PG/IIS/MTM/202/23

M.Sc. 2nd Semester Examination, 2023

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(*Numerical Analysis*)

PAPER – MTM-202

Full Marks : 40

Time : 2 hours

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

1. Answer any *four* questions : 4×2

   (*a*) State the sufficient condition for convergence of the Gauss-Seidal interation method to solve a system of non-linear equations containing three equations and three variables.

   (*b*) Write the merits and demerits of the LU-*decomposition* method to solve a system of linear equations.

(*c*) Discretise the following equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

using the finite difference method.

(*d*) Why spline interpolation is a very powerful method ? Explain.

(*e*) What are the differences between interpolation and approximation ?

(*f*) What is Gaussian quadrature ? State its fundamental theorem.

2. Answer any *four* questions : 4×4

(*a*) Explain a finite difference method to solve the wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, \quad t > 0, \quad 0 < x < 1$$

where initial conditions $u(x,0) = f(x)$ and

$\left(\frac{\partial u}{\partial t}\right)_{(x,0)} = g(x), \quad 0 < x < 1$ and

boundary conditions $u(0,t) = \varphi(t)$ and

$$u(1,t) = \psi(t),\ t \geq 0.$$

(*b*) Suppose for a system of linear equations $AX = B$, the matrix A is decomposed as $A = LU$, where L and U are the lower and upper triangular matrices and they are known. Explain a suitable method to solve the equation $AX = B$ with the help of the matrices L and U.

(*c*) Find the inverse of the following matrix using partial pivoting.

$$\begin{pmatrix} 0 & 2 & 4 \\ 1 & 2 & -5 \\ 4 & 2 & 6 \end{pmatrix}$$

(*d*) Find all the eigenvalues and corresponding eigenvectors of the matrix

$$\begin{pmatrix} 2 & 3 \\ 3 & 3 \end{pmatrix}.$$

(*e*) Describe the approximate of a continuous function $f(x)$ using orthogonal polynomials.

(*f*) Find the value of $\int_0^2 \frac{x}{1+x^3}\,dx$ using 6-point Gauss-Legendre quadrature formula.

3. Answer any *two* questions : 8×2

(*a*) Describe Braistow's method to find all roots of an algebraic equation of degree n. 8

(*b*) Explain the successive overrelaxation method to solve a system of linear equations. 8

(*c*) Define spline interpolation. Fit a cubic spline for the points (0,1), (1,0), (2,1), (3,2), (4,3) with the conditions $y''(0) = y''(4) = 0$. $2 + 6$

(*d*) Derive the Milne's predictor and corrector formulae. Explain how many staring values are required to obtain the solution by this method and how the starting points are obtained ? $6 + 2$
