First Order Reactant in Magneto-hydrodynamic Turbulence Before The Final Period of Decay in Presence of Dust Particle in a Rotating System

M.A.K. Azad, M. A. Aziz and M.S.Alam Sarker

Department of Applied Mathematics,
University of Rajshahi-6205, Bangladesh

ABSTRACT

Using Deissler's approach, the decay for the concentration fluctuation of a dilute contaminant undergoing a first order chemical reaction in dusty fluid MHD turbulence at times before the final period in a rotating system is studied and have considered correlations between fluctuating quantities at two and three point. Two and three point correlation equations are obtained and the set of equations is made to determinate by neglecting the quadruple correlations in comparison to the second and third order correlations. The correlation equations are converted to spectral form by taking their Fourier transforms. Finally we obtained the Decay law of magnetic energy for the concentration fluctuations before the final period in a rotating system in presence of dust particle by integrating the energy spectrum over all wave numbers.

Keywords: MHD Turbulence, First order Reactant, Dust particle, Rotating system, Decay before the final period .

1. Introduction

  Deissler [1,2] developed a theory "decay of homogeneous turbulence for times before the final period". Using Deissler's theory, Loeffler and Deissler [3] studied the decay of temperature fluctuations in homogeneous turbulence before the final period. In their approach they considered the two and three-point correlation equations and solved these equations after neglecting fourth and higher order correlation terms. Using Deissler theory, Kumar and Patel [4] studied the first-order reactant in homogeneous turbulence before the final period of decay for the case of multi-point and single-time correlation. Kumar and Patel [5] extended their problem [4] for the case of multi-point and multi-time concentration correlation. Patel [6] also studied in detail the same problem to carry out the numerical results. Sarker and Kishore [7] studied the decay of MHD turbulence at time before the final period

using chandrasekher's relation [8]. Sarker and Islam [9] studied the decay of MHD turbulence before the final period for the case of multi-point and multi-time. Azad and Sarker[13] studied the Decay of dusty fluid MHD turbulence before the final period in a rotating system for the case of multi-point and multi-time. Sarker and Islam[11] studied the Decay of dusty fluid turbulence before the final period in a rotating system. Islam and Sarker [10] studied the first order reactant in MHD turbulence before the final period of decay for the case of multi-point and multi-time. Sarker and Islam [14] also studied the first order reactant in MHD turbulence before the final period of decay.

   In our work, we studied the magnetic field fluctuation of concentration of a dilute contaminant undergoing a first order chemical reaction in dusty fluid MHD turbulence before the final period of decay in a rotating system. Here, we have considered the two-point and three-point correlation equations and solved these equations after neglecting the fourth-order correlation terms. Finally we obtained the decay law for magnetic field energy fluctuation of concentration of dilute contaminant undergoing a first order chemical reaction in dusty fluid MHD turbulence in a rotating system comes out to be

$$\langle h^2 \rangle = \exp[-R(t-t_0)]\left[A(t-t_0)^{-3/2} + \exp[-\{2 \in_{mkl} \Omega_m - fs\}]B(t-t_0)^{-5} \right]$$

where $\langle h^2 \rangle$ denotes the total energy (mean square of the magnetic field fluctuations of concentration), t is the time and A,B and t_0 are constants.

2. Basic Equations:

         The equation of motion and continuity for viscous, incompressible MHD dusty fluid turbulent flow in a rotating system are given by Chandrasekhar [8] as

$$\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_k}\left(u_i u_k - h_i h_k\right) = -\frac{\partial w}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_k \partial x_k} - 2 \in_{mki} \Omega_m u_i + f(u_i - v_i) \qquad (1)$$

$$\frac{\partial h_i}{\partial t} + \frac{\partial}{\partial x_k}(h_i u_k - u_i h_k) = \lambda \frac{\partial^2 h_i}{\partial x_k \partial x_k} \qquad\qquad (2)$$

$$\frac{\partial v_i}{\partial t} + v_k \frac{\partial v_i}{\partial x_k} = -\frac{k}{m_s}(v_i - u_i), \qquad\qquad \text{--------- (3)}$$

with

$$\frac{\partial u_i}{\partial x_i} = \frac{\partial v_i}{\partial x_i} = \frac{\partial h_i}{\partial x_i} = 0 \qquad\qquad \text{---------- (4)}$$

The subscripts can take on the values 1, 2 or 3.

Here, u_i, turbulent velocity component; h_i, magnetic field fluctuation component, v_i, dust velocity component

$$W(\hat{x},t) = \frac{p}{\rho} + \frac{1}{2}\langle h^2 \rangle + \frac{1}{2}\left|\hat{\Omega} \times \hat{x}\right|^2,$$ total MHD pressure inclusive of potential and

centrifugal force;

$p(\hat{x},t)$ = hydrodynamic pressure, ρ = fluid density,

$p_M = \dfrac{\nu}{\lambda}$, magnetic prandtl number,

$p_r = \dfrac{\nu}{\gamma}$, prandtl number,

ν = kinematic viscosity,

$\gamma = \dfrac{K}{\rho c_p}$, thermal diffusivity,

$\lambda = (4\pi\mu\sigma)^{-1}$, magnetic diffusivity,

c_p = heat capacity at constant pressure,

Ω_m = constant angular velocity components,

$\in_{m\,ki}$ = alternating tensor,

$f = \dfrac{kN}{\rho}$, dimension of frequency; N, constant number density of dust particle,

$m_s = \dfrac{4}{3}\pi R^3{}_s \rho_s$, mass of single spherical dust particle of radius R_s,

ρ_s = constant density of the material in dust particle,

x_k = Space co-ordinate, the subscripts can take on the values 1, 2 or 3.

3. Two-point Correlation and Spectral Equations

Under the condition that (I) the turbulence and the concentration magnetic field are homogeneous (ii) the chemical reaction has no effect on the velocity field and (iii) the reaction rate and the magnetic diffusivity are constant, the induction equation of a magnetic field fluctuation of concentration of a dilute contaminant undergoing a

first order chemical reaction at the points p and p' separated by the vector $\hat{r}$ could be written as

$$\frac{\partial h_i}{\partial t} + u_k \frac{\partial h_i}{\partial x_k} - h_k \frac{\partial u_i}{\partial x_k} = \lambda \frac{\partial^2 h_i}{\partial x_k \partial x_k} - Rh_i \qquad \text{---------- (5)}$$

and $\dfrac{\partial h_j^{'}}{\partial t} + u_k^{'} \dfrac{\partial h_j^{'}}{\partial x_k^{'}} - h_k^{'} \dfrac{\partial u_j^{'}}{\partial x_k^{'}} = \lambda \dfrac{\partial^2 h_j^{'}}{\partial x_k^{'} \partial x_k^{'}} - R h_j^{'}$ ---------- (6)

Where R is the constant reaction rate.

Multiplying equation (5) by $h_j^{'}$ and (6) by h_i , adding and taking ensemble average, we get

$$\dfrac{\partial \langle h_i h_j^{'} \rangle}{\partial t} + \dfrac{\partial}{\partial x}[\langle u_k h_i h_j^{'} \rangle - \langle h_k u_i h_j^{'} \rangle] + \dfrac{\partial}{\partial x_k^{'}}[\langle u_k^{'} h_i h_j^{'} \rangle - \langle h_k^{'} u_j^{'} h_i \rangle].$$

$$= \lambda [\dfrac{\partial^2 \langle h_i h_j^{'} \rangle}{\partial x_k \partial x_k} + \dfrac{\partial^2 \langle h_i h_j^{'} \rangle}{\partial x_k^{'} \partial x_k^{'}}] - 2R \langle h_i h_j^{'} \rangle \qquad \text{------------(7)}$$

Angular bracket $\langle .... \rangle$ is used to denote an ensemble average and the continuity equation is
Using the transformations,

$$\dfrac{\partial}{\partial r_k} = -\dfrac{\partial}{\partial x_k} = \dfrac{\partial}{\partial x_k^{'}} \qquad \text{------------(8)}$$

and the Chandrasekhar relations [8]

$$\langle u_k h_i h_j^{'} \rangle = -\langle u_k^{'} h_i h_j^{'} \rangle, \langle u_j^{'} h_i h_k^{'} \rangle = -\langle u_i h_k h_j^{'} \rangle \qquad \text{---------------(9)}$$

Equation (7) becomes

$$\left[\dfrac{\partial \langle h_i h_j^{'} \rangle}{\partial t} + 2 \dfrac{\partial}{\partial r_k}[\langle u_k^{'} h_i h_j^{'} \rangle - \langle u_i h_k h_j^{'} \rangle] \right].$$

$$= 2\lambda \dfrac{\partial^2 \langle h_i h_j^{'} \rangle}{\partial r_k \partial r_k} - 2R \langle h_i h_j^{'} \rangle \qquad \text{------------------(10)}$$

 Now we write equation (10) in spectral form in order to reduce it to an ordinary differential equation by use of the following three-dimensional Fourier transforms.

$$\langle h_i h'_j(\hat{r})\rangle = \int_{-\infty}^{\infty} \langle \psi_i \psi'_j(\hat{k})\rangle \exp\left[\hat{i}(\hat{k},\hat{r})\right]d\hat{k}, \qquad \text{---------- (11)}$$

$$\langle u_i h_k h'_j(\hat{r})\rangle = \int_{-\infty}^{\infty} \langle \alpha_i \psi_k \psi'_j(\hat{k})\rangle \exp\left[\hat{i}(\hat{k},\hat{r})\right]d\hat{K}, \qquad \text{---------- (12)}$$

Interchanging i and j, points p and p' then,

$$\langle u'_k h_i h'_j(\hat{r})\rangle = \langle u_k h_i h'_j(-\hat{r})\rangle = \int_{-\infty}^{\infty} \langle \alpha_k \psi_i \psi'_j(-\hat{k})\rangle \exp\left[\hat{i}(\hat{k},\hat{r})\right]d\hat{k} \text{ ---------- (13)}$$

Substituting of equation (11) to (13) in to equation (10) leads to the Spectral equation

$$\frac{\partial\langle \psi_i \psi'_j(\hat{k})\rangle}{\partial t} + 2\lambda K^2 \langle \psi_i \psi'_j(\hat{k})\rangle + 2R\langle \psi_i \psi'_j(\hat{k})\rangle$$
$$= 2ik[\langle \alpha_i \psi_i \psi'_j(\hat{k})\rangle - \langle \alpha_k \psi_i \psi'_j(-\hat{k})\rangle] \qquad \text{------------(14)}$$

$$\Rightarrow \quad \frac{\partial\langle \psi_i \psi'_j(\hat{k})\rangle}{\partial t} + 2\lambda[K^2 + \frac{R}{\lambda}]\langle \psi_i \psi'_j(\hat{k})\rangle$$
$$= 2ik[\langle \alpha_i \psi_i \psi'_j(\hat{k})\rangle - \langle \alpha_k \psi_i \psi'_j(-\hat{k})\rangle] \qquad \text{----------(15)}$$

The tensor equation (15) becomes a scalar equation by contraction of the indices i and j

$$\frac{\partial\langle \psi_i \psi'_i(\hat{k})\rangle}{\partial t} + 2\lambda[K^2 + \frac{R}{\lambda}]\langle \psi_i \psi'_i(\hat{k})\rangle$$
$$= 2ik[\langle \alpha_i \psi_i \psi'_i(\hat{k})\rangle - \langle \alpha_k \psi_i \psi'_i(-\hat{k})\rangle] \qquad \text{-------------(16)}$$

The term on the right hand side of equation (16) is called energy transfer term while the second term on the left hand side is the dissipation term.

4. Three-point Correlation and Spectral Equations

Similar Procedure can be used to find the three points correlation equation. For this purpose we take the momentum equation of dusty fluid MHD turbulence in a rotating system at the point P and the induction equations of magnetic field fluctuation, governing the concentration of a dilute contaminant undergoing a first order chemical reaction at P' and P'' separated by the vectors $\hat{r}$ and $\hat{r}'$ as

$$\frac{\partial u_l}{\partial t} + u_k \frac{\partial u_l}{\partial k_k} - h_k \frac{\partial h_l}{\partial x_k} = -\frac{\partial w}{\partial x_l} + v \frac{\partial^2 u_l}{\partial x_k \partial x_k} - 2 \in_{mkl} \Omega_m u_l + f(u_l - v_l),$$

$$\text{------- (17)}$$

$$\frac{\partial h_i^{'}}{\partial t} + u_k^{'} \frac{\partial h_i^{'}}{\partial x_k^{'}} - h_k^{'} \frac{\partial u_i^{'}}{\partial x_k^{'}} = \lambda \frac{\partial^2 h_i^{'}}{\partial x_k^{'} \partial x_k^{'}} - R h_i^{'} \qquad \text{------- (18)}$$

$$\frac{\partial h_j^{''}}{\partial t} + u_k^{''} \frac{\partial h_j^{''}}{\partial x_k^{''}} - h_k^{''} \frac{\partial u_j^{''}}{\partial x_k^{''}} = \lambda \frac{\partial^2 h_j^{''}}{\partial x_k^{''} \partial x_k^{''}} - R h_j^{''} \qquad \text{------------(19)}$$

where $W(\hat{x},t) = \dfrac{P}{\rho} + \dfrac{1}{2}\langle h^2 \rangle + \dfrac{1}{2}\left|\hat{\Omega} \times \hat{x}\right|^2$, total MHD pressure inclusive of potential and centrifugal force $P(\hat{x},t)$, hydrodynamic pressure; Ωm, constant angular velocity components; $\in_{mki}$, alternating tensor, $f = \dfrac{kN}{\rho}$, dimension frequency; N, constant number density of dust particle.

　　　　Multiplying equation (17) by $h_i^{'} h_j^{''}$ (18) by $u_l h_j^{''}$ and (19) by $u_l h_i^{'}$, adding and taking ensemble average, we obtain

$$\frac{\partial \langle u_l h_i^{'} h_j^{''} \rangle}{\partial t} + \frac{\partial}{\partial x_k}[\langle u_k u_l h_i^{'} h_j^{''} \rangle - \langle h_k h_l h_i^{'} h_j^{''} \rangle] + \frac{\partial}{\partial x_k^{'}}[\langle u_l u_k^{'} h_i^{'} h_j^{''} \rangle - \langle u_l u_i^{'} h_k^{'} h_j^{''} \rangle].$$

$$+ \frac{\partial}{\partial x_k^{''}}[\langle u_l u_k^{''} h_i^{'} h_j^{''} \rangle - \langle u_l u_j^{''} h_i^{'} h_j^{''} \rangle]$$

$$= -\frac{\partial \langle w h_i^{'} h_j^{''} \rangle}{\partial x_l} + v \frac{\partial^2 \langle u_l h_i^{'} h_j^{''} \rangle}{\partial x_k \partial x_k} + \lambda \left[\frac{\partial^2 \langle u_l h_i^{'} h_j^{''} \rangle}{\partial x_k^{'} \partial x_k^{'}} + \frac{\partial^2 \langle u_l h_i^{'} h_j^{''} \rangle}{\partial x_k^{''} \partial x_k^{''}} \right]$$

$$- 2R \langle u_l h_i^{'} h_j^{''} \rangle - 2 \in_{mki} \Omega_m \langle u_l h_i^{'} h_j^{''} \rangle + f(\langle u_l h_i^{'} h_j^{''} \rangle - \langle v_l h_i^{'} h_j^{''} \rangle)$$

$$\text{------ (20)}$$

Using the transformations

$$\frac{\partial}{\partial x_k} = -(\frac{\partial}{\partial r_k} + \frac{\partial}{\partial r_k^{'}}), \frac{\partial}{\partial x_k^{'}} = \frac{\partial}{\partial r_k}, \frac{\partial}{\partial x_k^{''}} = \frac{\partial}{\partial r_k^{'}} \qquad \text{---------(21)}$$

into equation (20)

$$\frac{\partial \langle u_l h_i' h_j'' \rangle}{\partial t} - \lambda \left[(1+P_M) \frac{\partial^2 \langle u_l h_i' h_j'' \rangle}{\partial r_k \partial r_k} + (1+P_M) \frac{\partial^2 \langle u_l h_i' h_j'' \rangle}{\partial r_k' \partial r_k'} + 2P_M \frac{\partial^2 \langle u_l h_i' h_j'' \rangle}{\partial r_k \partial r_k} \right]$$

$$= \frac{\partial}{\partial r_k} \langle u_l u_k h_i' h_j'' \rangle + \frac{\partial}{\partial r_k'} \langle u_l u_k h_i' h_j'' \rangle - \frac{\partial}{\partial r_k'} \langle h_l h_k h_i' h_j'' \rangle - \frac{\partial}{\partial r_k'} \langle h_l h_k h_i' h_j'' \rangle$$

$$- \frac{\partial}{\partial r_k} \langle u_l u_k' h_i' h_j'' \rangle + \frac{\partial}{\partial r_k} \langle u_l u_i' h_k' h_j'' \rangle - \frac{\partial}{\partial r_k'} \langle u_l u_k'' h_i' h_j'' \rangle + \frac{\partial}{\partial r_k'} \langle u_l u_j'' h_i' h_j'' \rangle$$

$$+ \frac{\partial}{\partial r_l} \langle w h_i' h_j'' \rangle + \frac{\partial}{\partial r_l'} \langle w h_i' h_j'' \rangle - 2R \langle u h_i' h_j'' \rangle - 2 \in_{mkl} \Omega_m \langle u_l h_i' h_j'' \rangle$$

$$+ f[\langle u_l h_i' h_j'' \rangle - \langle v_l h_i' h_j'' \rangle] \qquad \text{----------(22)}$$

In order to write the equation (22) to spectral form, we can define the following six dimensional Fourier transforms:

$$\langle u_l h_i'(\hat{r}) h_j''(\hat{r}') \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle \phi_l \beta_i'(\hat{k}) \beta_j''(\hat{k}) \rangle \exp\left[\hat{i}(\hat{k}.\hat{r} + \hat{k}'\hat{r}')\right] d\hat{k} d\hat{k}' \qquad \text{--------- (23)}$$

$$\langle u_l u_k' h_i'(\hat{r}) h_j''(\hat{r}') \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle \phi_l \phi_k'(\hat{k}) \beta_i'(\hat{k}) \beta_j''(\hat{k}') \rangle \exp\left[\hat{i}(\hat{k}.\hat{r} + \hat{k}'\hat{r}')\right] d\hat{k} d\hat{k}'$$

$$\text{-------- (24)}$$

$$\langle u_l u_i'(\hat{r}) h_k'(\hat{r}) h_j''(\hat{r}') \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle \phi_l \phi_i'(\hat{k}) \beta_k'(\hat{k}) \beta_j''(\hat{k}') \rangle \exp\left[\hat{i}(\hat{k}.\hat{r} + \hat{k}'\hat{r}')\right] d\hat{k} d\hat{k}'$$

$$\text{-------- (25)}$$

$$\langle u_l u_k h_i'(\hat{r}) h_j''(\hat{r}') \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle \phi_l \phi_k \beta_i'(\hat{k}) \beta_j''(\hat{k}') \rangle \exp\left[\hat{i}(\hat{k}.\hat{r} + \hat{k}'\hat{r}')\right] d\hat{k} d\hat{k}'$$

$$\text{--------- (26)}$$

$$\langle h_l h_k h_i'(\hat{r}) h_j''(\hat{r}') \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle \beta_l \beta_k \beta_i'(\hat{k}) \beta_j''(\hat{k}') \rangle \exp\left[\hat{i}(\hat{k}.\hat{r} + \hat{k}'\hat{r}')\right] d\hat{k} d\hat{k}'$$

$$\text{-------- (27)}$$

$$\langle wh_i'(\hat{r})h_j''(\hat{r}')\rangle = \int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty}\langle \gamma\beta_i'(\hat{k})\beta_j''(\hat{k}')\rangle \exp\left[\hat{i}(\hat{k}.\hat{r}+\hat{k}'\hat{r}')\right]d\hat{k}d\hat{k}'$$

- ------- (28)

$$\langle v_l h_i'(\hat{r})h_j''(\hat{r}')\rangle = \int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty}\langle \delta_l\beta_i'(\hat{k})\beta_j''(\hat{k}')\rangle \exp\left[\hat{i}(\hat{k}.\hat{r}+\hat{k}'\hat{r}')\right]d\hat{k}d\hat{k}'$$

------- (29)

Interchanging the points p' and p'' along with the indices i and j, result in the relations

$$\langle u_l u_k'' h_j'' h_i'\rangle = \langle u_l u_k' h_i' h_j''\rangle, \quad \langle u_l u_j'' h_i' h_j''\rangle = \langle u_l u_i' h_k' h_j''\rangle, \quad \langle u_l u_j'' h_i'\rangle = \langle u_l u_i' h_j''\rangle$$

--------(30)

By use of this facts(30) and equations (23)-(29), the equation (22) may be transformed as

$$\frac{\partial\langle \phi_l\beta_i'\beta_j''\rangle}{\partial t}+\lambda\left[(1+P_M)k^2+(1+P_M)k'^2+2P_Mk_kk_k'+2\frac{R}{\lambda}+\frac{2\in_{mkl}\Omega_m}{\lambda}-\frac{f}{\lambda}\right]\langle \phi_l\beta_i'\beta_j''\rangle$$

$$=i(k_k+k_k')\langle \phi_l\phi_k\beta_i'\beta_j''\rangle-i(k_k+k_k')\langle \beta_l\beta_k\beta_i'\beta_j''\rangle-i(k_k+k_k')\langle \phi_l\phi_k'\beta_i'\beta_j''\rangle+i(k_k+k')\langle \phi_l\phi_i'\beta_i'\beta_j''\rangle$$

$$+i(k_l+k_l')\langle \gamma\beta_i'\beta_j''\rangle-f\langle \delta_l\beta_i'\beta_j''\rangle .$$

------------ (31)

The tensor equation (31) can be converted to scalar equation by contraction of the indices i and j

$$\frac{\partial\langle \phi_l\beta_i'\beta_i''\rangle}{\partial t}+\lambda\left[(1+P_M)(k^2+k'^2)+2P_Mk_kk_k'+2\frac{R}{\lambda}+\frac{2\in_{mkl}\Omega_m}{\lambda}-\frac{f}{\lambda}\right]\langle \phi_l\beta_i'\beta_i''\rangle$$

$$=i(k_k+k_k')\langle \phi_l\phi_k\beta_i'\beta_i''\rangle-i(k_k+k_k')\langle \beta_l\beta_k\beta_i'\beta_i''\rangle-i(k_k+k_k')\langle \phi_l\phi_k'\beta_i'\beta_i''\rangle+i(k_k+k')\langle \phi_l\phi_i'\beta_i'\beta_i''\rangle$$

$$+i(k_l+k_l')\langle \gamma\beta_i'\beta_i''\rangle-f\langle \delta_l\beta_i'\beta_i''\rangle .$$

------------(32)

To relate the terms on right hand side of equation (32) derived from the quadruple correlation   terms and from the pressure force term in equation (22), we take the derivative with respect to x_l of the momentum equation (17) for the point p, and combine with the continuity equation to give

$$-\frac{\partial^2 \langle w \rangle}{\partial x_l \partial x_l} = \frac{\partial^2}{\partial x_l \partial x_k}\left(u_l u_k - h_l h_k\right).$$ ------------ (33)

Multiplying equation (33) by $h'_i h''_j$ taking time averages and writing this equation in terms of the independent variables $\hat{r}$ and $\hat{r}'$

$$-\left[\frac{\partial^2}{\partial r_l \partial r_l} + 2\frac{\partial^2}{\partial r_l \partial r'_l} + \frac{\partial^2}{\partial r'_l \partial r'_l}\right]\langle wh'_i h''_j\rangle = \left[\frac{\partial^2}{\partial r_l \partial r_k} + \frac{\partial^2}{\partial r'_l \partial r_k} + \frac{\partial^2}{\partial r_l \partial r'_k} + \frac{\partial^2}{\partial r'_l \partial r'_k}\right]$$

$$\times\left(\langle u_l u_k h'_i h''_j\rangle - \langle h_l h_k h'_i h''_j\rangle\right).$$ ------------ (34)

Now taking the Fourier transforms of equation (34) we get

$$-\langle \gamma \beta'_i \beta''_j\rangle = \frac{\left(k_l k_k + k'_l k_k + k_l k'_k + k'_l k'_k\right)\left(\langle \phi_l \phi_k \beta'_i \beta''_j\rangle - \langle \beta_l \beta_k \beta'_i \theta \beta''_j\rangle\right)}{k^2 + 2k_l k'_l + k^{'2}}$$

--------------(35)

Thus the equations (34) and (35) are the spectral equation corresponding to the three-point correlation equations. Equation (35) can be used to eliminate $\langle \gamma \beta'_i \beta''_j\rangle$ from the equation (32).

5. Solution for times before the final period

The equation for final period of decay is obtained by considering the two-point correlations after neglecting the 3rd order correlation terms. In order to study the decay for times before the final period, the three point correlations are considered and the quadruple correlation terms are neglected because the quadruple correlation terms decays faster than the lower-order correlation terms. For a better equation of the MHD homogeneous turbulence decay from its initial period to its final period , three-point correlation equations are to be considered. Here, we neglect the quadruple correlation terms since the decay faster than the lower order correlation terms.

Putting the value of $\langle \gamma \beta'_i \beta''_j\rangle$ from equation (35) into equation (32 and neglecting all the quadruple correlation terms, we have

$$\frac{\partial \langle \phi_l \beta_i' \beta_i'' \rangle}{\partial t} + \lambda \left[(1+P_M)(k^2 + k'^2) + 2P_M k_k k_k' + 2\frac{R}{\lambda} + \frac{2 \in_{mkl} \Omega_m}{\lambda} - \frac{f}{\lambda} \right] \langle \phi_l \beta_i' \beta_i'' \rangle$$

$$+ f \langle \delta_l \beta_i' \beta_i'' \rangle = 0$$

$$\Rightarrow$$

$$\frac{\partial \langle \phi_l \beta_i' \beta_i'' \rangle}{\partial t} + \lambda \left[(1+P_M)(k^2 + k'^2) + 2P_M k_k k_k' + 2\frac{R}{\lambda} + \frac{2 \in_{mkl} \Omega_m}{\lambda} - \frac{fs}{\lambda} \right] \langle \phi_l \beta_i' \beta_i'' \rangle = 0$$

-------(36)

where $\langle \delta_l \beta_i' \beta_i'' \rangle = C \langle \phi_l \beta_i' \beta_i'' \rangle$ and 1-C=S, C and S are arbitrary constants.

Taking inner multiplication by k_l, we get

$$\frac{\partial (k_l \langle \phi_l \beta_i' \beta_i'' \rangle)}{\partial t} + \lambda \left[(1+P_M)(k^2 + k'^2) + 2P_M k_k k_k' + \frac{2R}{\lambda} + \frac{2 \in_{mkl} \Omega_m}{\lambda} - \frac{fs}{\lambda} \right] (K_l \langle \phi_l \beta_i' \beta_i'' \rangle) = 0$$

-----------(37)

Integrating the equation (37) between t_0 and t, and gives

$$k_l \langle \phi_l \beta_i' \beta_i'' \rangle = k_l [\langle \phi_l \beta_i' \beta_i'' \rangle_0] \exp \{ -\lambda [((1+P_M)(k^2 + k'^2) + 2P_M k_k k' \cos \theta$$

$$+ \frac{2R}{\lambda} + \frac{2 \in_{mkl} \Omega_m}{\lambda} - \frac{fs}{\lambda}](t\text{-}t_0) \}$$ -------(38)

where θ is the angle between $\hat{k}$ and $\widehat{k}'$ and $\langle \phi_l \beta_i' \theta_i'' \rangle_0$ is the value of $\langle \phi_l \beta_i' \theta_i'' \rangle$ at t= t_o.

   Now, by letting $r' = 0$ in equation (23) and comparing with equations (12) and (13), we get

$$\langle \alpha_i \psi_k \psi_i'(\hat{k}) \rangle = \int_{-\infty}^{\infty} \langle \phi_l \beta_i'(\hat{k}) \beta_i''(\hat{k}') \rangle d\hat{k}',$$ -------- (39)

$$\langle \alpha_k \psi_i \psi_i'(-\hat{k}) \rangle = \int_{-\infty}^{\infty} \langle \phi_l \beta_i'(-\widehat{k}) \beta_i''(-\widehat{k}') \rangle d\hat{k}'$$ -------- (40)

Substituting equation (38) - (40) in equation (16), we get

$$\frac{\partial \langle \psi_i \psi_i'(\hat{k}) \rangle}{\partial t} + 2\lambda[k^2 + \frac{R}{\lambda}]\langle \psi_i \psi_i'(\hat{k}) \rangle = \int_{-\infty}^{\infty} 2ik_l \left[\langle \phi_l \beta_i'(\hat{k})\beta_i''(\hat{k}') \rangle - \langle \phi_l \beta_i'(-\hat{k}')\beta_i''(-\hat{k}') \rangle \right]$$

$$\times \exp\left[-\lambda\{(1+p_M)(k^2 + k'^2) + 2P_M kk' \cos\theta + \frac{2R}{\lambda} + 2\frac{\in_{mkl}\Omega_m}{\lambda} - \frac{fs}{\lambda}\}(t-t_0) \right] d\hat{K}'.$$

----- (41)

Now, $d\hat{k}'$ can be expressed in terms of k' and θ that is
$d\hat{K}' = -2\pi k'^2 d(\cos\theta)dk'$ (cf. Deissler [2]).

With the above relation, equation (41) to give

$$\frac{\partial \langle \psi_i \psi_i'(\hat{k}) \rangle}{\partial t} + 2\lambda[k^2 + \frac{R}{\lambda}]\langle \psi_i \psi_i'(\hat{k}) \rangle = 2 \int_{-\infty}^{\infty} 2ik_l \left[\langle \phi_l \beta_i'(\hat{k})\beta_i''(\hat{k}') \rangle - \langle \phi_l \beta_i'(-\hat{k}')\beta_i''(-\hat{k}') \rangle \right]_0$$

$$\times k'^2 \left[\int_{-1}^{1} \exp\left\{ -\lambda(t-t_0)[(1+P_M)(k^2 + k'^2) + 2P_M kk' \cos\theta + \frac{2R}{\lambda} + 2\frac{\in_{mkl}\Omega_m}{\lambda} - \frac{fS}{\lambda}] \right\} d(\cos\theta) \right] d\hat{K}'$$

---------(42)

In order to find the solution completely and following Loeffler and Deissler [3] we assume that

$$ik_l \left[\langle \phi_l \beta_i'(\hat{K})\beta_i''(\hat{K}') \rangle - \langle \phi_l \beta_i'(-\hat{k})\beta_i''(-\hat{k}') \rangle \right]_0 = -\frac{\xi_0}{(2\pi)^2} \left[k^2 k'^4 - k^4 k'^2 \right]$$

----------- (43)

where ξ_0 is a constant. The negative sign is placed in front of ξ_0 in order to make the transfer of energy from small large wave numbers for positive value of ξ_0.

Substituting equation (43) into equation (42) and completing the integration with respect to $\cos\theta$, one obtains

$$\frac{\partial(2\pi\langle\psi_i\psi'_i(\hat{K})\rangle)}{\partial t} + 2\lambda[k^2 + \frac{R}{\lambda}](2\pi\langle\psi_i\psi'_I(\hat{K})\rangle)' = -\frac{\xi_0}{v(t-t_o)}\int_0^\infty\left(k^3k'^5 - k^5k'^3\right)$$

$$\times\left[\exp\left\{-\lambda(t-t_o)[(1+P_M)(k^2+k'^2) - 2P_M kk' + \frac{2R}{\lambda} + 2\frac{\in_{mkl}\Omega_m}{\lambda} - \frac{fS}{\lambda}]\right\}\right.$$

$$\left.- \exp\left\{-\lambda(t-t_o)[(1+P_M)(k^2+k'^2) + 2P_M kk' + \frac{2R}{\lambda} + 2\frac{\in_{mkl}\Omega_m}{\lambda} - \frac{fS}{\lambda}]\right\}\right]dK'.$$

---------- (44)

Multiplying both sides of equation (44) by k^2, we get

$$\frac{\partial H}{\partial t} + 2\lambda\left(k^2 + \frac{R}{\lambda}\right)H = G,$$

-------- (45)

where, H$=2\pi k^2\langle\psi_i\psi'_k(\hat{K})\rangle$ is the magnetic energy spectrum function and G is the magnetic energy transfer term is given by

$$G = \frac{\xi_0}{v(t-t_0)}\int_0^\infty(k^3k^5 - k^5k^3)\left[\exp\left\{-\lambda(t-t_0)[(1+P_M)(k^2+k^2) - 2P_M kk' + \frac{2R}{\lambda} + \frac{2\in_{mkl}\Omega_m}{\lambda} - \frac{fS}{\lambda}\right\}\right.$$

$$\left.- \exp\left\{-\lambda(t-t_0)[(1+P_M)(k^2+k'^2) + 2P_M kk' + \frac{2R}{\lambda} + \frac{2\in_{mkl}\Omega_m}{\lambda} - \frac{fS}{\lambda}]\right\}\right]dK'$$

--------(46)

Integrating equation (46) with respect to K', we have

$$G = \frac{-\xi_0 P_M\sqrt{\pi}}{4\lambda^{3/2}(t-t_0)^{3/2}(1+P_M)^{5/2}}\exp[-\{2R + \frac{2\in_{mkl}\Omega_m}{\lambda} - \frac{fS}{\lambda}\}(t-t_0)]\exp\left[-\lambda(t-t_0)(\frac{1+2P_M}{1+P_M})k^2\right]$$

$$\times\left[\frac{15P_M k^4}{4v^2(t-t_0)^2(1+P_M)} + \frac{1}{(t-t_0)}\left\{\frac{5P_M^2}{v(1+P_M)^2} - \frac{3}{2v}\right\}k^6 + \frac{P_M}{(1+P_M)}\left\{\frac{P_M^2}{(1+P_M)^2} - 1\right\}k^8\right]$$

--------------(47)

The series of equation (42) contains only even powers of k and start with k^4 and the equation represents the transfer function arising owing to consideration of magnetic field at three points at a time.

It is interesting to note that if we integrate equation (47) over all wave numbers, we find that

$$\int_0^\infty Gdk = 0$$ -------- (48)

which indicates that the expression for G satisfies the condition of continuity and homogeneity.

The linear equation (45) can be solved to give

$$H = \exp\left[-2\lambda(k^2 + \frac{R}{\lambda})(t-t_o)\right]\left[\int G\exp\left[2\lambda(k^2 + \frac{R}{\lambda})(t-t_o)\right]dt\right.$$

$$+ J(k)\exp\left[-2\lambda(k^2 + \frac{R}{\lambda})(t-t_o)\right]$$ -------- (49)

where J(K)=$\dfrac{N_0 k^2}{\pi}$ is a constant of integration and can be obtained as by

Corrsin[12].   Substituting the values of G from equation (47) in to equation (49) and integrating with respect to t, we get

$$H = \frac{N_0 k^2}{\pi}\exp\left[-2\lambda(k^2 + \frac{R}{\lambda})(t-t_o)\right] + \frac{\xi_0\sqrt{\pi}\,p_M}{4\lambda^{3/2}(1+P_M)^{7/2}}$$

$$\times \exp[-\{2R + 2\in_{mkl}\Omega_m - fS\}(t-t_o)]\exp\left[-\lambda k^2\left\{\frac{1+2p_M}{(1+p_M)}\right\}(t-t_o)\right]$$

$$\left[\frac{3p_{M^r}k^4}{2\lambda^2 P_M(t-t_o)^{5/2}} + \frac{(7p_M - 6)k^6}{3\lambda(1+p_M)(t-t_o)^{3/2}} - \frac{4(3p_M^2 - 2p_M + 3)k^8}{3(1+p_M)^2(t-t_o)^{1/2}}\right]$$

$$+\frac{8\lambda^{1/2}(3p_M^2-2p_M+3)k^9}{3(1+p_M)^{5/2}}N(\omega)\Bigg]$$

--------- (50)

where $\quad N(\omega)=e^{-\omega^2}\int_o^\omega e^{x^2}dx,\qquad \omega=k\sqrt{\dfrac{\lambda(t-t_o)}{(1+p_M)}}.$

By setting $\quad \hat{r}=0,\; j=i,\; d\hat{K}=-2\pi k^2 d(\cos\theta)d\hat{k}\quad$ and $H=2\pi k^2\left\langle\psi_i\psi'_j(\hat{K})\right\rangle$ in equation (9), we get the expression for magnetic energy

decay with the fluctuating concentration as $\quad \dfrac{\langle h^2\rangle}{2}=\dfrac{\langle h_i h'_i\rangle}{2}=\int_0^\infty Hd\,\hat{k}.$

---------- (51)

Substituting equation (50) in to (51) and after integration with respect to k, we get

$$\frac{\langle h^2\rangle}{2}=\exp[-2R(t-t_0)]\Bigg[\frac{N_o(t-t_o)^{-3/2}}{8\lambda^{3/2}\sqrt{2\pi}}+\exp\left[-\{2\in_{mkl}\Omega_m-fS\}\right]\times\frac{\xi_0\pi(t-t_o)^{-5}}{4\lambda^6(1+p_M)(1+2p_M)}$$

$$\times\left\{\frac{9}{16}+\frac{5p_M(7p_M-6)}{16(1+2p_M)}-\frac{35p_M(3p_M^2-2p_M+3)}{8(1+2p_M)^2}+\frac{8p_M(3p_M^2-2p_M+3)}{3.2^6.(1+2p_M)^3}\sum_{n=0}^\infty\frac{1.3.5....(2n+9)}{n!(2n+1)2^{2n}(1+p_M)^n}\right\}\Bigg]$$

or

$$\frac{\langle h^2\rangle}{2}=\exp[-2R(t-t_0)]\Bigg[\frac{N_o(t-t_o)^{-3/2}}{8\lambda^{3/2}\sqrt{2\pi}}+\exp\left[-\{2\in_{mkl}\Omega_m-fS\}\right]\xi_0 Z(t-t_o)^{-5}\Bigg]$$

----------(52)

where

$$Z = \frac{\pi}{(1+p_M)(1+2p_M)^{5/2}}[\frac{9}{16} + \frac{5p_M(7p_M-6)}{16(1+2p_M)} - \frac{35p_M(3p_M^2-2p_M+3)}{8(1+2p_M)^2}$$
$$+ \frac{8p_M(3p_M^2-2p_M+3)}{3.2^6.(1+2p_M)^3} + ----]$$

Thus the decay law for magnetic energy fluctuation of dusty fluid MHD turbulence in a rotating system governing the concentration of a dilute contaminant undergoing a first order chemical reaction before the final period may be written as

$$\langle h^2 \rangle = \exp[-2R(t-t_0)]\left[A(t-t_0)^{-3/2} + \exp[-\{2 \in_{mkl} \Omega_m - fs\}]B(t-t_0)^{-5} \right]$$

---------(53)

where, $A = \dfrac{N_0}{8\lambda^{3/2}\sqrt{2\pi}}$, $B = \xi_0 Z$

6. Results and discussion

In equation (53) we obtained the decay law for magnetic energy fluctuation of dusty fluid MHD turbulence governing the concentration of a dilute contaminant undergoing a first order chemical reaction before the final period in a rotating system considering three-point correlation after neglecting quadruple correlation terms. If the system is non-rotating and the fluid is clean i.e. $\Omega_m = 0$, and f=0 then the equation (53) becomes

$$\langle h^2 \rangle = \exp[-2R(t-t_0)]\left[A(t-t_0)^{-3/2} + B(t-t_0)^{-5} \right]$$

----------(54)

which was obtained earlier by Sarker and Islam [14]

In absence of chemical reaction, i.e, R=0 then the equation (54) becomes

$$\langle h^2 \rangle = \left[A(t-t_0)^{-3/2} + B(t-t_0)^{-5} \right]$$

-----------(55)

which was obtained earlier by Sarker and Kishor [7].

This study shows that due to the effect of rotation of fluid in the magnetic field with chemical reaction of the first order in the concentrarion in presence of dust particle in a rotating system the magnetic field fluctuation i,e.the turbulent energy decays more rapidly than the energy for non-rotating clean fluid and the faster rate is governed by $\exp[-\{2 \in_{mkl} \Omega_m\} - fs]$. Here the chemical reaction (R≠0) in MHD turbulence causes the concentration to decay more they would for non-rotating clean fluid and it is governed by $\exp[-\{2RT_M + 2 \in_{mkl} \Omega_m - fs\}]$.

The first term of right hand side of equation (53) corresponds to the energy of magnetic field fluctuation of concentration for the two-point correlation and the second term represents magnetic energy for the three-point correlation. In equation (53), the term associated with the three-point correlation die out faster than the two-point correlation. For large times the last term in the equation (53) becomes negligible, leaving the -3/2 power decay law for the final period. If higher order correlations are considered in the analysis, it appears that more terms of higher power of time would be added to the equation (53).

REFERENCES

1. Deissler,R.G (1958): On the decay of homogeneous turbulence before the final period, Phys.Fluid,1,111-121.
2. Deissler,R.G(1960): Decay law of homogeneous turbulence for time before the final period. ,Phys.   Fluid,3, 176-187 .
3. Loefter,A.L and Deissler,R.G.(1961): Decay of temperature fluctuations in homogeneous turbulence   before the final period, Int. J. Heat mass Transfer,1,312-324.
4. Kumar,P. and Patel, S.R.(1974): First order reactant in homogeneous turbulence before the final period for the case of multi-point and single time, physics of fluids, 17, 1362.
5. Kumar,P. and Patel, S.R.(1975): First order reactant in homogeneous turbulence before the final period for the case of multi-point and multi-time,Int. J. Engng. Sci, 13, 305-315.
6. Patel, S.R.(1976): First order reactant in homogeneous turbulence Numerical results, Int. j. Engng. Sci,14, 75.
7. Sarker,M.S.A and Kishore,N (1991): Decay of MHD turbulence before the final period,Int. J. Engng   Sci. 29,1479-1485.
8. Chandrasekhar, S. (1951a): Proc ,Roy ,Soc , London, A204 ,435 -449 .
9. Sarker,M.S.A and Islam,M.A (2001): Decay of MHD turbulence before the final period for the case of multi-point and multi-time,Indian J. Pure appl. Math,32(7),1065-1076.
10. Islam,M.A and Sarker,M.S.A (2001): First order reactant in MHD turbulence before the   final period of deay for the case of multi-point and multi-time, Indian J. Pure appl. Math, 32(8), 1173-1184.
11. Sarker.M.S.A and Islam.M.A (2001): Decay of dusty fluid turbulence before the final period   in a rotating system , J. Math and Math. Sci, 16, 35-48.
12. Corrsin, S. (1951b): J. Applied physics, 22, 469.
13. Azad, M.A.K and Sarker.M.S.A (2003): Decay of dusty fluid MHD turbulence before the final period in a rotating system for the case of multi-point and multi-time , Bihar   Mathematical Society, 23, 1-20.
14. Sarker, M.S.A. and Islam, M.A. (2001): Ph.D. Thesis, Dept. of Mathematics, R.U., 56-70.