M.Sc. 3rd Semester Examination, 2022

PHYSICS

(*Quantum Mechanics*-III/*Statistical Mechanics*-I)

PAPER – PHS-301.1 & 301.2

Full Marks : 40

Time : 2 hours

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

PAPER—PHS-301.1

(*Quantum Mechanics*-III)

[*Marks* : 20]

Answer any two of the following : 2 × 2

1. Write down the following Lippmann-Schwinger equation in the position basis in one dimension and identify the Green's function $G_{\pm}(x, x')$.

$$\left|\psi^{(\pm)}\right\rangle = \left|\phi\right\rangle + \frac{1}{E - E_0 \pm i \in} V \left|\psi^{(\pm)}\right\rangle$$

Find the momentum representation of the Green's function.

2. By acting on the singlet state, check that the permutation operator P_{12}^{spin} for a two spin-1/2 particle state can be written as

$$\frac{1}{2}\left(1 + \frac{4}{\hbar^2} S_1 . S_2\right).$$

3. Starting from

$$S(\{\psi_i\}) = \left\langle\phi\middle|H_e\middle|\phi\right\rangle + \sum_{i=1}^{N} \epsilon_i \left(1 - \left\langle\psi_i \mid \psi_i\right\rangle\right)$$

extermize S to derive the Hartree equation, where $|\phi\rangle$ and $|\psi_i\rangle$ represent the N and one particle states respectively.

4. Given that the scattering amplitude in the partial wave analysis is

$$f(\theta) = \frac{1}{k}\sum_{l=0}^{\infty}(2l+1)\, e^{i\delta_l} \sin \delta_l P_l(\cos\theta),$$

use the optical theorem to find the total cross section σ_{tot} in terms of the phase shift δ_l.

Answer any **two** of the following : 4×2

5. A particle of mass m is in a one-dimensional harmonic oscillator potential $V_1 = \frac{1}{2}kx^2$. The particle is initially in the ground state of V_1. The constant k is suddenly doubled to $2k$, so that the new potential is $V_2 = kx^2$. What is the probability of finding the particle in the ground and first excited states of the potential V_2 ?

6. Write down the wave functions including the spin of the electrons in an helium atom for the following electronic configuration $(1s)(2s)$ neglecting the e^2/r_{12} interaction term. Next

considering the electron-electron interaction, write down the expression for the difference in the energy levels ΔE, of the electronic states. You do not need to perform any integral. Draw a schematic diagram showing the energy levels of the electronic configurations.

7. The outgoing wave function of a particle of mass m and momentum $p = \hbar k$ scattering off a hard sphere of radius R is given by

$$\langle x | \psi^+ \rangle = 1/(2\pi)^{3/2} \sum_l i^l (2l+1) A_l(r) P_l(\cos(\theta))$$

for $r > R$. Find $A_l(r)$ in terms of the phase-shift δ_l. Further find δ_l in terms of R and k.

8. The ground state of a hydrogen atom is subjected to a time-dependent potential $V = V_0 \cos(kz - \omega t)$. Using time-dependent perturbation theory obtain (to leading order) an expression for the transition rate at which the electron is ionized and emitted with momentum p. You can take the final wave

function to be $\psi_f(x) = (1/L^{3/2})\, e^{ip.x/\hbar}$. Leave your answer in the form of an integral.

Answer any **one** of the following : 8×1

9. (*a*) Consider scattering of a particle from a potential $V(r) = V_0 e^{-(r^2/R^2)}$. Compute the differential cross section in the Born approximation (up to the first order). 4

(*b*) A particle, initially (i.e., $t \to \infty$) in its ground state in an infinite potential well whose walls are located at $x = 0$ and $x = a$, is subject at time $t > 0$ to a time-dependent perturbation $V(t) = \in \hat{x} e^{-t^2}$ where $\in$ is a small real number. Calculate the probability (up to the first order in perturbation theory) that the particle will be found in its first excited state after a sufficiently long time (i.e.,$t \to \infty$). 4

10. (*a*) Consider a system of two spin 1/2 particles and suppose that the system is in the state $|+-\rangle$ for $t \leq 0$. The Hamiltonian for the system for $t > 0$ is given by $H = (4\Delta/\hbar^2)S_1.S_2$. Assume that the $H = 0$ for $t \leq 0$. Find the probability as a function of time for finding particles in the two $|-\pm\rangle$ states by solving the problem using first order perturbation theory, treating H as a perturbation. 5

(*b*) Consider the plane wave solution to the Hartree-Fock equation neglecting the electron-electron interaction term as well as the potential due to the ion. Find an expression for the dispersion relation of the electrons. 3

List of Formulae

1. Hartree Fock equation :

$$-\frac{\hbar^2}{2m}\vec{\nabla}^2\psi_k(\vec{r}) + \left[\sum_i \int \frac{e^2 |\psi_i(\vec{r}')|^2}{|\vec{r}-\vec{r}'|} d\vec{r}\right]\psi_k(\vec{r}) + \upsilon_{ion}\psi_k(\vec{r})$$

$$-\left[\sum_i \delta_{s_i s_k} \int \frac{e^2}{|\vec{r}-\vec{r}'|}\psi_i^*(\vec{r}')\psi_k(\vec{r}')d\vec{r}'\right] = \epsilon_k\, \psi_k(\vec{r}')$$

2. Ground state wave function for Hydrogen-like atom :

$$\psi_{100}(x)=\frac{1}{\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2} e^{-Zr/a_0}$$

3. Wave functions of 1-D harmonic oscillator :

$$\psi(x)=\frac{1}{\sqrt{2^n n!}}\left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{\frac{-m\omega x^2}{2\hbar}} H_n\left(\sqrt{\frac{m\omega}{\hbar}}x\right);$$

$$H_n(x)=(-1)^n e^{x^2}\frac{d^n}{dx^n}(e^{-x^2})$$

4. Asymptotic forms of spherical Hankel functions :

$$h_l^{(1)}(r)\xrightarrow[r-\infty]{}\frac{1}{r}e^{i}[r-(l+1)\pi/2];\quad h_l^{(2)}(r)\xrightarrow[r-\infty]{}\frac{1}{r}e^{-i}[r-(l+1)\pi/2]$$

PAPER—PHS-301.2

(*Statistical Mechanics*-I)

[*Marks* : 20]

Answer any **two** of the following : 2×2

11. Show that the partition function of two independent (non-interacting) system i and j is given by $Q_{ij} = Q_i \times Q_j$.

12. Calculate the number of microstates available for 1D ultra relativistic gas with energy $E = |\vec{p}|c$.

13. For classical rotator

$$E = \frac{1}{2I}\left(p_\theta^2 + \frac{p_\phi^2}{\sin^2\theta}\right)$$

Find $\langle E \rangle$.

14. Prove that for free particle in a box in momentum representation

$$\hat{\rho}_{KK'} = e^{\frac{-\beta\hbar^2 K^2}{2m}} \delta_{KK'}.$$

Answer any **two** of the following : 4×2

15. Show that density matrix

$$\hat{\rho} = \frac{e^{-\beta\hat{H}}}{Tr(e^{-\beta\hat{H}})}.$$

16. The Hamiltonian of a relativistic free gas with N-particles is given by

$$H = \sum_{i=1}^{N} \epsilon_i \text{ where } \epsilon_i = c\left(p_i^2 + m^2c^2\right)^{1/2}$$

Find $\left\langle p_i \frac{\partial H}{\partial p_i} \right\rangle \Big/ \varepsilon_i$.

17. For a particle of mass m and total energy E moving in a spherical symmetric potential $V(r) = Ar^2$, where A is constant. Find the number of microstates.

Answer any **one** of the following : 8 × 1

18. Find the density matrix for a particle in a box with infinite potential (3-dimensional) in co-ordinate representation and explain the physical significance. 6 + 2

19. If Hamiltonian

$$H = \sum_i A_i p_i^2 + \sum_i B_i q_i^2 + \sum_i C_i p_i q_i$$

where q_i and p_i are the canonical variables.

Prove that $\left\langle p_i \frac{\partial H}{\partial p_i} \right\rangle = 3NKT$

and $\left\langle q_i \frac{\partial H}{\partial q_i} \right\rangle = 3NKT.$ 4 + 4
