

2.2. Trapezoidal number

The fuzzy number $\bar{a}$ is a trapezoidal number, denoted by $[a_1, a_2, a_3, a_4]$ its membership function μ_a is given by Fig.1.

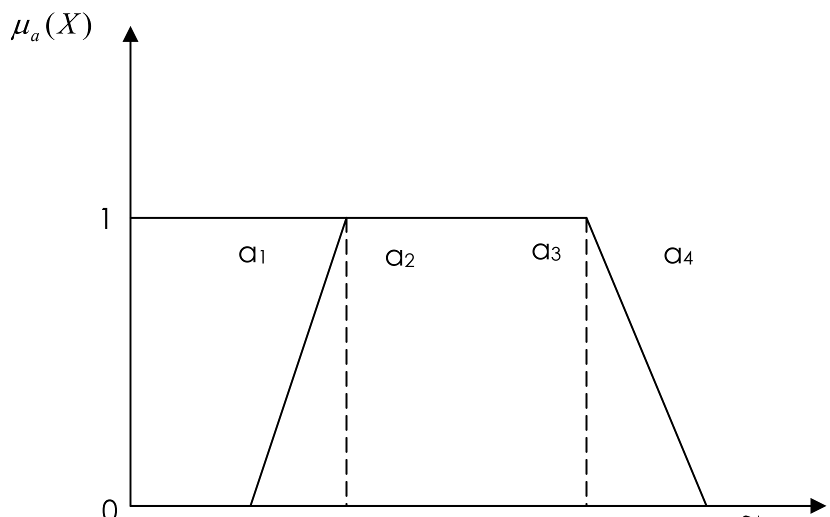

Fig. 1. Membership function of a fuzzy number $\tilde{a}$

2.3. α - level set

The α- level set of the fuzzy number $\bar{a}$ and $\bar{b}$ is defined as the ordinary set $L_\alpha(\bar{a}, \bar{b})$ for which the degree of their membership function exceeds the level $\alpha \in$ [0, 1]

$$L_\alpha(\bar{a}, \bar{b}) = \{a, b \in R^m / \mu_{\bar{a}}(a_i, b_j) \geq \alpha, i = 1,2.......m, j = 1,2.....n\}$$

2.4 Compromise solution

A feasible Vector $X^* \in S$ is called a **compromise solution** of p_1 iff $x^* \in$ E and $F(X^*) \le \underset{x \in s}{\wedge} F(X)$ where $\wedge$ stands for 'minimum' and E is the set of feasible solutions.

From a practical point of view the knowledge of the set of feasible solutions E is not always necessary. In such a case, a procedure is needed to determine a compromise solution. The purpose of this paper is to present a fuzzy programming approach to find an optimal compromise solution of a transportation problem with several objective in which the quantities are transported in two stages. Numerical example is given to illustrate the approach.

3. Fuzzy Programming approach for solving Multi-objective Two Stage Fuzzy Transportation Problem (MOTSFTP)

Let $\widetilde{b}_j$ be the minimum fuzzy requirement of a homogeneous product at the destination j and $\widetilde{a}_i$ the fuzzy availability of the same at source i. $F^k(x)=\{F^1(x),F^2(x),\ldots\ldots F^k(x)\}$ is a vector of K objective functions and the superscript on both $F^k(x)$ and c_{ij}^k are used to identify the number of objective functions k=1,2,3, k. without loss of generality it will be assumed in the whole paper that $a_i>0\ \forall i, b_j>0\ \forall j, c_{ij}^k>=0\ \forall i,j$ and $\sum_i a_i = \sum_j b_j$ .The Multi –objective Two-stage fuzzy Cost Minimization Transportation Problem deals with supplying the destinations their minimum requirements in stage-I and the quantity $\sum_i \bar{a}_i - \sum_j \bar{b}_j$ is supplied to the destinations in stage-II, from the sources which have surplus quantity left after the completion of stage-I.

Mathematically stated, the stage-I problem is

$$\min F^k(x) = \min_{X \in S_1}\left[\max_{|x|}\left(c_{ij}^{\ k}\left(X_{ij}\right)\right)\right] \tag{1}$$

where the set S_1 is given by

$$S_1 = \begin{cases} \sum_{j=1}^{n} x_{ij} \le \bar{a}_i & i = 1,2....m \\ \sum_{i=1}^{m} x_{ij} = \bar{b}_j & j = 1,2.....n \end{cases}$$

$x_{ij} \ge 0, \forall (i,j)$ corresponding to a feasible solution $X = (x_{ij})$ of the stage-I problem, the set $S_2 = \{\overline{X} = (x_{ij})\}$ of feasible solution of the stage-II problem is given by

$$S_2 = \begin{cases} \sum_{j=1}^{n} x_{ij} \le \bar{a}_i & i = 1,2....m \\ \sum_{i=1}^{m} x_{ij} \ge \bar{b}_j & j = 1,2.....n \end{cases}$$

$x_{ij} \ge 0, \forall (i, j)$ where $\tilde{a}_i$ is the quantity available at the i[th] source on completion so the stage-I, that is $\tilde{a}_i = \tilde{a}_i - \sum_j X_{ij}$. Clearly $\sum_i \tilde{a}_i = \sum_i \tilde{a}_i - \sum_j \tilde{b}_j$. Thus the state-II problem would be mathematically formulated as:

$$\min F^k(x) = \min_{X \in S_2} \left[\max_{|X|} \left(C_{ij}{}^k \left(X_{ij} \right) \right) \right] \tag{2}$$

We aim at finding that feasible solution $X = (X_{ij})$ of the stage-I problem corresponding to which the optimal cost for stage-II is such that the sum of the shipment is the least. The Multi-objective two stage fuzzy cost minimizing transportation problem can, therefore, be stated as,

$$\min \ F^k(x) = \min_{X \in S_1} \left[C_1^k(x) + \left[\min_{X \in S_2} C_2^k X \right] \right] \tag{3}$$

Also from a feasible solution of the problem (3) can be obtained. Further the problem (3) can be solved by solving following fuzzy cost minimizing Transportation problem.

$$\text{P1 :} \qquad \min \ F^k(x) = \min_{X \in S_2} \left[\max_{|X|} \left[C_{ij}^k \left[X_{ij} \right] \right] \right] \tag{4}$$

where S_2, the set of feasible solutions of (3), is defined as follows

$$S_2 = \begin{cases} \sum_{j=1}^{n} x_{ij} = \tilde{a}_i & i = 1,2......m \\ \sum_{i=1}^{m} x_{ij} = \tilde{b}_j & j = 1,2.....n \end{cases}$$

$X_{ij} \ge 0 \forall (i, j)$

where $\tilde{a}_i$, and $\tilde{b}_j$, represent fuzzy parameters involved in the constraints with their membership functions for $\mu_{\bar{a}}$ a certain degree α together with the concept of α

level set of the fuzzy numbers $\overline{a}_i, \overline{b}_j$. Therefore the problem of Two stage MOFCMTP can be understood as following non fuzzy α-general Two stage transportation problem (α-two stage MOFCMTP).

$$S = \begin{cases} \sum_{j=1}^{n} x_{ij} = \widetilde{a}_i & i = 1,2....m \\ \sum_{i=1}^{m} x_{ij} = \widetilde{b}_j & j = 1,2.....n \end{cases}$$

$$ai, bj \in L\alpha\left(\widetilde{a}_i, \widetilde{b}_j\right)$$

where $L\alpha\left(\widetilde{a}_i, \widetilde{b}_j\right)$ are the α-level set of the fuzzy number $\widetilde{a}_i, \widetilde{b}_j$ let $x\left(\widetilde{a}_i, \widetilde{b}_j\right)$ denote the constraint set of problem and supposed to be non empty. On the basis of the α-level sets of the fuzzy numbers, we give the concept of α-optimal solution in the following definition.

A point $X^* \in X\left(\widetilde{a}_i, \widetilde{b}_j\right)$ is said to be α-optimal solution (α-Two stage FCMTP), if and only if there does not exist another $x, y \in x(a,b), a, b \in L_\alpha\left(\widetilde{a}_i \widetilde{b}_j\right)$, such that $c_{ij}x_{ij} \leq c_{ij}x_{ij}{}^*$ with strict inequality holding for the at least one c_{ij} where for corresponding values of parameters $\left(\widetilde{a}, \widetilde{b}\right)$ are called α-level optimal parameters.

The problem (α-Two stage MOFCMTP) can be re written in the following equivalent form (α'-Two stage MOFCMTP)

$$S = \begin{cases} \sum_{j=1}^{n} x_{ij} = \widetilde{a}_i & i = 1,2....m \\ \sum_{i=1}^{m} x_{ij} = \widetilde{b}_j & j = 1,2.....n \end{cases}$$

$$h_i^O \leq a_i \leq H_i^O, h_j^O \leq b_j \leq H_j^O$$

$$x_{ij} \geq 0 \quad \forall \text{i, j}$$

It should be noted that the constraint $\left(a_i, b_j \in L_\alpha\left(\overline{a}_i, \overline{b}_j\right)\right.$ has been replaced by the constraint $h_i^O \leq a_i \leq H_i^O \; and \; h_j^O \leq b_j \leq H_j^O$ where h_i^O and H_i^O and h_j^O and H_j^O are lower and upper bounds and a_i, b_j are constants.

The parametric study of the problem (α'- Two stage MOFCMTP) where and h_i^O, H_i^O are assumed to be parameters rather than constants and (renamed h_i, H_i and h_j, H_j) can be understood as follows.

Let $X(h,H)$ denotes the decision space of problem (α'- Two Stage MOFCMTP), defined by

$$X(h,H) = \left(x_{ij}, a_i, b_j\right) \in R^{n(n+1)} \mid a_i - \sum_j x_{ij} \geq 0$$
$$b_j - \sum_i x_{ij} \geq 0,\ \mathrm{H_i} - a_i \geq 0, H_j - b_j \geq 0,$$
$$a_i - h_i \geq 0, b_j - h_j \geq 0, x_{ij} \geq 0, i \in I, j \in J$$

4. Solution Algorithm

Step 1 :Construct the Transportation problem

Step 2 : Supply and demand are trapezoidal fuzzy numbers (a_1, a_2, a_3, a_4) and (b_1, b_2, b_3, b_4) respectively in the formulation problem (Two Stage MOFCMTP).

Step 3 :Convert the problem (α-Two Stage MOFCMTP) in the form of the problem (α' - Two stage MOFCMTP)

Step 4 :Formulate the problem (α' - Two stage FCMTP) in the parametric form.

Step 5 :Apply VAM to get the basic feasible solution.

5. Geometric programming approach for solving MOTP

In 1970, Bellman and Zadeh [1] introduced three basic concepts; fuzzy goal (G), fuzzy constraints (C), and fuzzy decision (D) and explored the applications of these concepts to decision making under fuzziness. Their fuzzy decision is defined as follows:

$$D = G \cap C$$

This problem is characterized by the membership functions:

$$\mu_D(x) = \min(\mu_G(x), \mu_C(x))$$

To define the membership function of MOTP problem, let L_k, U_k be the lower and upper bounds of the objective functions $F^k(x)$. These values are determined as follows: Calculate the individual minimum of each objective function as a single objective transportation problem subject to the given set of constraints. Let $X^1, X^2, \ldots X^k$ be the respective optimal solutions for the K different transportation problems and evaluate each objective function at all these k optimal solutions. It is assumed here that at least two of these solutions are different for which the kth objective function has different bounded values. For each objective function $F^k(x)$, find the lower bound (minimum value) L_k and the upper bound (maximum value) U_k. On the basis of definitions L_k and U_k, Biswal [2] gives a membership function of a multi-objective geometric programming problem which can be implemented for the MOTP problem as follows:

$$U_k F^k(x)) = \begin{cases} 1 & \text{if } \mathrm{F}^{\mathrm{k}}(x) \le L_k \\ \dfrac{U_k - F^k(x)}{U_k - L_k} & \text{if } \mathrm{L}^{\mathrm{k}} < F^k(x) < U_k \\ 0 & \text{if } \mathrm{F}^{\mathrm{k}}(x) \ge U_k \end{cases} \quad (5)$$

where $L_k \neq U_k$, $\mathrm{k} = 1,2,....,\mathrm{k}$. If $\mathrm{L}_{\mathrm{k}} = U_k$ then $\mu_k(F^k(x)) = 1$ for any value of k.

Following the fuzzy decision of Bellman and Zadeh together with the linear membership function (5), a fuzzy optimization model of MOTP problem can be written as follows.

P2 : Max $\min\limits_{k=1,2,.....,k} \mu_k(F^k(x))$

Subject to
$$\sum_{j=1}^{n} x_{ij} = a_i, \qquad \mathrm{i} = 1,2,\ldots,\mathrm{m}$$
$$\sum_{i=1}^{n} x_{ij} = b_j, \qquad \mathrm{j} = 1,2,\ldots..,\mathrm{n}$$
$$x_{ij} \ge 0, \quad \mathrm{i} = 1, 2,....., \mathrm{m}$$
$$j = 1, 2, ..., \mathrm{n}$$

By introducing an auxiliary variable β, problem P2 can be transformed into the following equivalent conventional linear programming (LP) problem [26].

P3 : Max β

Subject to
$$\beta \le \mu_k(F^k(x)), \qquad k = 1,2,...k$$
$$\sum_{j=1}^{n} x_{ij} = a_i, \qquad \mathrm{i} = 1, 2, \ldots., \mathrm{m}$$
$$\sum_{i=1}^{n} x_{ij} = b_j, \qquad \mathrm{j} = 1,2,\ldots,\mathrm{n}$$
$$0 \le \beta \le 1,$$
$$x_{ij} \ge 0 \quad \forall \mathrm{i, j}$$

In problem P3, constraint (1) can be reduced to the following form.

$$\beta(U_k - L_k) \le (U_k - F^k(x)),$$
$$\beta(U_k - L_k) + F^k(x) \le U_k$$
$$\beta(U_k - L_k)/U_k + (1/U_k)F^k(x) \le 1.$$

Then, the solution procedure of the MOTP problem is summarized in the following steps.

Step 1 : Pick the first objective function and solve it as a single objective transportation problem subject to the constraints (2) – (4). Continue this process K times for K different objective functions. If all the solutions (i.e. $X^1 = X^2 = ... = X^k = \{x_{ij}\},\ i = 1,2,...m,\quad j = 1,2,...,n$) are the same, then one of them is the optimal compromise solution and go to step 6. Otherwise, go to step 2
Step 2 : Evaluate the kth objective function at the k optimal solutions $(k = 1,2,..., K)$. For each objective function, determine its lower and upper bounds (L_k and U_k) according to the set of optimal solutions.
Step 3 : Define the membership function as mentioned in Eq. (5)
Step 4 : Construct the fuzzy programming problem P2 and find its equivalent LP problem P3
Step 5 : Solve P3 by using an integer programming technique using a software package TORA to get an integer optimal solution and evaluate the K objective functions at this optimal compromise solution. Combining stage 1 and stage 2 ,we get an optimal solution.
Step 6 : Stop

This solution procedure requires the determination of upper and lower bounds of each objective (step 2) to construct the membership function of the MOTP problem (step 3). After that, Zadeh's min-operator is used to develop a linear compromise problem (P3) which is solved by using any integer programming technique.

6.Numerical Example

Consider the following multi-objective two stage cost minimizing transportation problem. Here supplies & demands are trapezoidal fuzzy numbers.
$a_1 = (4, 5, 7, 8)$ $a_2 = (6, 7, 8, 9)$ $a_3 = (5, 6, 7, 8)$ $a_4 = (4, 6, 8, 9)$
$b_1 = (1, 2, 4, 5)$ $b_2 = (4, 5, 6, 7)$ $b_3 = (3, 4, 5, 7)$ $b_4 = (4, 5, 6, 7)$ $b_5 = (2, 3, 4, 5)$ $b_6 = (3, 4, 5, 6)$
consider α - level set to be $\alpha = 0.75$ we get $4.5 \le a_1 \le 7.5$, $6.5 \le a_2 \le 8.5$, $5.5 \le a_3 \le 7.5$, $5.0 \le a_4 \le 8.5$, $1.5 \le b_1 \le 4.5$, $4.5 \le b_2 \le 6.5$, $3.5 \le b_3 \le 6.0$, $4.5 \le b_4 \le 6.5$, $2.5 \le b_5 \le 4.5$, $3.5 \le b_6 \le 5.5$.

The α - optimal parameters are
$a_1 = 6$ $a_2 = 8$ $a_3 = 7$ $a_4 = 7$
$b_1 = 3$ $b_2 = 5$ $b_3 = 5$ $b_4 = 6$ $b_5 = 4$ $b_6 = 5$
Penalties :

$$C^1 = \begin{pmatrix} 2 & 3 & 5 & 11 & 4 & 2 \\ 4 & 7 & 9 & 5 & 10 & 4 \\ 12 & 25 & 9 & 6 & 26 & 12 \\ 8 & 7 & 9 & 24 & 10 & 8 \end{pmatrix}$$

$$C^2 = \begin{pmatrix} 1 & 2 & 7 & 7 & 4 & 4 \\ 1 & 9 & 3 & 4 & 5 & 8 \\ 8 & 9 & 4 & 6 & 6 & 2 \\ 3 & 4 & 9 & 10 & 5 & 1 \end{pmatrix}$$

STAGE I

We take $a_1=3$, $a_2=4$, $a_3=3$, $a_4=3$

$b_1=1$, $b_2=2$, $b_3=3$, $b_4=3$, $b_5=2$, $b_6=2$

With respect to C^1, applying VAM, we get

$x_{12}=1 \quad x_{15}=2 \quad x_{21}=1 \quad x_{22}=1 \quad x_{26}=2$

$x_{34}=3 \quad x_{43}=3$

$\min z = 75$.

With respect to C^2, applying VAM we get

$x_{12}=2 \quad x_{15}=1 \quad x_{21}=1 \quad x_{24}=3$

$x_{33}=3 \quad x_{45}=1 \quad x_{46}=2$

$\min z = 40$

$F^1(X^1) = 75 \qquad F^1(X^2) = 82$

$F^2(X^1) = 81 \qquad F^2(X^2) = 40$

ie $75 \le F^1 \le 82$

$40 \le F^2 \le 81$

The member ship function of both $F^1(x)$ and $F^2(x)$ are

$$\mu_1(F^1(x)) = \frac{82 - F^1(x)}{82-75} = \frac{82-F^1(x)}{7}$$

$$\mu_2(F^2(x)) = \frac{81 - F^2(x)}{81-40} = \frac{81-F^2(x)}{41}$$

Now Solve Max β

S. to $x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} = 3$

$x_{21} + x_{22} + x_{23} + x_{24} + x_{25} + x_{26} = 4$

$x_{31} + x_{32} + x_{33} + x_{34} + x_{35} + x_{36} = 3$

$x_{41} + x_{42} + x_{43} + x_{44} + x_{45} + x_{46} = 3$

$x_{11} + x_{21} + x_{31} + x_{41} = 1$

$x_{12} + x_{22} + x_{32} + x_{42} = 2$

$x_{13} + x_{23} + x_{33} + x_{43} = 3$

$x_{14} + x_{24} + x_{34} + x_{44} = 3$

$x_{15} + x_{25} + x_{35} + x_{45} = 2$

$$x_{16} + x_{26} + x_{36} + x_{46} = 2$$

$0.0244\, x_{11} + 0.0366\, x_{12} + 0.0609\, x_{13} + 0.1341\, x_{14} + 0.0488\, x_{15} + 0.0244\, x_{16}$
$+ 0.0488\, x_{21} + 0.0854\, x_{22} + 0.1098\, x_{23} + 0.0609\, x_{24} + 0.1220\, x_{25} + 0.0488\, x_{26}$
$+0.1463\, x_{31} + 0.3048\, x_{32} + 0.1098\, x_{33} + 0.0732\, x_{34} + 0.3171\, x_{35} + 0.1463\, x_{36}$
$+0.0976\, x_{41} + 0.0854\, x_{42} + 0.1098\, x_{43} + 0.2927\, x_{44} + 0.1220\, x_{45} + 0.0976\, x_{46} +$
$0.0854\, \beta \le 1.$

$0.0123\, x_{11} + 0.0247\, x_{12} + 0.0864\, x_{13} + 0.0864\, x_{14} + 0.0494\, x_{15} + 0.0494\, x_{16}$
$+0.0123\, x_{21} + 0.111\, x_{22} + 0.0370\, x_{23} + 0.0494\, x_{24} + 0.0617\, x_{25} + 0.0988\, x_{26}$
$+0.0988\, x_{31} + 0.1111\, x_{32} + 0.0494\, x_{33} + 0.0741\, x_{34} + 0.0741\, x_{35} + 0.0247\, x_{36}$
$+0.0370\, x_{41} + 0.0494\, x_{42} + 0.1111\, x_{43} + 0.1235\, x_{44} + 0.0617\, x_{45} + 0.0123\, x_{46}$
$+ 0.5062\, \beta \le 1$

$$x_{ij} \ge 0 \text{ and integer, } \forall i, j$$

Using TORA program we have the following optimal compromise solution X^*

$x_{13} = 1$; $x_{15} = 2$; $x_{21} = 1$; $x_{24} = 2$; $x_{26} = 1$; $x_{33} = 2$; $x_{34} = 1$; $x_{42} = 2$; $x_{46} = 1$;

The overall satisfaction $\beta = 0.63394$
The optimum values of the objective functions after stage I are
$F^1(X^*) = 77$
$F^2(X^*) = 55$

STAGE II :

We take $a_1 = 3$ $a_2 = 4$ $a_3 = 4$ $a_4 = 4$
$b_1 = 2$ $b_2 = 3$ $b_3 = 2$ $b_4 = 3$ $b_5=2$ $b_6 = 3$

With respect to C^1, applying VAM we get

$x_{12} = 1$ $x_{15} = 2$ $x_{21} = 2$ $x_{22} = 2$
$x_{33} = 1$ $x_{34} = 3$ $x_{43} = 1$ $x_{46} = 3$

$\min z = 93$

With respect to C^2, applying VAM, we get

$x_{12} = 3$ $x_{21} = 2$ $x_{24} = 2$ $x_{33} = 2$
$x_{34} = 1$ $x_{35} = 1$ $x_{45} = 1$ $x_{46} = 3$

$\min z = 6 + 2 + 8 + 8 + 6 + 6 + 5 + 3$
$= 44$

$F^1(X^1) = 93$ $F^1(X^2) = 111$
$F^2(X^1) = 64$ $F^2(X^2) = 44$

i.e $93 \le F^1 \le 111$
$44 \le F^2 \le 64$

The membership function of both $F^1(x)$ & $F^2(x)$ are

$$\mu_1\left(F^1(x)\right) = \frac{111 - F^1(x)}{111 - 93} = \frac{111 - F^1(x)}{18}$$

$$\mu_1\left(F^2(x)\right) = \frac{64 - F^2(x)}{64 - 44} = \frac{64 - F^2(x)}{20}$$

Now Solve Max β

S. to $x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} = 3$

$x_{21} + x_{22} + x_{23} + x_{24} + x_{25} + x_{26} = 4$

$x_{31} + x_{32} + x_{33} + x_{34} + x_{35} + x_{36} = 4$

$x_{41} + x_{42} + x_{43} + x_{44} + x_{45} + x_{46} = 4$

$x_{11} + x_{21} + x_{31} + x_{41} = 2$

$x_{12} + x_{22} + x_{32} + x_{42} = 3$

$x_{13} + x_{23} + x_{33} + x_{43} = 2$

$x_{14} + x_{24} + x_{34} + x_{44} = 3$

$x_{15} + x_{25} + x_{35} + x_{45} = 2$

$x_{16} + x_{26} + x_{36} + x_{46} = 3$

$0.0180\,x_{11} + 0.0271\,x_{12} + 0.0450\,x_{13} + 0.0991\,x_{14} + 0.0360\,x_{15} + 0.0180\,x_{16}$
$+ 0.0360\,x_{21} + 0.0631\,x_{22} + 0.0811\,x_{23} + 0.0450\,x_{24} + 0.0901\,x_{25} + 0.0360\,x_{26}$
$+0.1081\,x_{31} + 0.2252\,x_{32} + 0.0811\,x_{33} + 0.0541\,x_{34} + 0.2342\,x_{35} + 0.1081\,x_{36}$
$+0.0721\,x_{41} + 0.0631\,x_{42} + 0.0811\,x_{43} + 0.2162\,x_{44} + 0.0901\,x_{45} + 0.0721\,x_{46} +$
$0.1622\,\beta \le 1.$

$0.0156\,x_{11} + 0.313\,x_{12} + 0.1094\,x_{13} + 0.1094\,x_{14} + 0.0625\,x_{15} + 0.0625\,x_{16}$
$+0.0156\,x_{21} + 0.1406\,x_{22} + 0.04690\,x_{23} + 0.0625\,x_{24} + 0.0781\,x_{25} + 0.125\,x_{26}$
$+0.125\,x_{31} + 0.1406\,x_{32} + 0.0625\,x_{33} + 0.0938\,x_{34} + 0.0938\,x_{35} + 0.0313\,x_{36}$
$+0.0469\,x_{41} + 0.0625\,x_{42} + 0.1406\,x_{43} + 0.1563\,x_{44} + 0.0781\,x_{45} + 0.0156\,x_{46} +$
$0.3125\,\beta \le 1$, $x_{ij} \ge 0$ and integer, $\forall i, j$

This problem is solved using TORA program yielding the following optimal compromise solution X^*

$x_{12}=2$; $x_{15}=1$; $x_{21}=2$; $x_{24}=1$; $x_{25}=1$; $x_{33}=2$; $x_{34}=2$; $x_{42}=1$; $x_{46}=3$

The overall satisfaction β =.089984
The optimum values of the objective functions after stage II are
$F^1(X^*) = 94$
$F^2(X^*) = 46$

The optimal values of the objective functions combining stage I and stage II are
$F^1(X^*) = 77+94=171$
$F^2(X^*) = 55+46=101$

7. Conclusion

Transportation models have wide applications in logistics and supply chain for reducing the cost. Some previous studies have devised solution procedures for fuzzy transportation problems. In this study , Fuzzy geometric programming approach is used to determine the optimal compromise solution of a multi-objective two stage fuzzy transportation problem, in which supplies, demands are trapezoidal fuzzy numbers and fuzzy membership of the objective function is defined. Tora software is used to find out the optimal compromise solution. This approach provides an analyst simple and easy mathematical programming problem. An illustrative example has been given to check the validity of the proposed method. In real world applications, the parameters in the transportation problem may not be known precisely due to uncontrollable factors. If the obtained results are crisp values, then it might lose some helpful information. Since the objective value is expressed by the membership function rather than by a crisp value, more information is provided for making decisions.

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