Chapter 5

Generalized trapezoidal intuitionistic fuzzy soft sets in risk analysis

[1]

5.1 Introduction

Decision-making is a challenging task in mathematics as well as in other fields such as, in economics, engineering, environmental science, social science, medical science, etc. Besides, in this universe, uncertainty is a prominent unavoidable component to be handled very seriously. Then, to handle uncertainty in our real-life, some traditional mathematical models have already been discussed by the researchers such as, fuzzy set theory [183], probability theory [85], theory of vague sets [68], theory of rough sets [128], etc. Furthermore, Molodtsov [118] pointed out some drawbacks of these existing theories and provided a new mathematical model 'soft set theory' to deal with uncertainty. The main advantages of soft set theory are, firstly, in this theory, objects are defined through some parameters where, parameters can be selected according to the requirement of a problem with the help of words, sentences, mappings, etc. and secondly, at the initial stage, no need to illustrate an exact description of an associated object rather one can illustrate it approximately. These benefits make this theory more worthy in practice.

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In literature, there exist several types of articles on soft set theory. First of all, Maji et al. [107] developed some basic set theoretic operations and properties on soft sets. Moreover, they also defined the idea of parameter reduction through soft set. Besides, Roy and Maji [139] elevated the concept of fuzzy soft set by combining fuzzy set theory with soft set theory. Further, by inspiring Maji's work, several generalizations of soft set theory including, intuitionistic fuzzy soft set [104], neutrosophic soft set [108], interval-valued fuzzy soft set [130], etc. have been saved in literature. Moreover, in reference [170], Xiao et al. introduced the concept of trapezoidal fuzzy soft set theory by combining the notion of fuzzy soft set with trapezoidal fuzzy number. They also solved a decision making problem where, basically, all the practical data are in linguistic form and to deal with these linguistic values, they transformed these linguistic values into trapezoidal fuzzy numbers and further used their proposed trapezoidal fuzzy soft set for getting the solution. Now, if non membership degree is added in a trapezoidal fuzzy soft set together with its membership degree, then we can solve more complicated real-life based problems. But, till now, no researcher has done this notion.

Furthermore, risk assessment is one of the crucial tool in mathematics as well as in real life. In 1984, Schmucker [143] first introduced the notion of fuzzy risk analysis. Then, various type of approaches have been developed to calculate the risk of a system by using different fuzzy numbers such as, triangular fuzzy number [169], trapezoidal fuzzy number [72, 174], trapezoidal intuitionistic fuzzy number [192], etc. Moreover, Patra and Mondal [125] proposed a methodological approach to analyze the fuzzy risk in a medical problem by using ranking method. They [126] also used similarity measure approach to analyze the fuzzy risk level in a production system. Furthermore, these exist few articles [44–46] where, soft set theory has been used in solving risk analysis. So, there is a scope of using soft set theory in solving different types of risk assessment based problems in practice.

Therefore, in this chapter, firstly, we have introduced the notion of generalized trapezoidal intuitionistic fuzzy soft set(*GTrIFSS*) where, the satisfaction of an object with respect to a parameter is in terms of generalized trapezoidal intuitionistic fuzzy number. Then, we have developed some basic set theoretic operations including complement, union and intersection operations, on generalized trapezoidal intuitionistic fuzzy soft sets. After that, we have defined hamming distance for two generalized trapezoidal intuitionistic fuzzy soft sets. Finally, a decision-making algorithm has been proposed to solve generalized trapezoidal intuitionistic fuzzy soft set based decision making problems in risk analysis with all linguistic information intuitively. A real-life related risk assessment problem regarding the detecting of diabetic patient has been discussed and solved through generalized trapezoidal intuitionistic fuzzy soft set.

The outline of the chapter is as follows:

Section 5.2 recalls some basic ideas. In Section 5.3, we have introduced the notion of generalized trapezoidal intuitionistic fuzzy soft set. Then, in Section 5.4, a new distance measure approach has been presented. After that, in Section 5.5, we have proposed a new methodological approach to solve generalized trapezoidal intuitionistic fuzzy soft set based decision-making problems. In Section 5.6, an experimental analysis regarding the detecting of diabetic patient has been discussed and solved based on our proposed methodological approach. Section 5.7 contains some conclusions of this chapter.

5.2 Some basic relevant notions

This section recalls some existing notions those have been used in our subsequent discussions.

(i) **Generalized trapezoidal fuzzy number (GTrFN) [170].**

A generalized trapezoidal fuzzy number (GTrFN) on $\mathbb{R}$ is also a fuzzy set. Mathematically, it can be denoted by, $G_{Tr} = (\alpha_1, \alpha_2, \alpha_3, \alpha_4; w_T)$ and its membership function $\mu_{G_{Tr}}$ can be defined as follows: $\forall x \in X$,

$$\mu_{G_{Tr}}(x) = \begin{cases} w_T(\frac{(x-\alpha_1)}{(\alpha_2-\alpha_1)}) & if,\ x \in [\alpha_1, \alpha_2) \\ w_T & if,\ x \in [\alpha_2, \alpha_3] \\ w_T(\frac{(x-\alpha_4)}{(\alpha_3-\alpha_4)}) & if,\ x \in (\alpha_3, \alpha_4] \\ 0 & for,\ x \geq \alpha_4 \end{cases}$$

where w_T is the maximum degree of membership of the generalized trapezoidal fuzzy number G_{Tr} and $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ are the real numbers which follow the relationship, $\alpha_1 \leq \alpha_2 \leq \alpha_3 \leq \alpha_4$.

(ii) **Generalized trapezoidal intuitionistic fuzzy number (GTrIFN) [59].**

A generalized trapezoidal intuitionistic fuzzy number (GTrIFN) on $\mathbb{R}$ is basically an intuitionistic fuzzy set. Mathematically, it can be written as, $\tilde{G}_{Tr} = ((\alpha_1, \alpha_2, \alpha_3, \alpha_4; w_T),$ $(\alpha_1^{'}, \alpha_2^{'}, \alpha_3^{'}, \alpha_4^{'}; w_T^{'}))$ and its membership function $\mu_{G_{Tr}}$ and non-membership function $\nu_{G_{Tr}}$ are defined as follows. For $x \in X$,

$$\mu_{\tilde{G}_{Tr}}(x) = \begin{cases} w_T(\frac{(x-\alpha_1)}{(\alpha_2-\alpha_1)}) & if,\ x \in [\alpha_1, \alpha_2) \\ w_T & if,\ x \in [\alpha_2, \alpha_3] \\ w_T(\frac{(x-\alpha_4)}{(\alpha_3-\alpha_4)}) & if,\ x \in (\alpha_3, \alpha_4] \\ 0 & for,\ x \geq \alpha_4 \end{cases}$$

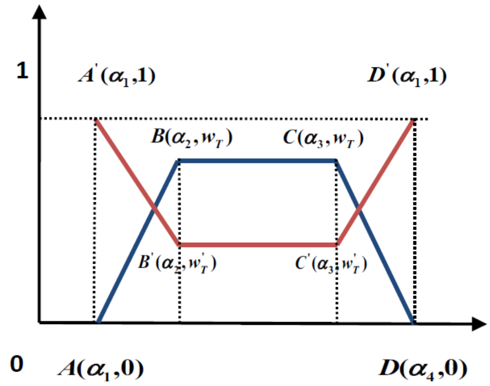

Figure 5.1: Graphical representation of a GTrIFN

$$\nu_{\tilde{G}_{Tr}}(x) = \begin{cases} \frac{(\alpha_2^{'}-x)+w_T^{'}(x-\alpha_1^{'})}{(\alpha_2^{'}-\alpha_1^{'})} & if,\ x \in [\alpha_1^{'}, \alpha_2^{'}) \\ w_T^{'} & if,\ x \in [\alpha_2^{'}, \alpha_3^{'}] \\ \frac{(x-\alpha_3^{'})+w_T^{'}(\alpha_4^{'}-x)}{(\alpha_4^{'}-\alpha_3^{'})} & if,\ x \in (\alpha_3^{'}, \alpha_4^{'}] \\ 1 & for,\ x \geq \alpha_4^{'} \end{cases}$$

where, w_T, $w_T^{'}$ are the maximum degree of membership and minimum degree of non-membership of the generalized trapezoidal intuitionistic fuzzy number $\tilde{G}_{Tr}$ such that, $w_T, w_T^{'} \in [0,1]$ and $0 \leq w_T + w_T^{'} \leq 1$ and also the real numbers $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_1^{'}, \alpha_2^{'}, \alpha_3^{'}, \alpha_4^{'}$ follow the relationship $\alpha_1 \leq \alpha_2 \leq \alpha_3 \leq \alpha_4$, $\alpha_1^{'} \leq \alpha_2^{'} \leq \alpha_3^{'} \leq \alpha_4^{'}$ with $\alpha_j \leq \alpha_j^{'}$, $j = 1, 2, 3, 4$.
The function $\Pi_{\tilde{G}_{Tr}}(x) = 1 - (\mu_{\tilde{G}_{Tr}}(x) + \nu_{\tilde{G}_{Tr}}(x))$ is called the hesitancy or indeterminacy of an element $x \in \tilde{G}_{Tr}$.

For the sake of simplicity in computation and without loss of any generality, throughout this chapter we have taken $\alpha_i = \alpha_i^{'}, i = 1, 2, 3, 4$, then symbolically the GTrIFN $\tilde{G}_{Tr}$ can be written as $\tilde{G}_{Tr} = (\alpha_1, \alpha_2, \alpha_3, \alpha_4; w_T, w_T^{'})$. Its graphical representation has been given in Figure 5.1.

(iii) **Arithmetic operations on GTrIFNs [121].**

Let, $\tilde{G}_A = (\alpha_1, \alpha_2, \alpha_3, \alpha_4; w_A, w_A^{'})$ and $\tilde{G}_B = (\beta_1, \beta_2, \beta_3, \beta_4; w_B, w_B^{'})$ be two GTrIFNs. Then, some arithmetic operations are as follows:
(i) $\tilde{G}_A + \tilde{G}_B = (\alpha_1 + \beta_1, \alpha_2 + \beta_2, \alpha_3 + \beta_3, \alpha_4 + \beta_4; (1 - w_A)(1 - w_B), (1 - w_A)(1 - w_B) - (1 - (w_A + w_A^{'}))(1 - (w_B + w_B^{'})))$,

(ii) For any $\lambda \in \mathbb{R}$,

$$\lambda\tilde{G}_A = \begin{cases} (\lambda\alpha_1, \lambda\alpha_2, \lambda\alpha_3, \lambda\alpha_4; 1-(1-w_A)^\lambda, (1-w_A)^\lambda - (1-(w_A+w_B)^\lambda))\text{if}, \lambda \geq 0 \\ (\lambda\alpha_4, \lambda\alpha_3, \lambda\alpha_2, \lambda\alpha_1; 1-(1-w_A)^\lambda, (1-w_A)^\lambda - (1-(w_A+w_B)^\lambda))\text{if}, \lambda \leq 0. \end{cases}$$

(iii) The complement of a generalized trapezoidal intuitionistic fuzzy number $\tilde{G}_A$ is denoted by, $\tilde{G}_A^c$ and is defined as follows:
$\tilde{G}_A^c = (1-\alpha_4, 1-\alpha_3, 1-\alpha_2, 1-\alpha_1; w_A^{'}, w_A)$.
(iv) The union of two generalized trapezoidal intuitionistic fuzzy numbers $\tilde{G}_A$ and $\tilde{G}_B$ is denoted by, $\tilde{G}_A \cup \tilde{G}_B$ and is defined as follows:
$\tilde{G}_A \cup \tilde{G}_B = (max(\alpha_1, \beta_1), max(\alpha_2, \beta_2), max(\alpha_3, \beta_3), max(\alpha_4, \beta_4); max(w_A, w_B),$
$min(w_A^{'}, w_B^{'}))$.
(v) The intersection of two generalized trapezoidal intuitionistic fuzzy numbers $\tilde{G}_A$ and $\tilde{G}_B$ is denoted by, $\tilde{G}_A \cap \tilde{G}_B$ and is defined as follows:
$\tilde{G}_A \cap \tilde{G}_B = (min(\alpha_1, \beta_1), min(\alpha_2, \beta_2), min(\alpha_3, \beta_3), min(\alpha_4, \beta_4); min(w_A, w_B),$
$max(w_A^{'}, w_B^{'}))$.

(iv) **Linguistic term set [171].**

Let $S = \{s_\alpha | \alpha = -\tau, .., -1, 0, 1, .., \tau\}$ be a discrete linguistic term set where, the term s_α indicates a possible evaluation for a linguistic variable. The mid level term s_0 represents the "indifferent performance" and the first level term and the last level term $s_{-\tau}$, s_τ represent the lower bound and upper bound of S respectively. The elements of the set S must satisfy the following features.
(1) ***Order relation:*** $s_\alpha \leq s_\beta$ if $\alpha \leq \beta$;
(2) ***Negative operator:*** $neg(s_\alpha) = s_\gamma$ where $\alpha = \tau - \gamma$.

Example 5.1. Now, consider a set of six linguistic terms as follows, $S = \{s_{-3} = very\ low\ s_{-2} = fairly\ low, s_{-1} = low\ , s_0 = medium, s_1 = high\ , s_2 = fairly\ high, s_3 = very\ high\}$.
Then, to deal with these linguistic variables, we have used generalized trapezoidal fuzzy numbers (GTrFNs) and generalized trapezoidal intuitionistic fuzzy numbers (GTrIFNs) individually as given in Table 5.1 and 5.2 respectively.
In Figure 5.2, graphical representation of a generalized trapezoidal intuitionistic fuzzy number has been given.

5.2.5 Trapezoidal fuzzy soft set (TrFSS) [170].

A pair (f_{Tr}, E) is said to be a trapezoidal fuzzy soft set (TrFSS) over a universal set X if and only if f_{Tr} is a mapping given by, $f_{Tr} : E \rightarrow P_{Tr}(X)$ where, $P_{Tr}(X)$ is the set of all trapezoidal fuzzy subsets of the set X.

Table 5.1: Six linguistic terms and their corresponding GTrFNs

Linguistic variable	*GTrFN*
$(VL)s_{-3}$	(0,0,0.02,0.07;1)
$(FL)s_{-2}$	(0.04,0.1,0.18,0.23;1)
$(L)s_{-1}$	(0.17,0.22,0.36,0.42;1)
$(M)s_0$	(0.32,0.41,0.58,0.65 ;1)
$(H)s_1$	(0.58,0.63,0.80,0.86;1)
$(FH)s_3$	(0.93,0.98,1.0,1.0;1)

Table 5.2: Six linguistic terms and their corresponding GTrIFNs

Linguistic variable	*GTrIFN*
s_{-3}	(0,0,0.02,0.07;1,0)
s_{-2}	(0.04,0.10,0.18,0.23;1,0)
s_{-1}	(0.17,0.22,0.36,0.42;1,0)
s_0	(0.32,0.41,0.58,0.65;1,0)
s_1	(0.58,0.63,0.80,0.86;1,0)
s_2	(0.72,0.81,0.92,0.97;1,0)
s_3	(0.93,0.98,1.0,1.0;1,0)

i.e., for each $e \in E$, $f_{Tr}(e)$ is called the set of all e-approximate elements of the trapezoidal fuzzy soft set (f_{Tr}, E).

Example 5.2. Now consider $X = \{u_1, u_2, u_3, u_4\}$ be the set of four alternatives and $E = \{e_1, e_2\}$ be the set of corresponding two parameters by which the elements of X have been defined. Now consider that, evaluations of all the alternatives with respect to every parameter are in terms of linguistic variables as given in Table 5.3. This a linguistic valued soft set. Now, by using Table 5.1, the corresponding trapezoidal fuzzy soft set has been given in Table 5.4.

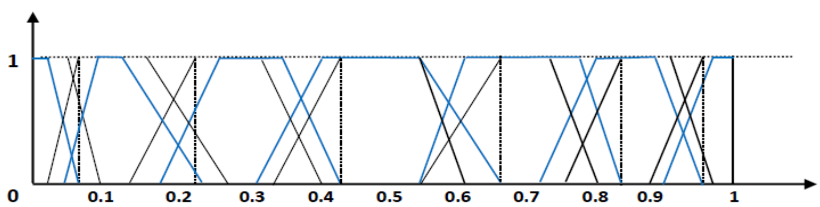

Figure 5.2: Graphical representation of six linguistic terms by GTrIFNs

Table 5.3: Linguistic valued soft set (Example 5.2)

	e_1	e_2
u_1	very low	Medium
u_2	High	Very high
u_3	Medium	Very low

Table 5.4: Trapezoidal fuzzy soft set (f_{Tr}, E) (Example 5.2)

	e_1	e_2
u_1	$(0, 0, 0.02, 0.07; 1)$	$(0.32, 0.41, 0.58, 0.65; 1)$
u_2	$(0.58, 0.63, 0.80, 0.86; 1)$	$(0.93, 0.98, 1.0, 1.0; 1)$
u_3	$(0.32, 0.41, 0.58, 0.65; 1)$	$(0, 0, 0.02, 0.07; 1)$

5.3 Generalized trapezoidal intuitionistic fuzzy soft set

Now, we have introduced the notion of generalized trapezoidal intuitionistic fuzzy soft set by considering all the parameters in a soft set in generalized trapezoidal intuitionistic fuzzy sense such that, the evaluation of an alternative or an object with respect to a parameter is in terms of generalized trapezoidal intuitionistic fuzzy number.

Definition 5.1. Let X be an initial universal set of alternatives and E be a set of parameters of the elements of X which are in generalized trapezoidal intuitionistic fuzzy words. Let $\tilde{P}_{Tr}(X)$ denotes the set of all generalized trapezoidal intuitionistic fuzzy subsets of the set X. Then a pair $(\tilde{f}_{Tr}, E)$ is said to be a generalized trapezoidal intuitionistic fuzzy soft set (GTrIFSS) over X if and only if $\tilde{f}_{Tr}$ is a mapping given by, $\tilde{f}_{Tr} : E \to \tilde{P}_{Tr}(X)$.

So, in other words, a generalized trapezoidal intuitionistic fuzzy soft set is also a parameterized family of subsets of the universe X and for each $e \in E$, $\tilde{f}_{Tr}(e)$ is called the set of e-approximate elements of the generalized trapezoidal intuitionistic fuzzy soft set $(\tilde{f}_{Tr}, E)$.

Mathematical representation of a GTrIFSS.

Now consider that, $X = \{x_1, x_2, .., x_m\}$ be an initial universal set and $E = \{e_1, e_2, .., e_n\}$ be the set of corresponding generalized trapezoidal intuitionistic fuzzy sense based parameters. Then, a generalized trapezoidal intuitionistic fuzzy soft set $(\tilde{f}_{Tr}, E)$ over the universal X can be represented as follows:

$$(\tilde{f}_{Tr}, E) = \{(e_j, \tilde{f}_{Tr}(e_j))\} = \{(e_j, (x_s, u_{sj}))|\forall e_j \in E, x_s \in X\} \tag{5.1}$$

where, u_{sj} is the rating of an alternative x_s corresponding to a parameter e_j in terms of generalized trapezoidal intuitionistic fuzzy number as, $u_{sj} = (\alpha_{(sj)_1}, \alpha_{(sj)_2}, \alpha_{(sj)_3}, \alpha_{(sj)_4}; w_{sj}, w'_{sj})$. Now, in the following, we have discussed this new generalization of soft set by using an example.

Example 5.3. Now let, $X = \{u_1, u_2, u_3\}$ be the set of three alternatives and $E = \{e_1, e_2, e_3\}$ be the set of corresponding three parameters of the elements of X. Now consider that, evaluations of all the alternatives with respect to every parameter are in terms of linguistic variables as given in Table 5.5. This is a linguistic valued soft set.

Table 5.5: Linguistic valued soft set (Example 5.3)

	e_1	e_2	e_3
u_1	very low	Medium	High
u_2	High	Very high	Low
u_3	Medium	Very low	High

Now, by using Table 5.2, the corresponding generalized trapezoidal intuitionistic fuzzy soft set has been given in Table 5.6.

Table 5.6: Generalized trapezoidal intuitionsitic fuzzy soft set $(\tilde{f}_{Tr}, E)$ (Example 5.3)

	e_1	e_2	e_3
u_1	$(0, 0, 0.02, 0.07; 1, 0)$	$(0.32, 0.41, 0.58, 0.65; 1, 0)$	$(0.58, 0.63, 0.80, 0.86; 1, 0)$
u_2	$(0.58, 0.63, 0.80, 0.86; 1, 0)$	$(0.93, 0.98, 1.0, 1.0; 1, 0)$	$(0.17, 0.22, 0.36, 0.42; 1, 0)$
u_3	$(0.32, 0.41, 0.58, 0.65; 1, 0)$	$(0, 0, 0.02, 0.07; 1, 0)$	$(0.58, 0.63, 0.80, 0.86; 1, 0)$

5.3.1 Some set theoretic operations on GTrIFSS

Definition 5.2. Consider that, $(\tilde{f}_{Tr}, E) = \{(e_j, (x_s, u_{sj}))|\forall e_j \in E, x_s \in X\}$ be a generalized trapezoidal intuitionistic fuzzy soft set over a universal set X where,
$u_{sj} = (\alpha_{(sj)_1}, \alpha_{(sj)_2}, \alpha_{(sj)_3}, \alpha_{(sj)_4}; w_{sj}, w'_{sj})$.
Then, the complement of $(\tilde{f}_{Tr}, E)$ is denoted by $(\tilde{f}_{Tr}, E)^c$ and is defined as follows:
$(\tilde{f}_{Tr}, E)^c = (\tilde{f}^c_{Tr}, E) = \{(e_j, (x_s, u^c_{sj}))|\forall e_j \in E, x_s \in X\}$ where, u^c_{sj} is the complement of the generalized trapezoidal intuitionistic fuzzy evaluation u_{sj}.

Example 5.4. Now, consider Example 5.3. Then, complement of the generalized trapezoidal intuitionistic fuzzy soft set (which has been given in Table 5.6) has been given in Table 5.7.

Definition 5.3. Consider two generalized trapezoidal intuitionistic fuzzy soft sets $(\tilde{f}_{Tr}, E)$ and $(\tilde{g}_{Tr}, E)$ over the universal set X as follows:
$(\tilde{f}_{Tr}, E) = \{(e_j, \tilde{f}_{Tr}(e_j))|\forall e_j \in E\} = \{(e_j, (x_s, u_{sj}))|\forall e_j \in E, x_s \in X\}$;
$(\tilde{g}_{Tr}, E) = \{(e_j, \tilde{g}_{Tr}(e_j))|\forall e_j \in E\} = \{(e_j, (x_s, v_{sj}))|\forall e_j \in E, x_s \in X\}$;

Table 5.7: Complement of the GTrIFSS $(\tilde{f}_{Tr}, E)$ (Example 5.3)

	e_1	e_2	e_3
u_1	$(0.93, 0.98, 1, 1; 0, 1)$	$(0.35, 0.42, 0.59, 0.68; 0, 1)$	$(0.14, 0.20, 0.37, 0.42; 0, 1)$
u_2	$(0.14, 0.20, 0.37, 0.42; 0, 1)$	$(0, 0, 0.02, 0.07; 0, 1)$	$(0.58, 0.64, 0.78, 0.83; 0, 1)$
u_3	$(0.35, 0.42, 0.59, 0.68; 0, 1)$	$(0.93, 0.98, 1, 1; 0, 1)$	$(0.14, 0.20, 0.37, 0.42; 0, 1)$

where, u_{sj} and v_{sj} are as follows:
$u_{sj} = (\alpha_{(sj)_1}, \alpha_{(sj)_2}, \alpha_{(sj)_3}, \alpha_{(sj)_4}; w_{sj}, w^{'}_{sj})$,
$v_{sj} = (\beta_{(sj)_1}, \beta_{(sj)_2}, \beta_{(sj)_3}, \beta_{(sj)_4}; W_{sj}, W^{'}_{sj})$.
Then, the union of $(\tilde{f}_{Tr}, E)$ and $(\tilde{g}_{Tr}, E)$ is denoted by, $(\tilde{f}_{Tr}, E) \cup (\tilde{g}_{Tr}, E) = (h_{Tr}, E)$ and is defined as $\tilde{h}_{Tr}(e_j) = \tilde{f}_{Tr}(e_j) \cup \tilde{g}_{Tr}(e_j)$ such that, $(\tilde{h}_T, E) = \{(e_j, (x_s, y_{sj}))|\forall e_j \in E, x_s \in X\}$ where,
$y_{sj} = (max(\alpha_{(sj)_1}, \beta_{(sj)_1}), max(\alpha_{(sj)_2}, \beta_{(sj)_2}), max(\alpha_{(sj)_3}, \beta_{(sj)_3}), max(\alpha_{(sj)_4}, \beta_{(sj)_4});$
$max(w_{sj}, W_{sj}), min(w^{'}_{sj}, W^{'}_{sj}))$.

Definition 5.4. The intersection operation of two GTrIFSSs $(\tilde{f}_{Tr}, E)$ and $(\tilde{g}_{Tr}, E)$ is denoted by, $(\tilde{f}_{Tr}, E) \cap (\tilde{g}_{Tr}, E) = (\tilde{H}_{Tr}, E)$ and is defined as $\tilde{H}_{Tr}(e_j) = \tilde{f}_{Tr}(e_j) \cap \tilde{g}_{Tr}(e_j)$ such that, $(\tilde{H}_{Tr}, E) = \{(e_j, (x_s, Y_{sj}))|\forall e_j \in E, x_s \in X\}$ where,
$Y_{sj} = (min(\alpha_{(sj)_1}, \beta_{(sj)_1}), min(\alpha_{(sj)_2}, \beta_{(sj)_2}), min(\alpha_{(sj)_3}, \beta_{(sj)_3}), min(\alpha_{(sj)_4}, \beta_{(sj)_4});$
$min(w_{sj}, W_{sj}), max(w^{'}_{sj}, W^{'}_{sj}))$.

Example 5.5. Now, consider two GTrIFSSs $(\tilde{f}_{Tr}, E)$ and $(\tilde{g}_{Tr}, E)$ as given in Tables 5.8 and 5.9.

Table 5.8: GTrIFSS $(\tilde{f}_{Tr}, E)$ (Example 5.5)

	e_1	e_2	e_3
u_1	$(0, 0, 0.02, 0.07; 1, 0)$	$(0.58, 0.63, 0.80, 0.86; 1, 0)$	$(0.93, 0.98, 1, 1; 1, 0)$
u_2	$(0.72, 0.81, 0.92, 0.97; 1, 0)$	$(0.17, 0.22, 0.36, 0.42; 1, 0)$	$(0.32, 0.41, 0.58, 0.65; 1, 0)$
u_3	$(0.32, 0.41, 0.58, 0.65; 1, 0)$	$(0.04, 0.10, 0.18, 0.23; 1, 0)$	$(0.04, 0.10, 0.18, 0.23; 1, 0)$

Table 5.9: GTrIFSS $(\tilde{g}_{Tr}, E)$ (Example 5.5)

	e_1	e_2	e_3
u_1	$(0.04, 0.10, 0.18, 0.23; 1, 0)$	$(0.32, 0.41, 0.58, 0.65; 1, 0)$	$(0.72, 0.81, 0.92, 0.97; 1, 0)$
u_2	$(0.93, 0.98, 1, 1; 1, 0)$	$(0.58, 0.63, 0.80, 0.86; 1, 0)$	$(0.04, 0.10, 0.18, 0.23; 1, 0)$
u_3	$(0.17, 0.22, 0.36, 0.42; 1, 0)$	$(0, 0, 0.02, 0.07; 1, 0)$	$(0.32, 0.41, 0.58, 0.65; 1, 0)$

Then, the union and intersection of these two generalized trapezoidal intuitionistic fuzzy soft sets have been given in Tables 5.10 and 5.11.

Table 5.10: Tabular form of $(\tilde{f}_{Tr}, E) \cup (\tilde{g}_{Tr}, E)$ (Example 5.5)

	e_1	e_2	e_3
u_1	$(0.04, 0.10, 0.18, 0.23; 1, 0)$	$(0.58, 0.63, 0.80, 0.86; 1, 0)$	$(0.93, 0.98, 1, 1; 1, 0)$
u_2	$(0.93, 0.98, 1, 1; 1, 0)$	$(0.58, 0.63, 0.80, 0.86; 1, 0)$	$(0.32, 0.41, 0.58, 0.65; 1, 0)$
u_3	$(0.32, 0.41, 0.58, 0.65; 1, 0)$	$(0.04, 0.10, 0.18, 0.23; 1, 0)$	$(0.32, 0.41, 0.58, 0.65; 1, 0)$

Table 5.11: Tabular form of $(\tilde{f}_{Tr}, E) \cap (\tilde{g}_{Tr}, E)$ (Example 5.5)

	e_1	e_2	e_3
u_1	$(0, 0, 0.02, 0.07; 0, 1)$	$(0.32, 0.41, 0.58, 0.65; 0, 1)$	$(0.72, 0.81, 0.92, 0.97; 0, 1)$
u_2	$(0.72, 0.81, 0.92, 0.97; 0, 1)$	$(0.17, 0.22, 0.36, 0.42; 1, 0)$	$(0.04, 0.10, 0.18, 0.23; 0, 1)$
u_3	$(0.17, 0.22, 0.36, 0.42; 0, 1)$	$(0, 0, 0.02, 0.07; 0, 1)$	$(0.04, 0.10, 0.18, 0.23; 0, 1)$

Definition 5.5. A generalized trapezoidal intuitionistic fuzzy soft set $(\tilde{f}_{Tr}, E)$, as defined in Definition 5.2., is said to be a null generalized trapezoidal intuitionistic fuzzy soft set if, $\forall e_j \in E$ and $x_s \in X$, $u_{sj} = (0, 0, 0, 0; 0, 1)$. Mathematically, a null generalized trapezoidal intuitionistic fuzzy soft set is denoted by, $(\tilde{f}_{Tr}, E)_\Phi$.

Definition 5.6. A trapezoidal intuitionistic fuzzy soft set $(\tilde{f}_{Tr}, E)$, as defined in Definition 5.2., is said to be an absolute generalized trapezoidal intuitionistic fuzzy soft set if, $\forall e_j \in E$ and $x_s \in X$, $u_{sj} = (1, 1, 1, 1; 1, 0)$. Mathematically, an absolute generalized trapezoidal intuitionistic fuzzy soft set is denoted by, $(\tilde{f}_{Tr}, E)_X$

Theorem 5.1. Union and intersection on generalized trapezoidal intuitionistic fuzzy soft sets satisfy the following properties:
(i) $(\tilde{f}_{Tr}, E) \cup (\tilde{f}_{Tr}, E) = (\tilde{f}_{Tr}, E)$;
(ii) $(\tilde{f}_{Tr}, E) \cap (\tilde{f}_{Tr}, E) = (\tilde{f}_{Tr}, E)$;
(iii) $(\tilde{f}_{Tr}, E) \cup (\tilde{f}_{Tr}, E)_\Phi = (\tilde{f}_{Tr}, E)$;
(iv) $(\tilde{f}_{Tr}, E) \cap (\tilde{f}_{Tr}, E)_\Phi = (\tilde{f}_{Tr}, E)_\Phi$;
(v) $(\tilde{f}_{Tr}, E) \cup (\tilde{f}_{Tr}, E)_\Phi = (\tilde{f}_{Tr}, E)_X$;
(v) $(\tilde{f}_{Tr}, E) \cap (\tilde{f}_{Tr}, E)_X = (\tilde{f}_{Tr}, E)$.

Proof: From Definition 5.3, Definition 5.4, Definition 5.5, Definition 5.6 the proofs of these theorems are straightforward.

Theorem 5.2. De Morgan's laws.
Let, $(\tilde{f}_{Tr}, E)$ and $(\tilde{g}_{Tr}, E)$ be two generalized trapezoidal intuitionistic fuzzy soft sets over the universal set X. Then, they satisfy the following properties:
(i) $\Big((\tilde{f}_{Tr}, E) \cup (\tilde{g}_{Tr}, E)\Big)^c = (\tilde{f}_{Tr}, E)^c \cap (\tilde{g}_{Tr}, E)^c$;

(ii) $\Big((\tilde{f}_{Tr}, E) \cap (\tilde{g}_{Tr}, E)\Big)^c = (\tilde{f}_{Tr}, E)^c \cup (\tilde{g}_{Tr}, E)^c$.

Proof: (i) Let, $(\tilde{f}_{Tr}, E) = \{(e_j, (x_s, u_{sj}))|\forall e_j \in E, x_s \in X\}$ and
$(\tilde{g}_{Tr}, E) = \{(e_j, (x_s, v_{sj}))|\forall e_j \in E, x_s \in X\}$ where,
$u_{sj} = (\alpha_{(sj)_1}, \alpha_{(sj)_2}, \alpha_{(sj)_3}, \alpha_{(sj)_4}; w_{sj}, w'_{sj})$ and $v_{sj} = (\beta_{(sj)_1}, \beta_{(sj)_2}, \beta_{(sj)_3}, \beta_{(sj)_4}; W_{sj}, W'_{sj})$.
Then, $(\tilde{f}_{Tr}, E)^c = \{(e_j, (x_s, u^c_{sj}))|\forall e_j \in E, x_s \in X\}$ and
$(\tilde{g}_{Tr}, E)^c = \{(e_j, (x_s, v^c_{sj}))|\forall e_j \in E, x_s \in X\}$.
Again let, $(\tilde{f}_{Tr}, E) \cup (\tilde{g}_{Tr}, E) = (\tilde{h}_{Tr}, E)$ where, $(\tilde{h}_{Tr}, E) = \{(e_j, (x_s, u_{sj} \cup v_{sj}))|\forall e_j \in E, x_s \in X\}$. Then, $(\tilde{h}_{Tr}, E)^c = \{(e_j, (x_s, (u_{sj} \cup v_{sj})^c))|\forall e_j \in E, x_s \in X\}$.
So, to prove first part of the theorem, we only prove that, $(u_{sj} \cup v_{sj})^c = u^c_{sj} \cap v^c_{sj}$.
Now, $u_{sj} \cup v_{sj} = (max(\alpha_{(sj)_1}, \beta_{(sj)_1}), max(\alpha_{(sj)_2}, \beta_{(sj)_2}), max(\alpha_{(sj)_3}, \beta_{(sj)_3}), max(\alpha_{(sj)_4}, \beta_{(sj)_4}); max(w_{sj}, W_{sj}), min(w'_{sj}, W'_{sj}))$.
Then,

$$\begin{aligned}(u_{sj} \cup v_{sj})^c &= (1 - max(\alpha_{(sj)_4}, \beta_{(sj)_4}), 1 - max(\alpha_{(sj)_3}, \beta_{(sj)_3}), 1 - max(\alpha_{(sj)_2}, \beta_{(sj)_2}),\\ &\quad 1 - max(\alpha_{(sj)_1}, \beta_{(sj)_1}); min(w'_{sj}, W'_{sj}), max(w_{sj}, W_{sj}))\\ &= (min((1 - \alpha_{(sj)_4}), (1 - \beta_{(sj)_4})), min((1 - \alpha_{(sj)_3}), (1 - \beta_{(sj)_3})),\\ &\quad min((1 - \alpha_{(sj)_2}), (1 - \beta_{(sj)_2})), min((1 - \alpha_{(sj)_1}), (1 - \beta_{(sj)_1}));\\ &\quad min(w'_{sj}, W'_{sj}), max(w_{sj}, W_{sj})) = u^c_{sj} \cap v^c_{sj}\end{aligned}$$

(ii) The proof of this part is same as the above.

5.4 Hamming distance of two generalized trapezoidal intuitionistic fuzzy soft sets

Definition 5.7. A function $D : \tilde{\Delta}_{Tr}(X) \times \tilde{\Delta}_{Tr}(X) \to [0, 1]$ where, $\tilde{\Delta}_{Tr}(X)$ is the set of all generalized trapezoidal intuitionistic fuzzy soft sets over the universe X, is said to be a distance measure for generalized trapezoidal intuitionistic fuzzy soft sets if it satisfies the following conditions.

(i) $D((\tilde{f}_{Tr}, E), (\tilde{f}_{Tr}, E)) = 0$;

(ii) $D((\tilde{f}_{Tr}, E), (\tilde{g}_{Tr}, E)) = D((\tilde{g}_{Tr}, E), (\tilde{f}_{Tr}, E))$;

(iii) For any $(\tilde{f}_{Tr}, E), (\tilde{g}_{Tr}, E), (\tilde{h}_{Tr}, E) \in \tilde{\Delta}_{Tr}(X)$,
$D((\tilde{f}_{Tr}, E), (\tilde{h}_{Tr}, E)) \leq D((\tilde{f}_{Tr}, E), (\tilde{g}_{Tr}, E)) + D((\tilde{g}_{Tr}, E), (\tilde{h}_{Tr}, E))$.

Definition 5.8. Let, $(\tilde{f}_{Tr}, E)$ and $(\tilde{g}_{Tr}, E)$ be two generalized trapezoidal intuitionistic fuzzy soft sets over the universal set X as given in Definition 5.3. Then, based on reference [134], hamming distance measure between these two GTrIFSSs is as follows:

$$D((\tilde{f}_{Tr},E),(\tilde{g}_{Tr},E)) = \frac{1}{8mn}(\mid \alpha_{(sj)_1}-\beta_{(sj)_1}\mid + \mid \alpha_{(sj)_2}-\beta_{(sj)_2}\mid + \mid \alpha_{(sj)_3}-\beta_{(sj)_3}\mid + \mid \alpha_{(sj)_4}-\beta_{(sj)_4}\mid + \mid w_A\alpha_{(sj)_1}-w_B\beta_{(sj)_1}\mid + \mid w_A\alpha_{(sj)_2}-w_B\beta_{(sj)_2}\mid + \mid w_A\alpha_{(sj)_3}-w_B\beta_{(sj)_3}\mid + \mid w_A\alpha_{(sj)_4}-w_B\beta_{(sj)_4}\mid + \mid w'_A\alpha_{(sj)_1}-w'_B\beta_{(sj)_1}\mid + \mid w'_A\alpha_{(sj)_2}-w'_B\beta_{(sj)_2}\mid + \mid w'_A\alpha_{(sj)_3}-w'_B\beta_{(sj)_3}\mid + \mid w'_A\alpha_{(sj)_4}-w'_B\beta_{(sj)_4}\mid) \quad (5.2)$$

Theorem 5.3. (i) from Equation 5.2. it is straightforward that $D((\tilde{f}_{Tr},E),(\tilde{f}_{Tr},E))=0$.
(ii) It is straightforward.
(iii) Now consider that, $(\tilde{h}_{Tr},E)=\{(e_j,(x_s,x_{sj}))\mid\forall e_j\in E, x_s\in X\}$ where, $x_{sj}=(\gamma_{(sj)_1},\gamma_{(sj)_2},\gamma_{(sj)_3},\gamma_{(sj)_4};\bar{w}_{sj},\bar{w}'_{sj})$.
Then,

$$D((\tilde{f}_{Tr},E),(\tilde{h}_{Tr},E)) = \frac{1}{8mn}(\mid \alpha_{(sj)_1}-\gamma_{(sj)_1}\mid + \mid \alpha_{(sj)_2}-\gamma_{(sj)_2}\mid + \mid \alpha_{(sj)_3}-\gamma_{(sj)_3}\mid + \mid \alpha_{(sj)_4}-\gamma_{(sj)_4}\mid + \mid w_{sj}\alpha_{(sj)_1}-\bar{w}_{sj}\gamma_{(sj)_1}\mid + \mid w_{sj}\alpha_{(sj)_2}-\bar{w}_{sj}\gamma_{(sj)_2}\mid + \mid w_{sj}\alpha_{(sj)_3}-\bar{w}_{sj}\gamma_{(sj)_3}\mid + \mid w_{sj}\alpha_{(sj)_4}-\bar{w}_{sj}\gamma_{(sj)_4}\mid + \mid w'_{sj}\alpha_{(sj)_1}-\bar{w}'_{sj}\gamma_{(sj)_1}\mid + \mid w'_{sj}\alpha_{(sj)_2}-\bar{w}'_{sj}\gamma_{(sj)_2}\mid + \mid w'_{sj}\alpha_{(sj)_3}-\bar{w}'_{sj}\gamma_{(sj)_3}\mid + \mid w'_{sj}\alpha_{(sj)_4}-\bar{w}'_{sj}\gamma_{(sj)_4}\mid)$$

$$= \frac{1}{8mn}(\mid \alpha_{(sj)_1}-\beta_{(sj)_1}+\beta_{(sj)_1}-\gamma_{(sj)_1}\mid + \mid \alpha_{(sj)_2}-\beta_{(sj)_2}+\beta_{(sj)_2}-\gamma_{(sj)_2}\mid + \mid \alpha_{(sj)_3}-\beta_{(sj)_3}+\beta_{(sj)_3}-\gamma_{(sj)_3}\mid + \mid \alpha_{(sj)_4}-\beta_{(sj)_4}+\beta_{(sj)_4}-\gamma_{(sj)_4}\mid + \mid w_{sj}\alpha_{(sj)_1}-W_{sj}\beta_{(sj)_1}+W_{sj}\beta_{(sj)_1}-\bar{w}_{sj}\gamma_{(sj)_1}\mid + \mid w_{sj}\alpha_{(sj)_2}-W_{sj}\beta_{(sj)_2}+W_{sj}\beta_{(sj)_2}-\bar{w}_{sj}\gamma_{(sj)_2}\mid + \mid w_{sj}\alpha_{(sj)_3}-W_{sj}\beta_{(sj)_3}+W_{sj}\beta_{(sj)_3}-\bar{w}_{sj}\gamma_{(sj)_3}\mid + \mid w_{sj}\alpha_{(sj)_4}-W_{sj}\beta_{(sj)_4}+W_{sj}\beta_{(sj)_4}-\bar{w}_{sj}\gamma_{(sj)_4}\mid + \mid w'_{sj}\alpha_{(sj)_1}-W'_{sj}\beta_{(sj)_1}+W'_{sj}\beta_{(sj)_1}-\bar{w}'_{sj}\gamma_{(sj)_1}\mid + \mid w'_{sj}\alpha_{(sj)_2}-W'_{sj}\beta_{(sj)_2}+W'_{sj}\beta_{(sj)_2}-\bar{w}'_{sj}\gamma_{(sj)_2}\mid + \mid w'_{sj}\alpha_{(sj)_3}-W'_{sj}\beta_{(sj)_3}+W'_{sj}\beta_{(sj)_3}-\bar{w}'_{sj}\gamma_{(sj)_3}\mid + \mid w'_{sj}\alpha_{(sj)_4}-W'_{sj}\beta_{(sj)_4}+W'_{sj}\beta_{(sj)_4}-\bar{w}'_{sj}\gamma_{(sj)_4}\mid).$$

$$\leq \frac{1}{8mn}(\mid \alpha_{(sj)_1}-\beta_{(sj)_1}\mid + \mid \beta_{(sj)_1}-\gamma_{(sj)_1}\mid + \mid \alpha_{(sj)_2}-\beta_{(sj)_2}\mid + \mid \beta_{(sj)_2}-\gamma_{(sj)_2}\mid + \mid \alpha_{(sj)_3}-\beta_{(sj)_3}\mid + \mid \beta_{(sj)_3}-\gamma_{(sj)_3}\mid + \mid \alpha_{(sj)_4}-\beta_{(sj)_4}\mid + \mid \beta_{(sj)_4}-\gamma_{(sj)_4}\mid + \mid w_{sj}\alpha_{(sj)_1}-W_{sj}\beta_{(sj)_1}\mid + \mid W_{sj}\beta_{(sj)_1}-\bar{w}_{sj}\gamma_{(sj)_1}\mid + \mid w_{sj}\alpha_{(sj)_2}-W_{sj}\beta_{(sj)_2}\mid + \mid W_{sj}\beta_{(sj)_2}-\bar{w}_{sj}\gamma_{(sj)_2}\mid + \mid w_{sj}\alpha_{(sj)_3}-W_{sj}\beta_{(sj)_3}\mid + \mid W_{sj}\beta_{(sj)_3}-\bar{w}_{sj}\gamma_{(sj)_3}\mid + \mid w_{sj}\alpha_{(sj)_4}-W_{sj}\beta_{(sj)_4}\mid + \mid W_{sj}\beta_{(sj)_4}-\bar{w}_{sj}\gamma_{(sj)_4}\mid + \mid w'_{sj}\alpha_{(sj)_1}-W'_{sj}\beta_{(sj)_1}\mid + \mid W'_{sj}\beta_{(sj)_1}-\bar{w}'_{sj}\gamma_{(sj)_1}\mid + \mid w'_{sj}\alpha_{(sj)_2}-W'_{sj}\beta_{(sj)_2}\mid + \mid W'_{sj}\beta_{(sj)_2}-\bar{w}'_{sj}\gamma_{(sj)_2}\mid + \mid w'_{sj}\alpha_{(sj)_3}-W'_{sj}\beta_{(sj)_3}\mid + \mid W'_{sj}\beta_{(sj)_3}-\bar{w}'_{sj}\gamma_{(sj)_3}\mid + \mid w'_{sj}\alpha_{(sj)_4}-W'_{sj}\beta_{(sj)_4}\mid + \mid W'_{sj}\beta_{(sj)_4}-\bar{w}'_{sj}\gamma_{(sj)_4}\mid) =$$

$$D((\tilde{f}_{Tr},E),(\tilde{g}_{Tr},E)) + D((\tilde{g}_{Tr},E),(\tilde{h}_{Tr},E)).$$

Example 5.6. Now, consider two generalized trapezoidal intuitionistic fuzzy soft sets $(\tilde{f}_{Tr}, E)$ and $(\tilde{g}_{Tr}, E)$ over X as given in Tables 5.8 and 5.9.
Then, by using Equation 5.2, the hamming distance between $(\tilde{f}_{Tr}, E)$ and $(\tilde{g}_{Tr}, E)$ is as follows:
$D((\tilde{f}_{Tr}, E), (\tilde{g}_{Tr}, E)) = 0.22.$

5.5 A new decision-making approach for solving problems under generalized trapezoidal intuitionistic fuzzy soft environment

In this section, we have proposed a new algorithm for selecting the best alternative out of some considered parameters which are in generalized trapezoidal intuitionistic fuzzy sense.

Let us consider, a decision system which consists of total m alternatives as, $X = \{x_1, x_2, .., x_m\}$ and n corresponding parameters as, $E = \{e_1, e_2, .., e_n\}$ which are in generalized trapezoidal intuitionistic fuzzy sense. Since each of the criterion comes from different environments, all of them may not have the same character. Some of them have the criteria that 'larger is better' and some of them have the criteria that 'smaller is better'. Thus the criterion set may be split into two subsets A and B such that the parameters in the A set has the character 'larger is better' and the parameters in the B set has the character 'smaller is better' with $A \cup B = E$ and $A \cap B = \phi$. It is also assumed that, the weights of the associated parameters are, $\Psi = \{\varpi_1, \varpi_2, .., \varpi_n\}$ where, $0 \leq \varpi_j \leq 1$ and $\sum_{j=1}^{n} \varpi_j = 1$.
Now, the tabular representation of generalized trapezoidal intuitionistic fuzzy soft set has been given in Table 5.12.

Table 5.12: Tabular form of a GTrIFSS $(\tilde{f}_{Tr}, E)$ (in general case)

	e_1	e_2	$\ldots$	e_n
x_1	u_{11}	u_{12}	$\ldots$	u_{1n}
x_2	u_{21}	u_{22}	$\ldots$	u_{2n}
..			$\ldots$	
x_m	u_{m1}	u_{m2}	$\ldots$	u_{mn}

where, $u_{sj} = (\alpha_{(sj)_1}, \alpha_{(sj)_2}, \alpha_{(sj)_3}, \alpha_{(sj)_4}; w_{sj}, w'_{sj})$ is the evaluation of an alternative x_s over a parameter e_j. Now, based on the above data, our decision question is, select the best alternative from m alternatives.

To solve this problem now we have proposed a stepwise algorithm as given below.

Step 1. Categorize the criteria set into two sets A and B where, A is the set of benefited criteria i.e., their higher degree indicates goodness, and B is the set of cost criteria i.e., their lower degree indicates goodness. They will satisfy the property that, $A \cup B = E$ and $A \cap B = \phi$.

Step 2. Then, normalize all the decision data to eliminate different dimensions of all the evaluations. Normalized evaluation of an alternative x_s over a parameter e_j is denoted by, $\bar{u}_{sj} = \big((\bar{\alpha}_{(sj)_1}, \bar{\alpha}_{(sj)_2}, \bar{\alpha}_{(sj)_3}, \bar{\alpha}_{(sj)_4}); w_{sj}, w^{'}_{sj}\big)$ where,

$$(\bar{\alpha}_{(sj)_1}, \bar{\alpha}_{(sj)_2}, \bar{\alpha}_{(sj)_3}, \bar{\alpha}_{(sj)_4}) = \begin{cases} \left(\frac{\alpha_{(sj)_1}}{\alpha^{+}_{(sj)_4}}, \frac{\alpha_{(sj)_2}}{\alpha^{+}_{(sj)_3}}, \frac{\alpha_{(sj)_3}}{\alpha^{+}_{(sj)_2}}, \frac{\alpha_{(sj)_4}}{\alpha^{+}_{(sj)_1}}\right) \text{if, } e_j \in A \\ \left(\frac{\alpha^{-}_{(sj)_1}}{\alpha_{(sj)_4}}, \frac{\alpha^{-}_{(sj)_2}}{\alpha_{(sj)_3}}, \frac{\alpha^{-}_{(sj)_3}}{\alpha_{(sj)_2}}, \frac{\alpha^{-}_{(sj)_4}}{\alpha_{(sj)_1}}\right) \text{if, } e_j \in B. \end{cases} \tag{5.3}$$

and $\alpha^{+}_{(sj)_l} = max^{m}_{s=1}\{\alpha_{(sj)_l}\}; l = 1, 2, 3, 4,$
$\alpha^{-}_{(sj)_l} = min^{m}_{s=1}\{\alpha_{(sj)_l}\}; l = 1, 2, 3, 4.$

Step 3. Construct the weighted GTrIFSS $(\tilde{f}_{WTr}, E)$ as follows:

$$(\tilde{f}_{WTr}, E) = \{(e_j, (x_s, \varpi_j \bar{u}_{sj}))\} = \{(e_j, (x_s, \bar{\bar{u}}_{sj}))\}$$

Scalar multiplication of a generalized trapezoidal intuitionistic fuzzy number has been given in point (iii) in Section 5.2.
It has been assumed that, $\bar{\bar{u}}_{sj} = \big((\bar{\bar{\alpha}}_{(sj)_1}, \bar{\bar{\alpha}}_{(sj)_2}, \bar{\bar{\alpha}}_{(sj)_3}, \bar{\bar{\alpha}}_{(sj)_4}); w_{sj}, w^{'}_{sj}\big)$

Step 4. Now, we determine the model solution u_M for this decision-making problem as follows: $\forall j = 1, 2, .., n,$

$$u_M = \left\{\left(e_j, \left((\sum_{s=1}^{m} \frac{\bar{\bar{\alpha}}_{(sj)_1}}{m}, \sum_{s=1}^{m} \frac{\bar{\bar{\alpha}}_{(sj)_2}}{m}, \sum_{s=1}^{m} \frac{\bar{\bar{\alpha}}_{(sj)_3}}{m}, \sum_{s=1}^{m} \frac{\bar{\bar{\alpha}}_{(sj)_4})}{m}; \sum_{s=1}^{m} \frac{w_{sj}}{m}, \sum_{s=1}^{m} \frac{w^{'}_{sj}}{m}\right)\right)\right\} \tag{5.4}$$

Note: Some times this model solution is given in a decision-making problem.

Step 5. By using our proposed hamming distance, obtain the distance of every alternative from the model solution.

Step 6. Find the alternative with minimum distance value that will be the best or optimal solution for this problem.

5.6 An application of our proposed method in risk assessment of being effected by diabetes

In 1984, the idea of risk assessment was introduced by Schmucker [143] under fuzzy uncertain environment. Then, several researchers have worked on this field. Liou et al. [101] proposed a multi-criteria risk analysis process under linguistic environment. Then, Lee et al. [97] established a risk assessment to determine the possible risk on human health from ground water contamination by using fuzzy process. Now, we have illustrated a risk assessment approach in medical science to recognize the diabetic patient by using our proposed generalized trapezoidal intuitionistic fuzzy soft set based decision-making methodology.

Basically, due to the unbalanced of blood glucose level, peoples are affected by diabetes disease. Moreover, this disease is the major cause of heart attack, stroke, lower limb amputation, blindness, etc. Therefore, patients who have been suffered from this disease should take proper treatment and should follow proper medical guide lines. In the last few decades, Diabetes disease has broken out as a worldwide public health problem. According to the report of World Health Organization(WHO), in the year 2016, the number of diabetic patients was 422 million; in the year 2013, this number was 382 million and in the year 1980, this number was 180 million. Now a days, this disease has become an acute cause for direct death. Now, we have given a mathematical solution based on our proposed generalized trapezoidal intuitionistic fuzzy soft set for treating a diabetic patient which can help ta a medical specialist.

Corresponding Symptoms.

Since, in medical science, with respect to one symptom, several diseases may be happened, therefore exact disease diagnosis is a very important issue in medical science. Now, based on a doctors' suggestion, the related parameters for being a diabetic patient are as follows:

1. Age(e_1). Age is a big risk factor for diabetes. The risk for being diabetic a patient will be increased as older age. Under age 30, the risk is less and then, with respect to the increasing of age, the risk of a patient for being diabetic is going to increase.

2. Inactivity(e_2). The less activity creates greater risk of being a diabetic patient. A person who exercises his body every day nearly 4hrs have low risk to have this diabetes.

3. Family history(e_3). Diabetes is a heritable disease. If someone in the family was suffering from this disease, then the risk is very high to have this disease in the nest generation of this family.

4. Hypertension(e_4). Hypertension is a cause for high blood pressure as well as for diabetes. Generally, it seems that, when the systolic and diastolic pressures are between 100 to 140 and 70 to 90, respectively, the possibility of affecting by this disease is low. Again, when the systolic and diastolic pressure is in between 140 to 160 and 90 to 100, respectively then the risk of affecting by this disease is medium. When the systolic pressure is greater than 160 and the diastolic pressure is greater than 100, the risk of affecting by this disease is high.

5. Body weight(e_5). Some times, over body weight may be the caused for this disease. Therefore, adjusting body weight with the age is very important. Body weight is measured by the body mass index (BMI). If BMI is normal in a patient, that is, if BMI is less than $27kg/m^2$ then, the probability for affecting by this disease is very low. If a patient BMI rate is between 27 and 30 kg/m^2, then the probability for affecting by this disease is medium and a patient whose BMI rate is greater than 30 kg/m^2, then the patient has a high risk for affecting by this diabetes disease.

6. HDL-cholesterol(e_6). HDL-cholesterol means the high density lipoprotein-cholesterol. This is a good-behaved cholesterol. The high level HDL-cholesterol minimizes the risk of affecting by the diabetes disease. HDL level between $40 - 60$mg/dl is good for health, between $60 - 70$ mg/dl indicates low risk and below 40 mg/dl indicates high risk for being a diabetic patient.

Mathematical formulation of our case study.

Let us consider a set of five patients, $X = \{p_1, p_2, p_3, p_4, p_5\}$, as a universal set who have these above discussed symptoms with different levels. Now, considered that, the corresponding symptoms are the parameters as, E={age(e_1),inactivity(e_2), family history(e_3), hypertension(e_4), body weight(e_5), HDL-cholesterol(e_6)}.
Now, from above discussions, the parameters age (e_1), family history (e_3), hypertension (e_4), body weight (e_5) are the benefit parameters and rest of them, inactivity (e_2) and HDL-cholesterol (e_6), are cost parameters. Now the profile of these five patients with respect to the parameters have been given in Table 5.13. These are the row data. Therefore, we have transformed these data into the linguistic variables as given in Table 5.14. After that, to detect the patient who have been affected by the diabetes disease by using our proposed approach, we have transformed this linguistic values (given in Table 5.14) into generalized trapezoidal intuitionisic fuzzy soft set by using Table 5.2, as given in Table 5.15.

Table 5.13: Profile of these five patients (case study)

	e_1	e_2	e_3	e_4	e_5	e_6
p_1	40	$2\frac{1}{2}$hr	father has	120-80	70	45mg/dl
p_2	50	1 hr	grand father had	140-110	67	50mg/dl
p_3	20	3hr	no	110-80	55	66mg/dl
p_4	45	$\frac{2}{3}$hr	father has	130-90	61	38mg/dl
p_5	67	2 hr	father has	140-90	72	49mg/dl

Table 5.14: Linguistic soft set $(\tilde{F}^L, E)$ (case study)

	e_1	e_2	e_3	e_4	e_5	e_6
p_1	medium	fairly high	low	medium	very high	medium
p_2	fairly high	low	high	fairly high	high	medium
p_3	very low	very high	very low	low	medium	fairly high
p_4	high	very low	medium	medium	high	fairly low
p_5	very high	high	very high	high	very high	medium

Table 5.15: Tabular form of GrTIFSS $(\tilde{f}_{Tr}, E)$ (case study)

	e_1	e_2	e_3
p_1	(0.32,0.41,0.58,0.65;1,0)	(0.72,0.81,0.92,0.97;1,0)	(0.17,0.22,0.36,0.42;1,0)
p_2	(0.72,0.81,0.92,0.97;1,0)	(0.17,0.22,0.36,0.42;1,0)	(0.58,0.63,0.80,0.86;1,0)
p_3	(0,0,0.02,0.07;1,0)	(0.93,0.98,1,1;1,0)	(0,0,0.02,0.07;1,0)
p_4	(0.58,0.63,0.80,0.86;1,0)	(0,0,0.02,0.07;1,0)	(0.32,0.41,0.58,0.65;1,0)
p_5	(0.93,0.98,1,1;1,0)	(0.58,0.63,0.80,0.86;1,0)	(0.93,0.98,1,1;1,0)

	e_4	e_5	e_6
p_1	(0.32,0.41,0.58,0.65;1,0)	(0.93,0.98,1,1;1,0)	(0.32,0.41,0.58,0.65;1,0)
p_2	(0.72,0.81,0.92,0.97;1,0)	(0.58,0.63,0.80,0.86;1,0)	(0.32,0.41,0.58,0.65;1,0)
p_3	(0.17,0.22,0.36,0.42;1,0)	(0.32,0.41,0.58,0.65;1,0)	(0.72,0.81,0.92,0.97;1,0)
p_4	(0.32,0.41,0.58,0.65;1,0)	(0.58,0.63,0.80,0.86;1,0)	(0.04,0.1,0.18,0.23;1,0)
p_5	(0.58,0.63,0.80,0.86;1,0)	(0.93,0.98,1,1;1,0)	(0.32,0.41,0.58,0.65;1,0)

Again, with the help of the medical science, the model solution for this diabetes disease decision-making has been given in Table 5.16.

Table 5.16: Model solution u_M (in linguistic form)

e_1	e_2	e_3	e_4	e_5	e_6
high	very low	very high	high	high	low

It is also considered that, weights of the parameters in this disease diagnosis decision-making are, $\Psi = \{0.1, 0.2, 0.1, 0.2, 0.1, 0.3\}$.

Solution:

Step 1. Categorize the parameter set E into two subsets A and B where, $A = \{e_1, e_3, e_4, e_5\}$ is the set of benefit parameters (positive parameters) and $B = \{e_2, e_6\}$ is the set of cost parameters (negative parameters).

Step 2. Now, we have normalized all the data of Table 5.15 by using Equation 5.3. Normalized generalized trapezoidal intuitionistic fuzzy soft set has been given in Table 5.17.

Table 5.17: Normalized generalized trapezoidal intuitionistic fuzzy soft set $(\tilde{f}_{Tr}, E)$

	e_1	e_2	e_3
p_1	(0.32,0.41,0.59,0.70;1,0)	(0,0,0.02,0.97;1,0)	(0.17,0.22,0.37,0.45;1,0)
p_2	(0.72,0.81,0.94,1 ;1,0)	(0,0,0.09,0.42;1,0)	(0.58,0.63,0.82,0.92;1,0)
p_3	(0, 0, 0.02,0.08;1,0)	(0,0,0.09,1 ;1,0)	(0,0,0.02,0.08;1,0)
p_4	(0.58,0.63,0.82,0.92;1,0)	(0,0,0 ,0 ;1,0)	(0.32,0.41,0.59,0.70;1,0)
p_5	(0.93,0.98,1 , 1 ;1,0)	(0,0,0.03,0.12;1,0)	(0.93,0.98,1,1;1,0)

	e_4	e_5	e_6
p_1	(0.33,0.44,0.72,0.90;1,0)	(0.93,0.98,1,1;1,0)	(0.06,0.17,0.44,0.72;1,0)
p_2	(0.74,0.88,1,1 ;1,0)	(0.58,0.63,0.82,0.92;1,0)	(0.06,0.17,0.44,0.72;1,0)
p_3	(0.18,0.24,0.44,0.58;1,0)	(0.32,0.41,0.59,0.70;1,0)	(0.04,0.11,0.22,0.32;1,0)
p_4	(0.33,0.44,0.72,0.90;1,0)	(0.58,0.63,0.82,0.92;1,0)	(0.17,0.56,1 ,1 ;1,0)
p_5	(0.60,0.68,0.99,1 ;1,0)	(0.93,0.98,1,1;1,0)	(0.06,0.17,0.44,0.72;1,0)

Step 3. In this disease diagnosis decision-making problem, given weights of the parameters are, $\Psi = \{0.1, 0.2, 0.1, 0.2, 0.1, 0.3\}$. Then, the weighted generalized trapezoidal intuitionistic fuzzy soft sets $(\tilde{f}_{WTr}, E)$ has been given in Table 5.18.

Table 5.18: Weighted generalized trapezoidal intuitionistic fuzzy soft set $(\tilde{f}_{WTr}, E)$

	e_1	e_2	e_3
p_1	(0.032,0.041,0.059,0.07;1,0)	(0,0,0.004,0.02;1,0)	(0.017,0.022,0.037,0.045;1,0)
p_2	(0.072,0.081,0.094,0.1 ;1,0)	(0,0,0.018,0.082;1,0)	(0.058,0.063,0.082,0.092;1,0)
p_3	(0, 0, 0.002,0.008;1,0)	(0,0,0.004,0.016 ;1,0)	(0,0,0.002,0.008;1,0)
p_4	(0.058,0.063,0.082,0.092;1,0)	(0,0,0 ,0 ;1,0)	(0.032,0.041,0.059,0.070;1,0)
p_5	(0.093,0.098,0.1 , 0.1 ;1,0)	(0,0,0.006,0.024;1,0)	(0.093,0.098,0.1,0.1;1,0)

e_4	e_5	e_6
(0.066,0.088,0.144,0.18;1,0)	(0.093,0.098,0.1,0.1;1,0)	(0.018,0.051,0.132,0.216;1,0)
(0.148,0.176,0.2,0.2 ;1,0)	(0.058,0.063,0.082,0.092;1,0)	(0.018,0.051,0.132,0.216;1,0)
(0.036,0.048,0.088,0.116;1,0)	(0.032,0.041,0.059,0.070;1,0)	(0.012,0.033,0.066,0.096;1,0)
(0.066,0.088,0.144,0.18;1,0)	(0.058,0.063,0.082,0.092;1,0)	(0.051,0.168,0.3 ,0.3 ;1,0)
(0.12,0.136,0.198,0.2 ;1,0)	(0.093,0.098,0.1,0.1;1,0)	(0.018,0.051,0.132,0.216;1,0)

Step 4. In this problem, model solution has been provided initially as given in Table 5.16. Then, by using Table 5.2, all generalized trapezoidal intuitionsitic fuzzy transformations

have been given in Table 5.19.

Table 5.19: Model solution u_M (in GTrIFN form)

	e_1	e_2	e_3	e_4	e_5	e_6
u_M	(0.58,0.63,0.80,0.86;1,0)	(0,0,0.02,0.07;1,0)	(0.93,0.98,1,1;1,0)	(0.58,0.63,0.80,0.86;1,0)	(0.58,0.63,0.80,0.86;1,0)	(0.17,0.22,0.36,0.42;1,0)

Step 5. Then, by using the Equation 5.2, the distance of each of the patients from the model solution is as follows:
$D(p_1, u_M) = 0.0909$, $D(p_2, u_M) = 0.0662$, $D(p_3, u_M) = 0.166$, $D(p_4, u_M) = 0.1041$, $D(p_5, u_M) = 0.0593$.
Let, the threshold value of the distance measure for this decision-making is less than 0.1.

Step 6. So, from the above results, we have seen that, the patients p_5, p_1 and p_2 have the risk of being diabetic and the patient p_5 has the highest risk of being diabetic.

5.7 Conclusion

In this chapter, we have introduced the notion of generalized trapezoidal intuitionistic fuzzy soft sets as an extension of soft sets. Additionally, some basic set theoretic operations and properties has been developed on generalized trapezoidal intuitionistic fuzzy soft sets. Moreover, we have also introduced the notion of hamming distance for generalized trapezoidal intuitionsitic fuzzy soft sets. Further, a mathematical model has been proposed to solve a generalized trapezoidal intuitionisitic fuzzy soft set based decision-making problems by using this distance measure approach. After that, our proposed framework has been applied on disease diagnosis in medical science which can also be helped to a clinician.

As a further research, one can extend this work to other models like, interval-valued fuzzy soft set, type-2 fuzzy soft set, interval type-2 fuzzy soft set.
However, due to the increasing complexity in real-life, some time evaluation of an alternative in a soft set (Ex: Fuzzy soft set, intuitionistic fuzzy soft set, etc.) can be changed with respect to some possible states. So, as a further research one can solve such type of problems.