M.Sc. 2nd Semester Examination, 2023

APPLIED MATHEMATICS

(*General Topology*)

PAPER – MTM-206

Full Marks : 20

Time : 2 hours

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

1. Answer any *two* questions : 2×2

(*a*) Is the collection $\tau = \{U : X - U$ is infinite or empty or all of $X\}$ a topology on X?

(*b*) Show that the order topology on $\mathbb{Z}_+$ is the discrete topology.

(*c*) Define homeomorphism of two topological space with an example.

(*d*) Define basis for a topology on a set X.

2. Answer any *two* questions : 4×2

(*a*) Let A be a subset of a topological space X. Then show that $x \in \bar{A}$ if and only if every open set U containing x intersects A. 4

(*b*) Let $f : X \to Y$ be a function, Then show that the following are equivalent : 4

(*i*) For every closed set V of Y, $f^{-1}(V)$ is closed in X.

(*ii*) For every $A \subseteq X, f(\bar{A}) \subseteq \overline{f(A)}$.

(*c*) Define convergence of a sequence of points in a topological space X. Show that in a Hausdorff space X, a sequence can converge to at most one point of X. 4

(*d*) Show that $\mathbb{R}^{\omega}$ in the box topology is not connected. 4

3. Answer any *one* question : 8×1

(*a*) (*i*) Let X be a Hausdorff space and S be a compact subspace of X. Then show that S is closed in X. 5

(*ii*) Show that every compact Hausdorff space is normal. 3

(*b*) (*i*) Define regular space with example. Show that a subspace of a regular space is regular. 4

(*ii*) Define Lindelof space. Give example to show that the product of two Lindelof space need not be Lindelof. 4

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