2023

M.Sc.

4th Semester Examination

PHYSICS

PAPER : PHS-402.1 & 402.2

Full Marks : 40

Time : 2 hours

The figures in the right-hand margin indicate marks. Candidates are required to give their answers in their own words as far as practicable. Illustrate the answers wherever necessary.

SECTION—I

(NUCLEAR PHYSICS—II)

PHS-402.1

1. Answer *any* **two** questions from the following : $2\times2=4$

(a) What is the isotopic spin of deuteron? Justify your answer.

(b) Calculate the energy of the proton detected at 90° when 2·1 MeV deuterons are incident on ^{27}Al to produce ^{28}Al with an energy difference $Q = 5{\cdot}5$ MeV.

(c) Why are the velocities of thermal neutrons in a reactor given by Maxwell distribution?

(d) Write the disadvantages of reactor sources used for productions of neutrons.

2. Answer *any* **two** questions from the following : 4×2=8

(a) *(i)* A crystal spectrometer for neutrons uses a rock salt crystal with a grating space of 2·18 Å. The neutrons are detected at an angle of 15°. What is the energy of the neutrons selected in the first order diffraction? 4

(ii) What do you understand by the level width (Γ) and level separation (D) between the levels of a continuum in nuclear reactions? 2+2=4

(b) The neutron and proton in the deuteron nuclide interact through a square well potential of width $b = 1 \cdot 9$ fm, and depth $V_0 = 40$ MeV in an $l = 0$ state. Calculate the probability that the proton moves within the range of the deuteron. Use the approximation that $m_n = m_p = m$, $kb = \pi/2$, $k = \sqrt{m(V_0 - \varepsilon_d)}/\hbar$, and ε_d is the binding energy of the deuteron. 4

(c) *(i)* What are the basic principles of velocity selector for neutron monochromator?

(ii) Correlates the fertile and fissile materials. 2+2=4

(d) *(i)* What do you mean by nuclear reaction cross-section?

(ii) How are the magic numbers explained using shell model? 1+3=4

3. Answer *any* **one** question from the following : 8×1=8

(a) *(i)* In n-p scattering, S-wave scattering is predominant in the energy range below 10 MeV. Write the comments on this observation. 8

(ii) Derive the relation between refractive index (μ) and scattering length (a) of a material with nuclei per unit volume (N) due to neutron for wavelength (λ). 8

(iii) How would the limitations of liquid-drop model be resolved in single-particle shell model? 2+4+2=8

(b) *(i)* In case of harmonic oscillator shell model, establish the relation between the harmonic oscillator frequency and the nuclear size. 8

(ii) Find the spin and parity of $_{15}P^{30}$ nucleus by using Nordheim's rule for shell model. 8

(iii) Calculate the slowing down time in graphite for neutrons having an initial energy of 2 MeV and final thermal energy 0·025 eV. Given, λ_s = 2·6 cm and ξ = 0·155. 3+2+3=8

SECTION—II

(QUANTUM FIELD THEORY)

PHS-402.2

1. Answer *any* **two** questions from the following : $2\times2=4$

(a) Show that if ψ satisfies the Dirac equation, it also satisfies the Klein-Gordon equation.

(b) For a complex scalar field theory, using the mode expansion for $\phi(x)$ write down $\langle 0|\phi(x)\phi^{\dagger}(y)|0\rangle$ as an integral over three-momenta.

(c) The Lagrangian density for a massive vector field A^{μ} is given by

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A_{\mu}A^{\mu}$$

Derive the equation of motion for the vector field A^{μ} and show that the equation of motion implies $\partial_{\mu}A^{\mu}=0$.

(d) Define $j_5^{\mu}(x)$ as the Noether current corresponding to the transformation $\psi(x)\rightarrow e^{i\alpha\gamma^5}\psi(x)$, ($\alpha$ is a constant) for a free Dirac theory with mass m. Compute $j_5^{\mu}(x)$ and find the condition under which $\partial_{\mu}j_5^{\mu}(x)=0$.

2. Answer *any* **two** questions from the following : $4\times2=8$

(a) In real scalar field theory with $\mathcal{L}_{int} = \frac{\lambda}{3!}\phi^3$, write down the expression for $\langle\Omega|T\{\phi(x_1)\phi(x_2)\}|\Omega\rangle$ up to order λ^2 in terms of the Feynman propagator. Draw the corresponding Feynman diagrams.

(b) Given that the Dirac field transforms as $\psi \to \Lambda_{\frac{1}{2}}\psi$, where $\Lambda_{\frac{1}{2}} = \exp\left[-\frac{i}{2}S^{\mu\nu}\omega_{\mu\nu}\right]$. Evaluate the matrix $\Lambda_{\frac{1}{2}}$ for Lorentz-boost along the z-axis. [Given, $S^{\mu\nu} = \frac{i}{4}[\gamma^\mu, \gamma^\nu]$]

(c) For a real scalar free field theory, the Hamiltonian operator in terms of creation and annihilation operators is

$$H = \int \frac{d^3p}{(2\pi)^3} E_p\, a_p^\dagger a_p$$

Using the mode expansion for $\phi(x, 0)$, show that $e^{iHt}\phi(x,0)e^{-iHt} = \phi(x,t)$.

(d) The integral representation of the Feynman propagator for a free real scalar field theory is given as

$$D_F(x-y)=\int \frac{d^4p}{(2\pi)^4}\frac{ie^{-ip\cdot(x-y)}}{p^2-m^2+i\varepsilon}$$

Perform the p_0 integral and show that $D_F(x-y)$ can be written as $\langle 0|T\{\phi(x)\phi(y)\}|0\rangle$.

3. Answer *any* **one** question from the following : 8×1=8

(a) (i) The action for a complex scalar field ϕ coupled to the electromagnetic field A_μ in 4 space time dimensions is given by,

$$S=\int d^4x\left[|D_\mu\phi|^2-m^2|\phi|^2-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}\right]$$

where the covariant derivative, $D_\mu=\partial_\mu+ieA_\mu$. Write down the dimension of the coupling constant e in the units of $\hbar=c=1$. Use the Noether's prescription to find the energy momentum tensor $T^{\mu\nu}$ for the above theory. Is $T^{\mu\nu}$ symmetric? If not, symmetrize it. 1+5 *(ii)* A solution for the free Dirac equation is written as $u^{s}(p)e^{-ip\cdot x}$. Show that

$$\sum_{s} u_{\alpha}^{s}(p)\bar{u}_{\beta}^{s}(p) = (\not{p} + m)_{\alpha\beta}$$ 2

(c) *(i)* Consider the free Dirac theory

$$S = \int d^4x \, \bar{\psi}(i\gamma^{\mu}\partial_{\mu} - m)\psi$$

Compute the conserved current j^{μ} corresponding to the global symmetry transformation $\psi(x) \to e^{i\alpha}\psi(x)$, ($\alpha$ is a constant). Next evaluate the normal ordered charge operator : Q : the free Dirac theory in terms of the creation and annihilation operators. 6

(ii) If $Pa_{\vec{p}}P^{-1} = a_{-\vec{p}}$, where P is the parity operator and $a_{\vec{p}}$ is the annihilation operator for a real scalar field. Show that

$$P\phi(t, \vec{x})P^{-1} = \phi(t, -\vec{x})$$ 2

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