PG/IIIS/MATH/302/23
(New & Old)

M.Sc. 3rd Semester Examination, 2023

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(*Transforms and Integral Equations*)

PAPER – MTM-302 (New & Old)

Full Marks : 50

Time : 2 hours

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

1. Answer any *four* questions : 2×4

(*a*) What do you mean by exponential order on Laplace transform ? Find the exponential order of the function e^{t^n} (n>1) (if exists).

(*b*) Define the term convolution on Fourier transform.

(*Turn Over*)

(*c*) Define the inversion formula for Fourier sine transform of the function $f(x)$. What happens if $f(x)$ is continuous ?

(*d*) What is the necessity to study Wavelets transform ?

(*e*) Find the Laplace transform of $f(x) = [x]$, where $[x]$ represents the greatest integer less than or equal to x.

(*f*) Define degenerate kernel with an example.

2. Answer any *four* questions : 4×4

(*a*) Form an integral equation corresponding to the differential equation

$$\frac{d^2y}{dx^2} - \sin(x)\frac{dy}{dx} + e^x y = x$$

with the initial conditions $y(0) = 1$, $y'(0) = -1$.

(*b*) If the Fourier transform of $f(x)$ is $\frac{\alpha}{1+\alpha^2}$, α being the transform parameter, then find $f(x)$.

(*c*) Solve the integral equation

$$y(x) = f(x) + \lambda \int_{-1}^{1} (xt + x^2t^2) y(t) dt.$$

(*d*) State initial value theorem in respect of Laplace transform. Evaluate $L\{J_0(t)\}$ by the help of initial value theorem, where $J_0(t)$ is the Bessel's function of order zero.

(*e*) If $L\{f(t)\} = F(p)$ which exists Real$(p) > \gamma$ and $H(t)$ is unit step function, then prove that for any α, $L\{H(t-\alpha) f(t-\alpha)\} = e^{-p\alpha} F(p)$ which exists for Real$(p) > \gamma$.

(*f*) Define wavelet transform. Write down the main advantages of wavelet theory. Compare the wavelet transform with Fourier transform.

3. Answer any *two* questions : 8×2

(*a*) (*i*) State Parseval's identity on Fourier transform. Use generalization of Parseval's relation to show that

$$\int_{-\infty}^{\infty}\frac{dx}{(x^2+a^2)(x^2+b^2)}=\frac{\pi}{ab(a+b)}, a,b>0.$$ 4

(*ii*) Find the resolvent kernel of the following integral equation and then solve it :

$$\varphi(x)=e^{x^2}+\int_0^x e^{x^2-t^2}\phi(t)dt.$$ 4

(*b*) (*i*) Solve the following ODE by Laplace transform technique :

$$ty''(t)+2y'(t)+ty(t)=\sin(t)$$

with initial condition $y(0)=1$. 6

(*ii*) Find the exponential Fourier transform of

$$f(t) = \begin{cases} 1-|t|, & |t|<1 \\ 0, & |t|>1 \end{cases}$$ 2

(c) (i) Solve the integral equation,

$$\frac{1}{\sqrt{\pi}}\int_0^x \frac{\varphi(t)}{\sqrt{x-t}}\,dt = f(x).$$ 4

(ii) If a real valued function $f(t)$ of real variable which is piecewise continuous in any finite interval of t and is of exponential order $0(e^{vt})$ as $t \to \infty$, when $t \geq 0$ then prove that the integral,

$$\int_0^\infty f(t)e^{-pt}\,dt,$$

converges in the domain Real$(p) > v$. 4

(d) Solve the following boundary value problem in the half plane $y > 0$, described by PDE :

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, -\infty < x < \infty, y > 0,$$

with boundary conditions

$$u(x,0) = f(x), -\infty < x < \infty.$$

u is bounded as $y \to \infty$; u and $\frac{\partial u}{\partial x}$ both vanish as $|x| \to \infty$. 8

[Internal Assessment – 10 Marks]