PG/IIIS/MATH/305B/23
(New & Old)

M.Sc. 3rd Semester Examination, 2023

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER – MTM-305B

Full Marks : 50

Time : 2 hours

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

PAPER – MTM-305B (New)

(*Dynamical Meteorology-I*)

1. Attempt any *four* questions out of *six* questions : 2×4

(*i*) Derive the hydrostatic equation in the atmosphere.

(*ii*) Find the water vapor content of an air column in the atmosphere.

(*iii*) Derive the required amount of heat transferred per unit mass of air during the isobaric process.

(*iv*) Show that the potential temperature is invariant during the adiabatic process.

(*v*) Define specific humidity and mixing ratio. Also find the relation between these two.

(*vi*) Differentiate between saturated adiabatic process and pseudo-adiabatic process.

2. Attempt any *four* questions out of *six* questions : 4×4

(*i*) Derive an expression for the density ρ of an air parcel at pressure p if it is adiabatically expands from a level where pressure and density are p_s and ρ_s respectively.

(*ii*) What do you mean by isobaric cooling ? Show that the relative increase in dewpoint temperature is about 5% of the sum of relative increase in mixing ratio and pressure.

(*iii*) Define specific entropy. Establish the relation between the specific entropy and the potential temperature.

(*iv*) Derive the adiabatic lapse rate for moist unsaturated air parcel.

(*v*) Discuss the variation of pressure with respect to altitude in the atmosphere.

(*vi*) Derive the equation of state for moist air in the atmosphere.

3. Attempt any *two* questions out of *four* questions : 8×2

(*i*) Derive the area equivalence of Tephigram and discuss its important features.

(*ii*) What is the concept of geopotential in atmosphere ? State and prove the Clausius-Clapeyron equation in the atmosphere.

(*iii*) Derive the thermodynamic equation for any isolated moist saturated air parcel.

(*iv*) Discuss about the phase change of an ideal gas and derive the relation of dependency of latent heat of evaporation with respect to temperature during the phase change of an air parcel.

[Internal Assessment – 10 Marks]

PAPER – MTM-305B (Old)

(*Advanced Optimization and Operations Research*)

1. Answer any *four* questions of the following : 2×4

(*a*) Explain the effects of deletion of an existing variable in the optimal solution of an LPP.

(*b*) Write the basic differences between Fibonacci method and Golden section method.

(*c*) Explain the concept of deviational variable in goal programming problem.

(*d*) Discuss the need of integer programming in mathematical programming.

(*e*) Write the initial criteria and achievement of the dual simplex method.

(*f*) Show that for a quadratic objective function optimum point can be obtained in a single step by Newton's method.

2. Answer any *four* questions of the following : 4×4

(*a*) Following is the optimal solution of an LPP

		c_j	4	6	2	0	0
c_B	X_B	b	y_1	y_2	y_3	y_4	y_5
4	x_1	1	1	0	−1	$\frac{4}{3}$	$-\frac{1}{3}$
6	x_2	2	0	1	2	$-\frac{1}{3}$	$\frac{1}{3}$
$z_j - c_j$		16	0	0	6	$\frac{10}{3}$	$\frac{2}{3}$

If the cost coefficient c_1 changes to 8, then find the optimal basic feasible solution of the modified problem.

(*b*) Describe the branch-and-bound method to find the optimal solution of an IPP.

(*c*) Maximize

$$f(x) = \begin{cases} \frac{2x}{3}, & x \leq 3 \\ 5 - x, & x > 3 \end{cases}$$

in the interval [1,4] by Fibonacci method for $n = 5$.

(*d*) Using steepest descent method

Minimize $f = x_1^2 + x_2^2 + 8x_1 + 10x_2 + 50$

(*e*) Write the procedure of Golden section method to optimize an unimodal minimization problem.

(*f*) Explain the cutting plane method for optimization problem with non-linear objective function and non-linear constraints.

3. Answer any *two* questions of the following : 8×2

(*a*) Find the optimal basic feasible solution of the following LPP by artificial constraint method

Maximize $z = -2x_1 + x_2$

subject to $x_1 + 4x_2 \geq 5$

$x_1 - 3x_2 \leq 1$

$x_1, x_2 \geq 0$

(*b*) Solve the following IPP using Gomory's method

Maximize $z = 5x_1 + 7x_2$

subject to $-2x_1 + 3x_2 \leq 6$

$$6x_1 + x_2 \leq 30$$

$x_1, x_2 \leq 30$ and integers.

(*c*) Graphically solve the following goal programming problem

Minimize $z = P_1 d_1^- + P_2 d_2^- + P_3 d_3^-$

Subject to $2x_1 + 3x_2 \leq 30;$

$$6x_1 + 4x_2 \leq 60;$$

$$x_1 + x_2 + d_1^- - d_1^+ = 10;$$

$$x_1 + d_2^- - d_2^+ = 7;$$

$$x_2 + d_3^- - d_3^+ = 8 \text{ and}$$

$$x_1, x_2, d_i^-, d_i^+ \geq 0 (i = 1, 2, 3).$$

(*d*) Using revised simplex method, solve the following

Maximize $Z = 3x_1 + 5x_2$

Subject to $x_2 \leq 6$

$x_1 \leq 4$

$3x_1 + 2x_2 \leq 18$ and

$x_1, x_2 \geq 0.$

[Internal Assessment — 10 Marks]