2022

M.Sc.

4th Semester Examination

PHYSICS

PAPER—PHS-401

Full Marks : 40

Time : 2 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

PHS–401.1 PARTICLE PHYSICS

[Marks : 20]

1. Answer any *two* questions : 2×2

(a) Using the angular momentum conservation determine if the process $\rho^0 \rightarrow \pi^0 + \pi^0$ is allowed.

(b) Consider the SU(n) Lie algebra where the generators satisfy

$$\left[t^a, t^b\right] = i f^{abc} t^c \text{ with } tr\left(t^a t^b\right) = \frac{1}{2}\delta^{ab}$$

[Repeated indices are summed over]. Show that

(i) $t_a^\dagger = t_a$ (ii) $tr(t_a) = 0$.

(c) In the $K^0 - \overline{K}^0$ system the CP even/odd states are $K_{L/S} = \frac{1}{\sqrt{2}}\left(K^0 \pm \overline{K}^0\right)$. Giving reasons identify the state which decays into two pions and the one into three pions.

(d) Using the conjugate representation of SU(2) Isospin show that $T_+ \overline{u} = -\overline{d}$.

2. Answer any *two* questions : 2×4

(a) Given that the fermion field $\psi(t, x)$ transforms Charge Conjugation (C) as $-i\gamma^2\psi^*(t, x)$, determine the transformations of $\overline{\psi}\gamma^\mu\psi$ under C.

(b) For the process $\pi^- + d \rightarrow n + n$, if the initial orbital angular momentum of $\pi^- + d$ pair is $l = 1$, find the possible values of spin and orbital angular momentum for the n + n pair. The spin-parity of pion π^- and deuteron d are 0^- and 1^+ respectively.

(c) The three states with $I_3 = Y = 0$ in $3 \otimes \bar{3}$ are $[u\bar{u} - d\bar{d}]$, $[d\bar{d} - s\bar{s}]$ and $[d\bar{d} - s\bar{s}]$. Determine the irreducible states i.e the wave functions of the π^0, η and the η' mesons in terms the constituent (u, d, s) quarks.

(d) Assuming only isospin symmetry find the ratio of the following cross sections :

$$\sigma(K^- + p \rightarrow \pi^+ + \Sigma^-) ;$$
$$\sigma(K^- + p \rightarrow \pi^0 + \Sigma^0) ;$$
$$\sigma(K^- + p \rightarrow \pi^- + \Sigma^+).$$

(Table of Clebsh-Gordan coefficients will be provided on request).

3. Answer any *one* question : 1×8

(a) (i) Starting from the fact that under Parity (P) the spin-1/2 creation operators transform as

$$Pa_p^{st}P^\dagger = \eta_a^* a_p^{st} \text{ and } Pb_p^{st}P^\dagger = \eta_b^* b_p^{st}.$$

Show that $P\psi(x)P^\dagger = \eta_a \gamma^0 \psi(-x, t)$. Further explain why a fermion-antifermion pair has odd intrinsic parity.

(ii) Define a G-parity operator as $G = Ce^{i\pi I_2}$ where C is the charge conjugation operator and I_2 is the second component of isospin. Show that $\pi^\pm$ are eigenstates of G. What are the eigenvalues ?

(b) (i) Consider a bound state of e^+ and e^- with S and P wave-functions. List the P (parity) and C (charge conjugation) quantum numbers for these states. Find the number of photons

each of these states can decay into (assuming that electromagnetic interaction is invariant under Parity and Charge Conjugation).

(ii) Consider a complex scalar field theory with a continuous Global symmetry defined by the Lagrangian

$$L = \partial_\mu \phi(x)^* \partial^\mu \phi(x) - \mu^2 \phi(x)^* \phi(x) - \lambda\left(\phi^* \phi\right)^2.$$

For the case of spontaneous symmetry breaking, $\mu^2 < 0$, $\lambda > 0$, by considering fluctuations about the minimum of the potential express the Lagrangian in terms of the fluctuation fields. What are the masses of the fields after spontaneous symmetry breaking.

PHS-401.2 STATISTICAL MECHANICS - II

[Marks : 20]

4. Answer any *two* questions : 2×2

(a) Plot the variation of chemical potential with temperature for FD and BE ideal gases.

(b) Write down the expression for number of particles having spins parallel or antiparallel to the magnetic field H.

(c) Show that correlation $\langle S_i S_j \rangle$ in Ising model is $\tanh^2 J_e\beta$.

(d) Plot the temperature dependence of the fugacity for FD and BE ideal gases.

5. Answer any *two* questions : 2×4

(a) Prove that Landau diamagnetism is represented by

$$X_\infty = -\langle n \rangle \frac{\mu^2}{3}\beta \quad \text{where} \quad \beta = \frac{1}{k_B T}.$$

(b) Prove that for Ising model long range order parameter

$$L(T) = \tanh \beta(J_e \gamma L + \mu_0 H)$$

where γ is the n.n.n.

(c) Prove that isothermal compressibility $K_T \sim (T - T_c)^{-1}$ for a gas of free Spin-o-Bosons in 3D.

(d) Show that zero point energy of solid $E_0 = \frac{9}{8} N k_B T_D$ where T_D is the Debye temperature.

6. Answer any *one* question : 1×8

(a) Spins on a 1D lattice have a 3 level Ising Hamiltonian in absence of magnetic field

$$\hat{H} = -Je \sum_{i=1}^{N} S_i S_{i+1} \; ; \; S_i = 1, 0, -1 \; ; \; Je > 0.$$

Calculate the exact partition function in terms of temperature and the number of sites.

(b) Find an expression of sp. ht. for 3D non-interacting Bosons gas at $T > T_c$ and $T < T_c$ where Tc is the condensation temperature.

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