Chapter 1

Introduction

Magnetohydrodynamics (MHD) deals with the study of motion of an electrically conducting fluids (liquid metals, plasmas and salt solutions) in the presence of magnetic fields which influence the fluid motion by carrying the magnetic field lines partially (depending upon the electrical conductivity of the fluid) along with it. The magnetic field affects the motion by producing a mechanical force, named the Lorentz force. The initial motion of the conductor is modified by this force. In magnetohydrodynamic heat transfer problems, the Lorentz force comes into play in the momentum equation and the term corresponding to Joule heating appears in the energy equation.

The study of MHD is important in the field of astrophysics, geophysics and plasma physics. Most of the baryonic matter in the universe is constituted by plasma which is electrically conducting, therefore it responds to a magnetic field well. As reported by Joseph Larmor [1], Sun spots are caused by the Sun's magnetic field. The solar wind is also governed by MHD. The electrically conducting fluid in the core of the Earth and other planets is a huge MHD fluid dynamo that generates and maintains the Earth's magnetic field due to the motion of molten iron. MHD is significant because of its frequent occurrence in designing MHD pump, MHD generator, MHD flow meter, MHD propulsion etc. MHD micro pumps are required in chemical, fabrication process, medical and biological applications such as micro syringes for diabetics since they are able to handle small and precise volumes. MHD generators are used as sources of sounding signals for exploration and assessment of underground petroleum deposits and natural gas fields. MHD flow meter is used to monitor fluctuations in the rate of blood flows in arteries during vascular surgery. A number of experimental methods of ship and spacecraft propulsion are based on MHD principles. MHD devices are essential for levitating, stirring, metallurgical processing(melting, refining and solidification) and in other applications such as casting, welding, jet printers, crystal growth, dispersion of metals etc. Electromagnetic stirring(EMS) improves quality and productivity in continuous casting. Again, MHD technologies and devices are wider applied in ferrous and non-ferrous metal works. MHD plays an important role in the field of aeronautics especially in missile aerodynamics. In high speed flights, Joule heating due to electric currents plays a very important role. In thermonuclear reactors, MHD is used to create and contain hot plasmas by electromagnetic forces. MHD principle is applied in biomedical engineering include cardiac MRI, ECG, NMR spectroscopy, thermo chemotherapy(MHTCT) of malignant tumors etc. Due to the presence of haem, blood is electrically conducting. Kolin [2] initiated the concept of electromagnetic field in medical research. MHD principle is helpful in treatment of cardiovascular disorder such as atherosclerotic plaques, hemorrhages and hypertension, aneurysms in brain circulation systems at and near the apex of bifurcation etc.

The concept of MHD was first initiated by Hartmann [3] who studied the subject as Hg-dynamics in his efforts to pump mercury by exploitation of hydrodynamical pressure and electromagnetic fields. Hannes Alfv́en [4] was the first to discover the transverse waves for which he was awarded the Nobel Prize in physics in 1970. The study of magnetohydrodynamics problems has been analyzed by many authors, viz. Bullard [5], Cole [6], Cowling [7], Meyer [8], Harris [9], Ferraro and Plumpton [10], Shercliff [11], Cramer and Pai [12], Davidson [13].

1.1 Basic equations in Magnetohydrodynamics (MHD)

The basic equations of MHD are the modified electrodynamic equations together with the modified hydrodynamic equations. The electrodynamic Maxwell's equations remain same while the Ohm's law (which relates the electric field to the electric current) has to be modified to include the induced current. The inclusion of the Lorentz force* modifies the momentum equation and the energy equation has to include the Joule dissipation into consideration. In MHD, the displacement current is neglected in the case of motion of fluid whose velocity is very small compared to the velocity of light. Also, the charge density is negligible for the fluids which are almost neutral. The basic equations for the flow of a viscous incompressible electrically conducting fluid are as follows:

The equation of continuity is

$$\nabla \cdot \vec{q} = 0, \tag{1.1}$$

where $\vec{q}$ is the fluid velocity vector. The equation of continuity is a statement about the conservation of mass of fluid.

The momentum equation of the magnetohydrodynamics in a rotating frame of reference is

$$\rho \left[\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla)\vec{q} + 2(\vec{\Omega} \times \vec{q}) \right] = -\nabla p + \mu \nabla^2 \vec{q} + (\vec{j} \times \vec{B}) + \rho\, \hat{g}, \tag{1.2}$$

where $\vec{\Omega}$ is the angular velocity, ρ the fluid density, p the modified fluid pressure including centrifugal force, μ the coefficient of viscosity, $\vec{j}$ the current density vector, $\vec{B}$ the magnetic induction vector and $\hat{g}$ the acceleration due to gravity. The term $(\vec{q}\cdot\nabla)\vec{q}$ represents convective acceleration†, $2(\vec{\Omega}\times\vec{q})$ the Coriolis acceleration, ∇p the fluid pressure gradient including centrifugal force and $\mu\nabla^2\vec{q}$ the viscous force. The equation (1.2) is known as the modified Navier-Stokes' equation.

Maxwell's equations are

$$\nabla \times \vec{B} = \mu_e \vec{j} \ \text{(Ampere's law)}, \tag{1.3}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \ \text{(Faraday's law of induction)}, \tag{1.4}$$

$$\nabla \cdot \vec{B} = 0 \ \text{(Gauss's law for magnetism)}, \tag{1.5}$$

$$\nabla \cdot \vec{D} = \rho_e \ \text{(Gauss's law)}, \tag{1.6}$$

where μ_e is the the magnetic permeability, $\vec{D}$ the displacement vector and ρ_e the electric charge density.

*Lorentz force is the force exerted on a charge particle in an electromagnetic field.

†The convective acceleration is an acceleration caused by a change in velocity over position.

The Ohm's law, on neglecting Hall currents is

$$\vec{j} = \sigma(\vec{E} + \vec{q} \times \vec{B}), \tag{1.7}$$

where σ is the electrical conductivity of the fluid.

The magnetic induction equation is

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{q} \times \vec{B}) + \nu_m \nabla^2 \vec{B}, \tag{1.8}$$

where $\nu_m = \frac{1}{\sigma \mu_e}$ is the magnetic diffusivity (or resistivity). It holds quite independently of the particular dynamical forces generating the flow (e.g. whether these are of thermal or compositional origin, whether the Lorentz force is or is not important, whether Coriolis force is present or not). It also holds whether the fluid is incompressible or not.

The constitutive equations are

$$\vec{B} = \mu_e \vec{H} \quad \text{and} \quad \vec{D} = \epsilon \vec{E}, \tag{1.9}$$

where $\vec{H}$ is the magnetic field vector and ϵ the dielectric constant.

The energy equation including viscous and Joule dissipations is

$$\rho c_p \left[\frac{\partial T}{\partial t} + (\vec{q} \cdot \nabla) T \right] = k \, \nabla^2 T + \mu \, \Phi + \frac{\vec{j}^2}{\sigma}, \tag{1.10}$$

where T is the fluid temperature, k the thermal conductivity, c_p the specific heat at constant pressure and Φ the viscous dissipation function. The last term on the right-hand-side is due to Joule dissipation.

1.2 Boundary Conditions

1.2.1 MHD boundary conditions

The flow field and the electromagnetic field are to be determined by solving the fundamental equations stated in the section(1.1) under appropriate boundary conditions for the flow field and electromagnetic field. The boundary conditions to be satisfied are the usual hydrodynamic boundary conditions imposed on the velocity field (such as the no-slip condition at a solid boundary for a viscous fluid), the continuity of the temperature distribution and the electromagnetic boundary conditions. The electromagnetic properties change abruptly at a solid boundary. Across such a surface of discontinuity, the electromagnetic variables must satisfy the following conditions.

(i) The normal component of the magnetic induction $\vec{B}$ is continuous, i.e.

$$(\vec{B}_2 - \vec{B}_1).\widehat{n} = 0, \tag{1.11}$$

where $\widehat{n}$ is the unit vector normal to the surface of discontinuity and the subscripts 1 and 2 refer to the medium on either side of the surface.

(ii) The magnetic field $\vec{H}$ satisfies the condition

$$\widehat{n} \times (\vec{H}_2 - \vec{H}_1) = \vec{j_s}, \tag{1.12}$$

where $\vec{j_s}$ is the surface current density. When the electrical conductivity of the solid is finite ($\sigma \neq \infty$) then $\vec{j_s} = \vec{0}$ but when $\sigma = \infty$ then $\vec{j_s}$ is different from zero.

(iii) The tangential component of the electric field is continuous, i.e.

$$\widehat{n} \times (\vec{E}_2 - \vec{E}_1) = \vec{0}. \tag{1.13}$$

(iv) The dielectric displacement $\vec{D} = \epsilon\vec{E}$ must satisfies the condition

$$(\vec{D}_2 - \vec{D}_1).\widehat{n} = \rho_{es}, \tag{1.14}$$

where ρ_{es} is the surface free charge density.

1.3 Hall currents

It has been observed that in an ionized gas if the density is low and/or the magnetic field is very strong the charged particles can gyrate round the lines of force several times before suffering collisions with other particles. This results in a drift of a charged particles(say electron) at right angles to the direction of the magnetic field and the electric current. The resulting current known as Hall current reduces the electrical conductivity normal to the magnetic lines of force so that electrical conductivity becomes an isotropic. The effects of Hall current is likely to be important in many astrophysical situations as well as in flows of plasma through MHD power generator.

A fully ionized gas can be considered to be a mixture of two different gases, one consisting of positively charged ions and the other negatively charged electrons. The mass of the electrons being very much less than that of ions, the velocity $\vec{q}$ of the ion gas can be taken as the velocity of the whole mixture. The current density of the ionized gas can be given as

$$\vec{j} = -e\, n_e \vec{V}, \tag{1.15}$$

where $\vec{V}$ is the velocity of the electron gas relative to the ion gas, $-e$ the charge of an electron and n_e the number of electrons per unit volume.

At each collision of an electron with an ion, the electron loses a quantity of momentum equal to the mean momentum $m_e\vec{V}$ relative to the ion gas, where m_e is the mass of the electron. The collision of an ion with an electron can be supposed to be smoothed into a continuous force. If τ_e be the mean interval between two successive collisions of an electron with ions, the total momentum lost by the electrons due to collisions with ions is $n_e m_e \vec{V}/\tau_e$ per unit volume and time. This loss of momentum subjects the electron to a drag force $-n_e m_e \vec{V}/\tau_e$ per unit volume.

The various forces acting on the electron gas are (per unit volume)

(i) The drag force due to collisions,

(ii) The gradient $-\nabla p_e$ of the electron pressure, p_e,

(iii) The electromagnetic force, $-\, e\, n_e \left(\vec{E} + \mu_e \vec{q} \times \vec{H} + \mu_e \vec{V} \times \vec{H}\right)$,

(iv) The force due to gravity, $n_e\, m_e\, g$.

Since electrons have negligible mass, the force due to gravity is negligible and the other three forces can be considered to be in equilibrium. Thus, neglecting ion-slip and thermo-electric effects, we have

$$0 = -\nabla p_e - e\, n_e \left(\vec{E} + \mu_e \vec{q} \times \vec{H} + \mu_e \vec{V} \times \vec{H}\right) - \frac{n_e m_e \vec{V}}{\tau_e}. \tag{1.16}$$

On the use of the equation (1.15), the equation (1.16) becomes

$$\frac{m_e}{e\,\tau_e}\vec{j} = \nabla p_e + e\,n_e\left(\vec{E} + \mu_e\vec{q}\times\vec{H}\right) - \mu_e\vec{j}\times\vec{H}. \tag{1.17}$$

This equation is known as generalized Ohm's law.
The equation (1.17) can be written as

$$\vec{j} = \sigma\left[\vec{E} + \mu_e\vec{q}\times\vec{H} - \frac{\mu_e}{e\,n_e}\vec{j}\times\vec{H} + \frac{1}{e\,n_e}\nabla p_e\right], \tag{1.18}$$

where $\sigma = e^2 n_e \tau_e / m_e$ is the electrical conductivity of the ionized gas.
For partially ionized gas, the electron pressure gradient ∇p_e is negligible and the equation (1.18) becomes

$$\vec{j} = \sigma\left[\vec{E} + \mu_e\,\vec{q}\times\vec{H} - \frac{\mu_e}{e\,n_e}\vec{j}\times\vec{H}\right]. \tag{1.19}$$

The term $\frac{\sigma\mu_e}{e\,n_e}(\vec{j}\times\vec{H})$ in the equation (1.19) is due to the Hall currents.

1.4 Porous Medium

A material which contains a large number of pores (void spaces) embedded in it is termed as a porous medium. These pores may be either connected or non-connected, distributed more or less frequently in either a regular or random manner in the material. The skeletal portion of the material is usually a solid and is often called as matrix or frame. Interconnected pores are called the effective pores while the non-interconnected ones are called ineffective pores. In most cases the embedded pores are filled with fluid (liquid or gas). The study of characteristic of porous media involving deformation of the solid frame is called poromechanics. Beach sand, soil granules, rocks like sand stone, limestone, dolomite etc. are the natural substances to be considered as porous media. Again, zeolite, biological tissues (lungs, bones, wood, cork) and concrete, cement, brick, paper cloth, filter paper and ceramics are considered as porous media. The concept of porous media is used in many areas such as filtration, mechanics (acoustics, soil mechanics, geomechanics, rock mechanics). Also it is used in geosciences (hydrogeology, petroleum geology, geophysics), engineering (petroleum engineering, bio-remediation, construction engineering), biology and biophysics, material science etc. Porosity and permeability often describe the distinctive nature of a porous medium.

1.4.1 Porosity

The porosity of a porous medium is defined mathematically as

$$\sigma^* = \frac{\text{pore volume}}{\text{matrix volume}}, \tag{1.20}$$

where the pore volume denotes the total volume of the pore space in the matrix and the matrix volume is the total volume of the matrix including the pore space. But in case of effective porosity, total volume of the interconnected pore space is considered. The porosity σ^* has to be averaged over many pores and the considered matrix volume must be larger than the pore size. Often the porosity σ^* can be chosen as a non-dimensional constant for the whole medium and the value of σ^* lies between 0 and 1. As particle size of surface soil increases, porosity usually decreases. For solid granite, porosity is less than 0.01 whereas for clay soil it ranges between 0.51 and 0.58.

1.4.2 Permeability

The porosity alone is not sufficient to characterize a porous material. To distinguish two porous media having the same porosity, an additional characteristic term is used named as permeability. The capability of the fluid to pass through a porous medium is described by the permeability k^*. The permeability is a quantity independent of fluid properties but depends on the geometry of the medium only. Intuitively the permeability depends on the porosity of the medium and equivalent diameter of the particle. The permeability is a statistical average of the fluid percolation of all the flow channels in the medium. This permeability constant has been first demonstrated by H. Darcy [14]. An empirical observation of Darcy known as Darcy's Law introduces permeability in terms of measurable quantity.

1.4.3 Darcy's Law

The French engineer H. Darcy [14] has established an empirical law for a homogeneous, isotropic porous medium with a linear relationship between the sea-page velocity of water and the pressure gradient which is expressed mathematically as

$$\vec{q^*} = -\frac{\text{constant}}{\mu}\left(\nabla p - \vec{F}\right), \tag{1.21}$$

where μ is the dynamical of the viscosity of the porous medium, p the mean pressure and $\vec{F}$ the body force per unit volume acting on the fluid. Later, Muskat [15] has proved that the constant in the equation (1.21) should be related to the permeability, k^* of the porous material and then the equation (1.21) takes the form

$$\vec{q^*} = -\frac{k^*}{\mu}\left(\nabla p - \vec{F}\right), \tag{1.22}$$

where $\vec{q^*}$ is the volume flow rate or fitter velocity. The negative sign indicates that the fluid velocity is in the opposite direction of increasing pressure gradient. The equation (1.22) is known as the Darcy's law for isotropic porous media in one-dimensional flow. However, it is now used to describe the flow field in two or three-dimensional problems. Darcy's law for a steady flow can be written as

$$\vec{q^*} = -\frac{k^*}{\mu}\nabla\left(p - \rho g z\right), \tag{1.23}$$

where z the height of the vertical column.

1.5 Heat Transfer

Heat transfer problems are of considerable interest in view of their wide applications in electric machines, extraction of heat energy from atomic pile, high speed aircraft, atmospheric re-entry vehicles and cooling of rotating turbine blades. A good knowledge of heat transfer characteristics is necessary for the successful design of heat exchangers such as boiler, condensers, radiators. Heat transfer also deals with the better variety of seeds, breeders and food processing. A keen analysis of heat flow is mandatory to maintain air conditioning of buildings. Depending upon the features of heat transfer process, the three distinct modes of heat transfer are conduction, convection and radiation.

1.5.1 Conduction

Heat transfer by conduction is accomplished through two mechanisms. One is molecular interactions whereby the exchange of thermal energy takes place due to direct impact of molecules or the kinetic motion. Molecules at a relatively higher temperature impart energy to adjacent molecules at lower temperature. Such kind of energy transfer always exist so long as there is a temperature gradient in a system comprising molecules of solid, liquid or gas. Another is in the case of metallic solids where heat conduction takes place by drifting the free electrons. The ability of metallic alloys to conduct heat varies directly as the concentration of the free electrons in them. Non-metals are bad conductors of heat due to low concentration of free electrons in them. Pure metals, molten metals(copper, platinum, gold etc.), mercury, sea water and gasses in ionized states are good conductors whereas fluids especially gasses are poor conductor of thermal energy.

1.5.2 Convection

Convection is the transfer of heat only through the fluid medium. It concerns the energy exchange between a solid surface and an adjacent fluid flowing on it. In the atmosphere, the Sun warms the surface of the Earth and the warmed air moves up and cool air comes in. Convection of heat transfer can be classified as:
(i)Forced convection
(ii)Free(Natural) convection
(i)Forced convection: A convection process which takes place due to motion created by an external agency such as pressure gradient or an agitator, is known as forced convection. In incompressible fluids with constant physical properties, such flows are characterised by the fact that the velocity distribution is not affected by the temperature field but the reverse is not true. In such flows, the heat transferred at the surface of the solid body is swept away by the fluid motion without in any way affecting the local density of the fluid. Hence in a forced convective flow the velocity field is independent of the temperature field so that the parts of the motion arising from the difference of density caused by thermal expansion can be neglected.
(ii)Free convection: On the other hand if the motion is caused by the action of body forces on the fluid, which arise as a result of density gradients due to changes in temperature, is called free convection. Such variation of density gives rise to buoyancy force which causes relative motion. In free convection flow the velocity field and the temperature field are coupled. In problems of free convection or combined free and forced convection it is customary to express the body force termed

as a buoyancy term. In this case, density ρ varies slightly from point to point because of the variation in the temperature T, we write $\rho = \rho_0 [1 - \beta(T - T_0)]$, where β is the coefficient of thermal expansion and ρ_0 is density at some reference temperature T_0. In well known Boussinesq approximation, the variation of ρ is taken into account only in the buoyancy term.

1.5.3 Radiation

The transfer of energy in an absorbing/emitting medium is of considerable interest in many physical and engineering problems e.g. in problems of furnaces, combustion chambers and high speed flow. Radiative heat transfer was first studied in the context of astophysical problems concerning the formation of absorption lines in stellar spectra, energy transfer in stars, planets and galaxies. A good account of radiative heat transfer can be found in the monograph by Chandrasekhar [16], Kourganoff [17] and Sobolev [18].
Two limiting cases of radiative heat tranfer processes are generally considered in the literature (i)optically thick limit and (ii)optically thin limit. The optical dimension of a system is defined as the ratio of characteristic physical dimension to the penetration length of radiation. For an optically thick medium, the penetration length is small and is equivalent to the radiation mean free path. Here the radiation can be considered as a diffusive process. For an optically thin medium just the reverse is true so that the medium does not absorb its own emmited radiation.

1.6 Earlier works relevant to the present investigations

A brief survey of the earlier investigations related to the problems considered in this thesis has been conducted.

1.6.1 Hydromagnetic Couette flow in a rotating system

In fluid dynamics, Couette flow is one of the fundamental flow that refers to the laminar flow of a viscous fluid between two parallel plates, one of which is moving and other is at rest. The viscous drag force acting on the fluid, is caused to drive the flow. Many interested investigators have attracted to study Couette flow in different geometries, due to its applications in many areas of science and technology such as MHD power generators and pumps, accelerators, polymer technology, aerodynamic heating, electrostatic precipitation, extrusion processing, metal forming, continuous casting, wire and glass fiber drawing, fluid droplets and sprays, plasma jets and chemical synthesis, petroleum industry and in purification of crude oil etc. Rotating fluid is related in designing thermosyphon tube, cooling turbine blades and to the subjects of Oceanography, Meteorology, Atmospheric Science and Limnology.

The theory of rotating fluids has been explained by Greenspan [19]. Jana and Datta [20] have investigated heat transfer characteristic of a viscous incompressible Couette flow in a rotating frame of reference. Further, Jana et al. [21], Seth and Maiti [22] have imposed a uniform transverse magnetic field in this flow geometry. An alternative solution for the oscillatory Ekman boundary layer flow bounded by two parallel plates has been proposed by Mazumder [23] and Ganapathy [24]. Seth and Mahto [25] and Seth et al. [26] have improved this work taking a uniform strength of magnetic field. Mondal and Mandal [27] have discussed an magnetohydrodynamic Couette flow of a viscous incompressible electrically conducting fluid in the presence of a uniform magnetic field by considering Hall current into account. Singh et al. [28] have studied this problem setting one plate of the channel into a uniformly accelerated motion. The hydromagnetic Couette flow with an inclined magnetic field in a rotating environment has been considered by Jana and Dogra [29], Seth et al. [30]. Singh [31] has examined an magnetohydrodynamic Couette flow of a viscous fluid in presence of a uniform magnetic field by considering different aspects of the problem. Ghosh and Pop [32] have studied the combined effects of Hall current and Coriolis force on an magnetohydrodynamic Couette flow in a rotating environment. Attia [33], Seth et al. [34] have presented the unsteady Couette flow with heat transfer considering ion-slip. The combined effects of Hall current and Coriolis force on an magnetohydrodynamic Couette flow in a rotating system has been considered also by Ghosh and Pop [35]. Kumar et al. [36] have discussed an magnetohydrodynamic Couette flow of a viscous incompressible electrically conducting fluid in the presence of a uniform magnetic field by considering different aspects of the problem. Das et al. [37] have studied the Couette flow of a viscous incompressible fluid in a rotating system in the absence of the magnetic field. Das et al. [38] have investigated the unsteady hydromagnetic Couette flow of a viscous incompressible electrically conducting fluid in a rotating system for small as well as large times. Guria et al. [39] have considered Couette flow in a rotating environment with inclined magnetic field where upper plate is held at rest and the lower plate oscillates non-torsionally. Seth et al. [40] have studied magnetohydrodynamic Couette flow in a rotating system in the presence of induced magnetic field. Seth et al. [41] have discussed an magnetohydrodynamic Couette flow of a viscous incompressible electrically conducting fluid in the presence of a uniform magnetic field by considering different aspects of the problem. Mathematical modeling of the oscillatory magnetohydrodynamic Couette flow in a rotating system by an oblique magnetic field has been studied by Bég et al. [42]. Effects of radiation on magnetohydrodynamic Couette flow have been discussed by Ghara et al. [43] and Sarkar et al. [44]. Unsteady Couette flow in a rotating frame of reference has been extended by Maji et al. [45] on taking Hall current into account. Das et al. [46] have analyzed the entropy generation in viscous incompressible Couette flow with suction/injection in a rotating frame of reference. Ali et al. [47] have analyzed numerically the unsteady magnetohydrodynamic Couette flow and heat transfer of nanofluid in a rotating system with convective cooling. Das et al. [48] have studied a transient hydromagnetic reactive Couette flow and heat transfer in rotating frame of reference. Hall effects on a rotating Couette flow have been examined by Das et al. [49] under the condition that magnetic field fixed to either fluid or to moving plate. Entropy generation in an unsteady hydromagnetic Couette flow of a viscous incompressible electrically conducting fluid between two infinite horizontal parallel plates subjected to asymmetric convective heat exchange in a rotating system have been analyzed by Das et al. [50]. Recently, Rajesh et al. [51] have analyzed oscillatory hydromagnetic Couette flow through a porous medium in a rotating system.

1.6.2 Hall effects on magnetohydrodynamic flow

The phenomenon of Hall effects becomes significant in partially ionized gases or fluids such as electrolysis, salt water and solar wind when intensity of the applied magnetic field is very strong or when the cyclotron frequency is high. Hall currents characterize the flow pattern in many cosmological objects such as dense molecular clouds and formation of white dwarfs. The influence of Hall currents is important in many engineering applications such as Hall generators, Hall sensors, Hall probes, turbine, space missions and in laboratory plasma.

Hall effects on the steady flow of an ionized gas between two parallel plates have been presented by Sato [52], Yamanishi [53]. Soundalgekar [54] has described the heat transfer characteristic between two horizontal plates taking Hall currents into account. The temperature distribution on MHD channel flow with Hall effects has been reported by Eraslan [55]. Pop [56] has used power series method to study the Hall effects on the MHD flow over an impulsively started flat plate. An investigation on the flow of a conducting liquid being permeated by a transverse magnetic field, past an infinite porous flat plate has been deemed by Gupta [57] taking Hall effects into account. Bhadram [58] has reported the influence of Hall effects on the hydromagnetic oscillatory flow in the presence or absence of rotation. The heat transfer characteristic on the hydromagnetic Ekman layer on a porous plate has been explained by Mazumder et al. [59] taking Hall currents into account. Jana and Datta [60] have obtained an exact solution for the problem of Hall effects on the hydromagnetic flow due to an impulsive start of a porous flat plate. Hall effects on hydromagnetic flow past an infinite porous flat plate have been described by Jana et al. [61]. Datta and Jana [62] have designed a hydromagnetic flow problem through a rotating channel taking Hall currents into account. Jana and Datta [63] have considered Hall effects on unsteady Couette flow. Hall effects on the unsteady MHD free and forced convective flow in a porous rotating channel have been reported by Sivaprasad et al. [64]. Kanch et al. [65] have considered Hall effects on hydromagnetic flow in a horizontal channel in the presence of inclined Magnetic field. Ghosh [66, 67] has designed MHD flow geometry in a rotating channel taking Hall currents into account. Guria and Jana [68] have analyzed Hall effects on the hydromagnetic convective flow through a rotating channel under general wall conditions. Seth and Ansari [69] have studied the magnetohydrodynamic convective flow in a rotating channel taking Hall currents into account. Hall effects on oscillatory hydromagnetic Couette flow in a rotating system have been described by Seth et al. [70]. Hall effects on MHD Couette flow in a rotating system partially filled with a porous medium have been presented by Chauhan and Agrawal [71]. Jha and Apere [72] have explained unsteady MHD Couette flow of a rotating fluid on taking Hall currents into account. The effects of rotation and radiation on the mixed convective MHD flow through a vertical channel embedded in a porous medium have been studied by Singh and Pathak [73] on taking Hall currents into account. The influence of Hall currents and rotation on MHD mixed convection in a rotating vertical channel have been explained by Guchhait et al. [74]. Sarkar et al. [75-78] have looked the study into MHD flow with Hall currents in several flow environment. Effects of Hall current and rotation on MHD natural convective flow past an impulsively moving vertical plate with ramped temperature in the presence of thermal diffusion and heat absorption have been analyzed by Seth et al. [79]. Das et al. [80] have examined the Hall effects on MHD free convection boundary layer flow past a vertical flat plate. Effects of Hall current and radiation on unsteady MHD flow past a heated moving vertical plate have been examined by Das et al. [81]. The influence of Hall current and wall conductance on MHD flow due to asymmetric heating of the walls has been explained by Das et al. [82]. Das et al. [83] have analyzed the impact of Hall currents on MHD flow through a porous medium with slip condition in a rotating frame of reference. Hall effects on hydromagnetic free convection in a heated vertical channel in presence of inclined magnetic field and thermal radiation have been reported by Guchhait et al. [84]. Seth et al. [85] have studied natural convective flow past an exponentially accelerated vertical ramped plate temperature taking Hall effects and heat absorption into account. An unsteady hydromagnetic flow of a viscous incompressible electrically conducting fluid past an accelerated porous flat plate in the presence of a uniform transverse magnetic field in a rotating system have been presented by Das et al. [86] taking the Hall effects into account. Hall effects on an unsteady magneto-convection and radiative heat transfer past a porous plate have been analyzed by Das et al. [87]. Das et al. [88] have considered the effects of Hall current to study rotating slip flow in a shrinking permeable channel. Padma and Suneetha [89] have reported Hall effects on MHD flow through porous medium in a rotating parallel plate channel.

The effect of Hall current on an unsteady MHD three dimensional flow past an impulsively started infinite horizontal porous plate relative to a rotating system has been studied by Prabhakar Reddy [90]. Recently, Das et al. [91] have reported rotational magneto-hydrodynamic Couette flow of nano-fluids taking Hall effects into account.

1.6.3 MHD flow through a vertical channel

MHD flow through a vertical channel have become more important in recent years because of its wide applications in MHD generators, MHD pumps, MHD accelerators, MHD flow-meters, astrophysics, geology, power generation, thermonuclear reactor technology, designing cooling systems with liquid metals, medicine etc.

An analytical study on free and forced convective flow between two vertical plates under transverse magnetic field has been made by Regirer [92], Mori [93], Yu [94] and many others. The flow of an electrically conducting fluid between two heated vertical plates has been investigated by Poots [95] taking Joule heating and viscous dissipation into account. Gupta and Gupta [96], Datta and Jana [97] have analyzed the effects of radiation on combined free and forced convective electrically conducting fluid flow between two vertical walls. The Hall effects on hydromagnetic convective flow through a vertical channel with conducting walls have been reported by Datta and Jana [98]. The magnetohydrodynamic fully developed flow of a viscous incompressible electrically conducting fluid in a vertical channel during combined convection under the influence of a constant pressure gradient and in the presence of a uniform transverse magnetic field has been studied by Ghosh and Nandi [99] and Ghosh et al. [100] taking asymmetric heating of the wall. Umavathi and Malashetty [101] have analyzed combined free and forced convective MHD flow in a vertical channel on taking viscous and Ohamic dissipations into account. The MHD free convective flow between two vertical walls has been discussed by Das et al. [102]. The effects of radiation on unsteady free convective flow between two vertical walls have been presented by Mandal et al. [103]. Sarkar et al. [104] have considered transient MHD natural convection flow of a viscous incompressible electrically conducting fluid confined between vertical walls heated/cooled asymmetrically. Singh [105] has evaluated an exact solution of an MHD mixed convective periodic flow through a rotating vertical channel with radiative heat transfer. The effects of Hall current and rotation on MHD free convection flow in a vertical rotating channel filled with porous medium have been studied by Singh and Pathak [106]. Das and Jana [107] have examined the Hall effects on an unsteady free convective flow of a viscous incompressible electrically conducting fluid between two heated vertical plates in the presence of a transverse applied magnetic field and heat generation. Adesanya et al. [108] have analyzed the hydromagnetic natural convective flow between two vertical parallel plates with time dependent periodic boundary conditions. MHD natural convection in a vertical parallel plate microchannel has been presented by Jha et al. [109]. Further, the effect of suction/injection on MHD natural convection flow in a vertical micro channel has been observed by Jha et al. [110]. The fully developed mixed convection flow in a vertical channel filled with nanofluids in the presence of a uniform transverse magnetic field has been studied by Das et al. [111]. Das et al. [112] have considered the transient natural convection in a vertical channel filled with nanofluids in the presence of thermal radiation. MHD flow through vertical channel with porous medium has been described by Dwivedi et al. [113]. Jha et al. [114] have investigated the effects of Hall current on MHD natural convection flow in a vertical microchannel. Recently, Goswami et al. [115] have considered unsteady magnetohydrodynamic flow of an incompressible, viscous fluid bounded by two non-conducting parallel plates placed vertically in presence of uniform inclined magnetic field.

1.6.4 Radiation effect on MHD flow

Radiation effect plays an important role, if the fluid temperature is high. In astrophysics and geophysics, it is mainly used to study the stellar and solar structures, interstellar matter, radio propagation through the ionosphere etc. In engineering, the problem is significant in MHD pumps, MHD journal bearings etc.

Takhar et al. [116] have studied the radiation effect on an MHD free convective flow past a semi-infinite vertical plate. The effect of radiation on MHD steady asymmetric flow of an electrically conducting fluid past a stretching porous sheet has been analyzed by Quaf [117]. The effect of thermal radiation on an MHD flow has been studied by Raptis et al. [118]. The radiation effect on the electrohydrodynamic flow in a vertical channel has been reported by Gbadeyan et al.

[119]. Duwairi and Duwairi [120] have reported the thermal radiation effects on the Rayleigh flow of viscous fluids under the effect of a transverse magnetic field. The impact of radiation on an unsteady MHD free convective flow has been observed by Perdikis and Raptis [121]. Mebine [122] has investigated the influence of radiation on an MHD Couette flow with heat transfer between two parallel plates. The effects of thermal radiation and viscous dissipation on magneto-hydrodynamic (MHD) unsteady free-convection flow over a semi-infinite vertical porous plate are analyzed by Zueco [123]. The free convective flow of heat generating/ absorbing fluid between vertical porous plates has been investigated by Jha and Ajibade [124]. The radiation effect on three dimensional vertical channel flow subjected to a periodic suction has been considered by Guria et al. [125]. Jha and Ajibade [126] have explored the unsteady free convective Couette flow due to heat generating/absorbing of the fluid. The thermal radiation effect on the unsteady hydromagnetic gas flow along an inclined plane with the indirect natural convection has been discussed by Ghosh et al. [127]. Kumar and Varma [128] have noticed the radiation effect on unsteady MHD flow of a viscous incompressible electrically conducting fluid past an exponentially accelerated vertical plate with variable temperature in the presence of heat generation and applied transverse magnetic field. Rajput and Kumar [129] have reported the combined effects of rotation and radiation on MHD flow past an impulsively started vertical plate with variable temperature in presence of grey, absorbing-emitting radiation but a non-scattering medium. Jana et al. [130] have analyzed the radiation effect on the unsteady MHD free convective flow past an exponentially accelerated vertical plate taking viscous and Joule dissipations into account. The effect of radiation on an MHD free convective flow past a vertical plate with oscillatory ramped plate temperature has been reported by Jana et al. [131]. The influence of radiation on a free convective MHD Couette flow with variable wall temperature has been investigated by Das et al. [132] in presence of heat generation. Further, Das et al. [133] have studied the impact of radiation on a unsteady MHD free convective Couette flow of heat generation/absorbing fluid. Combined effects of Hall current and radiation on MHD free convective flow in a vertical channel with an oscillatory wall temperature have been considered by Guchhait et al. [134]. Impact of radiation on MHD free convective flow of a viscous incompressible electrically conducting fluid past an oscillating vertical porous plate

with uniform suction or blowing at the plate in the presence of a uniform magnetic filed has been discussed by Das et al. [135]. Ogulu and Makinde [136] have studied the effect of thermal radiation absorption on an unsteady free convective flow past a vertical plate in the presence of a magnetic field and constant wall heat flux. Effects of thermal radiation and viscous dissipation on boundary layer flow of nanofluids over a permeable moving flat plate has been reported by Motsumi and Makinde [137]. Makinde and Tshehla [138] have investigated the unsteady hydromagnetic-free convection of an incompressible electrical conducting Boussinesqs radiating fluid past a moving vertical plate in an optically thin environment with the Navier slip, viscous dissipation and Ohmic and Newtonian heating. Das et al. [139] have reported the efffects of Hall current and thermal radiation on an oscillatory MHD convective flow in a vertical channel filled with porous medium. Natural convective magnetonano fluid flow and radiative heat transfer past a moving vertical plate has been carried out by Das and Jana [140]. Nandkeolyar and Das [141] have considered inclined magnetic field to study MHD free convective radiative flow past a flat plate with ramped temperature. Diffusion-thermo and thermal radiation of an optically thick grey gas in presence of magnetic field and porous medium have been interpreted by Sarkar et al. [142]. Ghosh et al. [143] have examined thermal radiation on transient laminar grey gas flow past an oscillating vertical plate with variable temperature. Das et al. [144] have studied the transient natural convection in a vertical channel filled with nanofluids in presence of thermal radiation. Das et al. [145] have examined the effect of wall thermal conductance on a fully developed mixed convective flow of nanofluids in a vertical channel in the presence of thermal radiation. Das et al. [146] have considered thermal radiation to analyze slip flow of nanofluid past a vertical plate with ramped wall temperature. Recently, Das et al. [147] have explained the interaction of convection and thermal radiation on an unsteady magnetohydrodynamic (MHD) boundary layer flow of a viscous incompressible electrically conducting fluid past a rotating vertical plate whose temperature varies linearly with time.

1.6.5 MHD boundary layer flow past a stretching surface

The fluid flow over a stretching surface have attracted considerable attention in recent times due to its numerous applications in manufacturing wire drawing, hot rolling, metal spinning, glass fiber production, paper production, drawing of plastic films and rubber sheets etc. The MHD flow over a stretching sheet has its broad range of applications in the polymer industry where one deals with stretching of plastic sheets. The cooling and drying of paper and textiles, water pipes, sewer pipes, irrigation channels, blood vessels could be considered as applications in the field of flow over a stretching sheets.

Crane [148] has initiated the investigation of boundary layer flow over a stretching sheet. After this famous work, a number of investigations on this topic has been carried out by many researchers. Carragher and Crane [149] have described the heat transfer characteristic on a continuous stretching sheet. Liquid film on a unsteady stretching sheet has been considered by Wang [150]. The effects of non-uniform surface temperature due to heat transfer from an arbitrarily stretching surface has been analyzed by Afzal [151]. Vajravelu and Hadjinicolaou [152] have reported heat transfer characteristics in the laminar boundary layer of viscous and heat absorbing fluid over a stretching sheet with viscous dissipation. Chiam [153] has examined the stagnation point flow towards a stretching plate. Kumaran and Ramanaiah [154] have considered boundary layer flow over a quadratic stretching sheet. Fluid flow with heat transfer over an exponentially stretching sheet with constant suction has been reported by Elbashbeshy [155]. The steady, laminar, axisymmetric flow of a Newtonian fluid due to a stretching sheet when there is a partial slip of the fluid past the sheet has been investigated by Donald Ariel [156]. By using similarity transform, an exact solution of the entrained flow due to a stretching surface with partial slip is solved by Wang [157]. The slip flow past a stretching surface has been considered by Andersson [158]. Elbashbeshy and Bazid [159] have considered fluid flow with heat transfer over an unsteady stretching surface with internal heat generation or absorption. Wang [160] has demonstrated natural convection on a vertical radially stretching sheet. Sajid et al. [161] have derived a series solution for unsteady axisymmetric flow and heat transfer of a viscous fluid over a radially stretching sheet. Ishak et al. [162] have demonstrated the flow behaviours due to a stretching sheet in presence of magnetic field. Fang et al. [163] have derived an exact solution to study the MHD slip flow over a stretching sheet. Kumaran et al. [164] have studied MHD flow past a stretching permeable sheet. Later, Kumaran et al. [165] have extended this work on taking boundary layer flow into account. The boundary layer flow of a nanofluid past a stretching sheet with a convective boundary condition has been discussed by Makinde and Aziz [166]. Computational modelling of nanofluids flow over a convectively heated unsteady stretching sheet has been considered by Makinde [167]. Makinde and Aziz [168] have described boundary layer flow of a nanofluid past a stretching sheet with convective boundary condition. An investigation of the magnetohydrodynamics (MHD) boundary layer slip flow over a vertical stretching sheet in nanofluid with non-uniform heat generation/absorbtion in the presence of a uniform transverse magnetic field has been carried out by Das et al. [169]. Impact of thermal radiation on the unsteady boundary layer flow of a nanofluid over a heated stretching surface has been reported by Das et al. [170]. The entropy generation during hydromagnetic boundary layer flow of a viscous incompressible electrically conducting fluid due to radial stretching sheet with Newtonian heating in the presence of a transverse magnetic field and the thermal radiation has been analyzed by Das et al. [171]. Das et al. [172, 173] have extended entropy analysis in hydromagnetic flow over a stretching sheet. Gireesha et al. [174] have examined the thermal radiation and Hall effects on boundary layer flow past a stretching non-isothermal surface embedded in porous medium. Effect of Lorentz forces on forced-convection nanofluid flow over a stretched surface has been reported by Sheikholeslami et al. [175]. Hydromagnetic slip flow of nanofluid over a curved stretching surface has been described by Abbas et al. [176] taking heat generation and thermal radiation into account. A numerical study of magnetohydrodynamic transport of nanofluids over a vertical stretching sheet has been examined by Akbar et al. [177] taking buoyancy effects into account. Ibrahim [178] has examined the melting heat transfer in magnetohydrodynamic (MHD) stagnation point flow of a nanofluid past a stretching sheet. Das et al. [179] have examined entropy analysis of MHD flow past a convectively heated stretching cylinder in presence of variable thermal conductivity. Recently, Alarifi et al. [180] have studied MHD flow and heat transfer over a vertical stretching sheet with heat source or sink effect.

1.7 Present Investigation

In this thesis, we have studied some MHD flow and heat transfer problems with or without Hall Current. Some problems are presented considering radiation effect. Some flow models are designed in rotating environment. The thesis consists of eight chapters, **Chapter 1** presents the introductory part. The definition of related key terms, parameters, basic equations are presented in this chapter. Also a review on earlier works related to the present investigations has been discussed precisely. In **Chapter 2-7**, model problems are mathematically developed and the impact of pertinent parameters are deliberated in details through graphs and tables. Furthermore, **Chapter 8** summarizes the thesis pointing out the main findings and outlining some directions for future research work in this field.

Six problems are considered in this thesis, namely (i) Radiation effect on MHD fully developed mixed convection in a vertical channel with asymmetric heating, (ii) Oscillatory MHD Couette flow in a rotating system, (iii) Hydromagnetic oscillatory reactive flow through a porous channel in rotating frame subject to convective heat exchange under Arrhenius kinetics, (iv) Layout of Boussinesq couple-stress fluid flow over an exponentially stretching sheet with slip in porous space subject to variable magnetic field, (v) Outlining impact of Hall currents on unsteady magneto-convection in a moving channel with Cogley-Vincent-Gilles heat flux model, (vi) Hall effects on unsteady MHD reactive flow through a porous channel with convective heating under Arrhenius reaction rate.

The effects of radiative heat transfer on MHD fully developed flow of a viscous incompressible electrically conducting fluid through a vertical channel with asymmetric heating of the walls in the presence of a uniform transverse magnetic field has been studied in **second chapter**. The Cogley heat flux model is considered in the energy equation and the temperature of the walls are assumed to be constant. The governing equations of the motion are a set of simultaneous ordinary differential equations and their exact solutions in dimensionless form have been obtained under relevant boundary conditions. The expressions for velocity profile, the induced magnetic field, temperature field and shear stresses have been obtained. To validate the results obtained, the governing equations of the flow has been derived taking a limiting consideration of radiation. The numerical values of velocity profile, the induced magnetic field, temperature and shear stresses are illustrated graphically to show interesting features of radiation, squared-Hartmann number, Grashof number and temperature difference ratio parameter. With increasing radiation parameter, the fluid temperature is lowering within the channel. The squared-Hartmann number slows down the fluid motion near the hot wall and enhances near the cold wall.

In **third chapter**, unsteady oscillatory Couette flow between two infinite horizontal parallel plates in a rotating system has been studied when one of the plate is held at rest and the other oscillates in its own plane. The governing equations are solved assuming the plate velocity in the form of Fourier series. Mathematical formulation of the problem contains three pertinent flow parameters, namely, Hartmann number, rotation parameter and frequency parameter. Asymptotic behaviours of the solution are analyzed taking very large values of frequency parameter, rotation parameter and Hartman number separately to gain the existence of different kinds of boundary layers. Asymptotic expansion ensures that either for large frequency parameter or for large rotation parameter, there exists a double-deck boundary layers whereas for large Hartmann number there exists a single-deck boundary layer. To validate the results obtained, the velocity fields are derived again under single plate oscillation. The numerical values of the steady and unsteady velocity fields are displayed graphically whereas that of shear stress at the stationary plate are presented in tabular form. The steady primary velocity increases while the steady secondary velocity decreases with an increase in Hartmann number. Phase angle causes to decrease the unsteady primary velocity while it causes to increase the unsteady secondary velocity.

The **fourth chapter** aims to study an unsteady hydromagnetic flow of a viscous incompressible electrically conducting reactive fluid in a porous channel with asymmetrical convective boundary conditions in a rotating frame of reference under Arrhenius kinetics, neglecting reactant consumption. The asymmetric convective heat exchange with the surrounding medium at the channel surfaces follows Newton's law of cooling. The chemical kinetics in the flow system is exothermic and assumed to follow Arrhenius rate law. The heat transfer characteristics of the flow are considered taking viscous and Joule dissipations into account. Expressions of the velocity components are obtained in closed form which are used to compute the wall shear stresses. The energy equation is tackled numerically using MATLAB. The effects of the pertinent parameters on the flow dynamics are analyzed graphically. Our results reveal that combined effect of magnetic field, rotation, suction/injection and convective heating have significant impact in controlling the flow characteristics in the channel. Biot number and Eckert number increase the fluid temperature across the channel. Fluid temperature decreases with an increase in Prandtl number while it increases with an increase in reaction parameter. An increase in reaction parameter reduce the rate of heat transfer at the upper plate of the channel.

The purpose of the **fifth chapter** is to analyze numerically the impacts of Joule heating, viscous dissipation, magnetic field and slip condition on flow of an electrically conducting Boussinesq couple stress fluid over an exponential stretching sheet embedded in a porous medium under the effect of variable magnetic field. The governing nonlinear partial differential equations are transformed to the nonlinear ordinary differential equations by using some appropriate dimensionless variables and then solved numerically by making the use of shooting iteration technique along with the fourth order Runge- Kutta integration scheme. The impacts of physical parameters on velocity and temperature profiles, stream lines, shear stress and rate of heat transfer are described quantitatively through graphs and tables. Results reveal that the fluid velocity profile is observed to reduce considerably within the boundary layer in the presence of magnetic field and slip condition. The enhanced radiation parameter is to decrease temperature profile. The thermal radiation is observed to influence the growth of thermal boundary layer thickness. The slip effect is favorable for fluid flow.

The candid intention of the **sixth chapter** is to explore the impacts of Hall currents with the buoyancy forces on the unsteady flow and heat transfer of a viscous incompressible electrically conducting fluid in a moving vertical channel under the influence of strong magnetic field and thermal radiation. The strong magnetic field causes the generation of Hall currents, which in turn give rise to the magnetic forces acting on the moving fluid in the channel. The Hall effect is taken into account by introducing the Hall parameter. The induced magnetic field is neglected while the electron-atom collision frequency is assumed to be relatively high, so that the Hall effect is assumed to exist. One of the channel walls has impulsive or uniformly accelerated motion. The flow is induced due to motion of the right wall of the channel and the entire flow field is subjected to a strong magnetic field. The fluid is considered to be a grey, absorbing-emitting but non-scattering medium. Cogley-Vincent-Gilles heat flux model is adopted to simulate the radiation component of heat transfer. A unified closed form analytical solution of governing equations has been obtained by employing the Laplace transform technique. The influences of intricate physical parameters on the velocity and temperature profiles, shear stresses and rate of heat transfer are analyzed graphically for both impulsive and accelerated motions of the right wall. The analysis reveals that Hall parameter has enhancing behaviour on the velocity profiles.

The **seventh chapter** deals with the study of an unsteady magnetohydrodynamic(MHD) flow and heat transfer of a reactive viscous incompressible electrically conducting fluid between two infinitely long parallel porous plates when one of the plate is set into impulsive/uniform accelerated motion in the presence of a uniform transverse magnetic field under Arrhenius reaction rate on taking Hall currents into account. The transient momentum equations are solved analytically using the Laplace transform technique and the velocity field and shear stresses are obtained in a unified closed form. The energy equation is tackled numerically using MATLAB. The effects of the pertinent parameters on the fluid velocity, temperature, the shear stress and the rate of heat transfer at the plates are presented in graphical form and discussed in detail. Our results reveal that the combined effects of magnetic field, suction/injection, exothermic reaction and variable thermal conductivity have significant impact on the hydromagnetic flow and heat transfer. The temperature of the reactive viscous fluid in the channel is reduced with increase in the Hall parameter or the variable thermal conductivity parameter, while it increases with the magnetic parameter or suction parameter.