M.Sc. 2nd Semester Examination, 2023

APPLIED MATHEMATICS

(*Fluid Mechanics*)

PAPER – MTM-201

Full Marks : 40

Time : 2 hours

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

1. Answer any *four* questions out of six questions : 2×4

   (*a*) Suppose a viscous fluid (of density 900 Kg/m^3 and viscosity 0.5 Ns/m^2) is flowing through a pipe of radius 20 mm at a speed 3 m/s. Calculate the Reynolds number for *this flow.*

(*b*) Find the substantial derivative of the steady state velocity field represented by the velocity vector

$$\vec{V} = (-3x, -3y, 6z).$$

(*c*) Find the thickness of the boundary layer for an incompressible viscous flow over a flat plate of length 10 meter at which the Reynold number is 10^4.

(*d*) Write the physical principles used for the equations of continuity, Navier-Stokes and energy, and then write the equations of continuity and Navier-Stokes for incompressible viscous two-dimensional flow.

(*e*) Give reason for the use of scaling or ordering terms in the fluid mechanics.

(*f*) Write the z-component of Reynolds Averaged Navier-Stokes (RANS) equations.

2. Answer any *four* questions out of six questions : 4×4

(*a*) For the Poisseuille flow in a channel, write the necessary assumptions, deduce the approximate equation, draw the flow configuration and hence write the required boundary condition on the configuration. Also write the complete set of Navier-Stokes equation for a laminar incompressible viscous flow in the channel, draw the flow configuration and show the boundary conditions at the inlet, outlet and channel walls. Finally show the velocity profile graphically for the Couette-Poisseuille flow for positive and negative pressure gradients.

(*b*) Draw an infinitesimally small moving element and show all the surface forces acting along the y-direction on the element. Finally find the net surface and body forces acting on that element.

(c) An incompressible velocity fields is given by $u = a(x^2 - y^2)$, $v = -2axy$ and $w = 0$. Determine under what conditions it is a solution to the Navier-Stokes momentum equation for the case of without any body forces. Assuming that these conditions are met, determine the resulting pressure distribution.

(d) Write the set of governing equations for the boundary layer flow along a flat plate along with the proper boundary conditions for the above set of equations. Show that the x-component of the momentum equations applied at the edge of the boundary layer reduces to the Bernoulli equation. Finally write the governing equations outside the Boundary layer.

(e) Make the two-dimensional unsteady energy equation

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial x^2}$$

in non-dimensional form (in terms of Reynolds number $\mathrm{Re} = \frac{UL}{\nu}$ and Prandtl number $\mathrm{Pr} = \frac{\nu}{\alpha}$) with the help of characteristics time, length and velocity as L/U, L and U, respectively, with non-dimensional temperature $\theta = \frac{T - T_0}{T_0}$ where T_0 is the reference temperature and symbols have their usual meaning.

(*f*) For typical horizontal length scale (L) of 700 km, horizontal speeds (U) are of the order of 0.1 ms^{-1} and a vertical scale length (H) of 1100 m, estimate a typical vertical speed (W).

3. Answer any *two* questions out of three questions : 8×2

(*a*) (*i*) An incompressible velocity fields is given by u is unknown, $v = c(x^2 - 2xy +$

y^2), $w = a - y^2$. What should be the form of the velocity component u ?

(*ii*) Derive the energy equation for non-Newtonian, incompressible, viscous fluid flow with negligible radiation effects. 2 + 6

(*b*) (*i*) Write the assumptions of boundary layer theory.

(*ii*) Based on the above assumptions, derive the set of governing equations for the boundary layer flow along a flat plate. Also write the proper boundary conditions for the above set of equations. 2 + 6

(*c*) (*i*) Write all the equations of motion in terms of eddy viscosities.

(*ii*) For the ocean with horizontal and vertical length scales 1000 KM and

2 KM, respectively and horizontal speed of order 0.15 m/s, scale all the above equations written in part-(i) and reduces to approximated equations with order of accuracy 1%. 2 + 6

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