2015

M.Sc. 1st Seme. Examination

PHYSICS

PAPER—PHS-101

Full Marks : 40

Time : 2 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Use separate Answer-scripts for Group-A & Group-B

Group-A

[Marks : 20]

Answer Q. No. *1* and any *one* from the rest.

1. Answer any *five* of the following : 5×2

(a) Find the residue of $f(z) = \frac{1-e^{2z}}{z^4}$ at all its poles in the finite plane.

(Turn Over)

(b) Locate and name the singularity of the function

$$f(z) = \frac{\text{Sin}\sqrt{Z}}{\sqrt{Z}}.$$

(c) Let X be an eigen vector of a hermitian matrix H. If Y is any vector orthogonal to X, show that HY is also orthogonal to X.

(d) Show that if the nonsingular matrices A and B commute, so also do A^{-1} and B.

(e) Prove that the diagonalizing matrix of a hermitian matrix is unitary.

(f) A differential equation has the form

$$(1 - x^2)\, y'' - 2xy' + n(n+1)y = 0$$

Where n is a constant. Examine and explain the nature of the points at $x = 0$ and $x = \pm 1$.

(g) Evaluate $\int_0^\infty x^{n-1}\, e^{-a^2x^2}\, dx$ interms of the gamma function.

2. (a) Evaluate $\int_0^{\pi/2} \sqrt{\text{Cot}\,\theta}\, d\theta$. 2

(b) State Cauchy's residue theorem. Using this, evaluate

$$\int_{-\infty}^{+\infty} \frac{e^{ikr}}{k^2 + \mu^2}\, dk.$$ 4

(c) The matrix $A = \begin{pmatrix} a & h \\ h & b \end{pmatrix}$ is transformed to the diagonal

form $D = T^{-1} AT$ where

$$T = \begin{pmatrix} \text{Cos}\theta & \text{Sin}\theta \\ -\text{Sin}\theta & \text{Cos}\theta \end{pmatrix}.$$

Find the value of θ which give this diagonal transformation. 4

3. (a) If f(z) is analytic function of z,

prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(z)|^2 = 4\,|f'(z)|^2$. 3

(b) Expand $f(z) = \frac{1}{(z-1)(z-2)}$ for $1 < |z| < 2$. 2

(c) $A = \begin{pmatrix} 1 & 0 \\ 2 & 4 \end{pmatrix}$. Find the eigen values of

$4A^{-1} + 3A + 2I$. 2

(d) Evaluate $\int_0^{\infty} \frac{dx}{1+x^6}$ using residue theorem. 3

Group-B

[Marks : 20]

Answer Q. No. *1* and any *one* from the rest.

1. Answer any *four* of the following : $4 \times 2\frac{1}{2}$

(a) A particle of mass m moves in one dimenssion such that it has the Lagrangian,

$$L = \frac{m^2 x^4}{12} + mx^2 v(x) - v^2(x)$$

where v(x) is some differentiable function of x. Find the equation of motion for x(t).

(b) A system is governed by Hamiltonian

$$H = \frac{1}{2}(p_x - ay)^2 + \frac{1}{2}(p_y - bx)^2$$

a and b are constants and p_x, p_y are momentums conjugate to x and y. Find the value of 'a' and 'b' so that the quantities $(p_x - 3y)$ and $(p_y + 2x)$ be conserved.

(c) Show that the maximum possible velocity for a particle of mass m is

$$v = \frac{mg}{K}$$

Which is falling from rest under the influence of gravity when frictional force is obtainable from a dissipation function $\frac{1}{2}Kv^2$ is present.

(d) A linear transformation of a generalized co-ordinate q and corresponding momentum p to Q and P is given by,

$$Q = q + p \text{ and } P = q + \alpha P$$

is canonical. Find the value of α.

(e) Derive the Hamiltonian corresponding to the Lagrangian,

$$L = \frac{1}{2}\left(\dot{q}_1^2 + \dot{q}_1\dot{q}_2 + \dot{q}_2^2\right) = V(q).$$

(f) Prove that if F(q, p, t) and G(q, p, t) are two integrals of motion then [F, G] is also integral of motion.

2. Find the Lagrangian of a particle of charge q, mass m and linear momentum p, enters an electromagnetic field of vector potential $\vec{A}$ and scalar potential ϕ. Hence derive the equation of motion.

If L is the Lagrangian of a system of n degrees of freedom satisfying Hamilton's variational principle, show that,

$$L' = L + \frac{dF(q_1, q_2, \dots q_n, t)}{dt}$$

will also satisfy Hamilton's variational principle, where F is any arbitrary well behaved function. 4+3+3

3. Outline the Hamilton-Jacobi equation. Explain the physical significance of Hamilton's principle function.

Obtain the relation between Hamilton's principle function and Hamilton's characteristic function.

Solve the harmonic oscillator problem by Hamilton-Jacobi method.

2+2+2+4