2022

M.Sc.

2nd Semester Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-206

GENERAL TOPOLOGY

Full Marks : 25

Time : 1 Hour

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer any *two* questions : 2×2

(a) If Y is a subspace of X and Z is a subspace of Y, then show that Z is a subspace of X.

(Turn Over)

(b) Is the space $\mathbb{R}_l$ connected ? Justify your answer.

(c) Give an example to show that a subspace of a normal space need not be normal.

(d) Show that if Y be a subspace of X and A is a subset of Y, then the topology A inherits a subspace of Y is the same as the topology it inherits as a subspace of X.

2. Answer any *two* questions : 2×4

(a) Let A be a subset of a topological space X. Then show that $x \in \overline{A}$ if and only if every open set U containing x intersects A.

(b) Show that $\mathbb{R}^{\omega}$ in the uniform topology satisfies the first countability axiom but it does not satisfy the second countability axiom.

(c) If L is a straight-line in the plane, describe the topology L interits as a subspace of $\mathbb{R}_l \times \mathbb{R}$ and as a subspace $\mathbb{R}_1 \times \mathbb{R}_1$. In each case it is a familiar topology.

(d) In the finite complement topology on $\mathbb{R}$, to what point or points does the sequence $x_n = \frac{1}{n^3}$ converge ? Justify.

3. Answer any *one* question : 1×8

(a) (i) Consider the box topology $\mathbb{R}^\omega$ in the product topology is connected.

(ii) If $A \subset X$, a retraction of X onto A is a continuous map $r : X \to A$ such that $r(a) = a$ for each $a \in A$. Show that a retraction is a quotient map. 4+4

(b) (i) Let X be a Hausdorff topological space and Y be a compact subspace of X. Show that Y is closed.

(ii) Let $\{A_\alpha\}$ be a collection of connected subspaces of X and A be a connected subspace of X. Show that if $A \cap A_\alpha \neq \phi$ for all α, then $A \cup (\cup_\alpha A_\alpha)$ is connected.

4+4

[Internal assessment – 05]

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