Chapter 5

Detor g-interior nodes and detor g-boundary nodes in m-polar fuzzy graphs

5.1 Introduction

Graph model can be used to represent electrical circuits. Minimizing the non-overlapping circuit is the main objective in such a system. Many problems in the real world involve multipolar information or multi-agents or multi-objects. Compared to a fuzzy graph, mPFG gives more accurate and exact results for real problems. Here, we present an application of mPFG about how a person can reach his destination in a short time using a strong path. Linda and Sunitha [75] given the concept of fuzzy detour g-distance. The notion of g-distance in fuzzy graphs was established by Rosenfeld and Bhutani [18]. Linda and Sunitha [76] founded the notation of g-boundary node, g-interior node, g-eccentric node. The length of longest $x - y$ path in a connected fuzzy graph G is the detour distance between two nodes x and y defined in [44]. Chartrand [47] defined the main concept of the detour center of a graph. The notion of detour number, detour set, detour nodes, detour basis in a graph were established by Chatrand et al. [46]. Interior nodes and boundary nodes are discussed in [45]. In this chapter, we introduced mPF detour g-distance, mPF detour g-interior node, mPF detour g-boundary node and explained their relations. Also, some properties of these parameters are investigated.

5.2 mPF detour g distance, mPF detour g periphery and mPF detour g eccentric subgraph

First we define mPF detour g distance and then mPF geodesic g distance. Then we defined mPF detour g periphery and discussed the characterization of mPF detour g eccentric node.

Definition 5.2.1. *The length of a $c-d$ strong mPFP P between c and d in connected mPFG G is an mPF detour g distance if there does not exist other strong mPFP longer than P between a and b and we denote it by $mPFD_g(c,d)$. Any $c-d$ strong mPFP with length $mPFD_g(c,d)$ is said to be a $c-d$ mPF g-detour.*

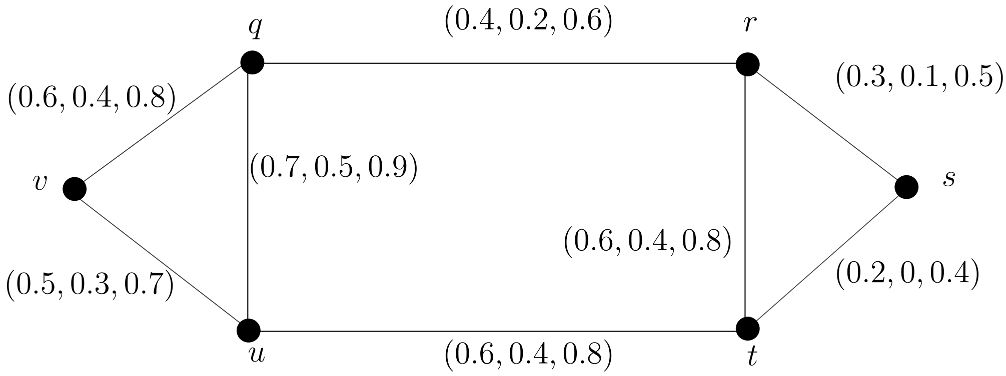

Figure 5.1: Connected 3PFG G.

Example 5.2.1. *Suppose G be a connected 3PFG of the graph $G^* = (V,E)$ where $V = \{q,r,s,t,u,v\}$ and $E = \{(r,t),(r,s),(q,r),(t,u),(u,v),(q,v),(s,t),(q,u)\}$ (see Fig. 5.1). For the 3PFG of Figure 5.1, it is seen that all arcs except (t,s), (q,r) and (v,u) are strong 3PFE and the 3PF detour g-distance of two nodes are given below: $3PFD_g(q,r) = 3$, $3PFD_g(q,v) = 1$, $3PFD_g(q,u) = 1$, $3PFD_g(q,t) = 2$, $3PFD_g(q,s) = 4$, $3PFD_g(v,u) = 2$, $3PFD_g(t,v) = 3$, $3PFD_g(v,s) = 5$, $3PFD_g(v,r) = 4$, $3PFD_g(u,t) = 1$, $3PFD_g(u,r) = 2$, $3PFD_g(u,s) = 3$, $3PFD_g(t,r) = 1$, $3PFD_g(t,s) = 2$ and $3PFD_g(r,s) = 1$.*

Definition 5.2.2. *The length of any smallest strong mPFP from s to t is the mPF geodesic distance, denoted by $mPFD_g(s,t)$.*

The mPF detour g eccentricity $e_{mPFD_g}(y)$ for a node y is an mPF detour g distance from y to a node maximum from y which implies $e_{mPFD_g}(y) = \max(mPFD_g(y, a))$, $\forall\ a \in G$. Suppose y be a node and each node whose mPF detour g distance is equal to $e_{mPFD_g}(y)$ then these vertex is called an mPF detour g eccentric node. The set of all mPF detour g eccentric vertices of x is denoted by $mPFD_g(x)$. The mPF detour g radius of G, symbolized as $rad_{mPFD_g}(G)$ and which is defined as $\min e_{mPFD_g}(x)$, $\forall\ x \in G$. If $e_{mPFD_g}(x) = rad_{mPFD_g}(G)$, then the vertex $x \in G$ is said to be the mPF detour g central node of G. The mPF detour g diameter of G is symbolized by $diam_{mPFD_g}(G)$, is defined as $\max e_{mPFD_g}(x)$, $\forall\ x \in G$. A node d in a G is an mPF detour g peripheral node of G if $e_{mPFD_g}(d) = diam_{mPFD_g}(G)$.

Example 5.2.2. *For the connected $mPFG$ G in Fig. 5.1, $e_{3PFD_g}(s) = 5$, $e_{3PFD_g}(r) = 4$, $e_{3PFD_g}(q) = 4$, $e_{3PFD_g}(t) = 3$, $e_{3PFD_g}(u) = 3$, $e_{3PFD_g}(v) = 5$ and $rad_{3PFD_g}(G) = 3$, $diam_{3PFD_g}(G) = 5$.*

Definition 5.2.3. *An $mPFG$ G is an mPF g-detour graph if $mPFD_g(s,t) = mPFD_g(s,t)$, $\forall\ (s,t) \in E$.*

Definition 5.2.4. *The mPF subgraph of an $mPFG$ G is induced by the only mPF detour g peripheral node of G, now the subgraph is called mPF detour g periphery of G and which is symbolized by $\big(Per_{mPFD_g}(G)\big)$.*

Definition 5.2.5. *If all vertex of a connected $mPFG$ G is mPF detour g eccentric node, then G is an mPF detour g eccentric graph. An mPF detour g eccentric subgraph of G is an $mPFSG$ of G, generated by the set of all mPF g-eccentric nodes of G is called, it is symbolized as $Ecc_{mPFD_g}(G)$.*

Example 5.2.3. *For the $3PF$ graph of Figure 5.2, nodes q, r, t are mPF detour g-periphery nodes since $e_{3PFD_g}(q) = 4$, $e_{3PFD_g}(r) = 4$, $e_{3PFD_g}(s) = 3$, $e_{3PFD_g}(t) = 4$, $e_{3PFD_g}(u) = 3$ and $diam_{3PFD_g}(G) = 4$. Here $Per_{3PFD_g}(G)$ of $mPFG$ shown in Figure 5.2.*

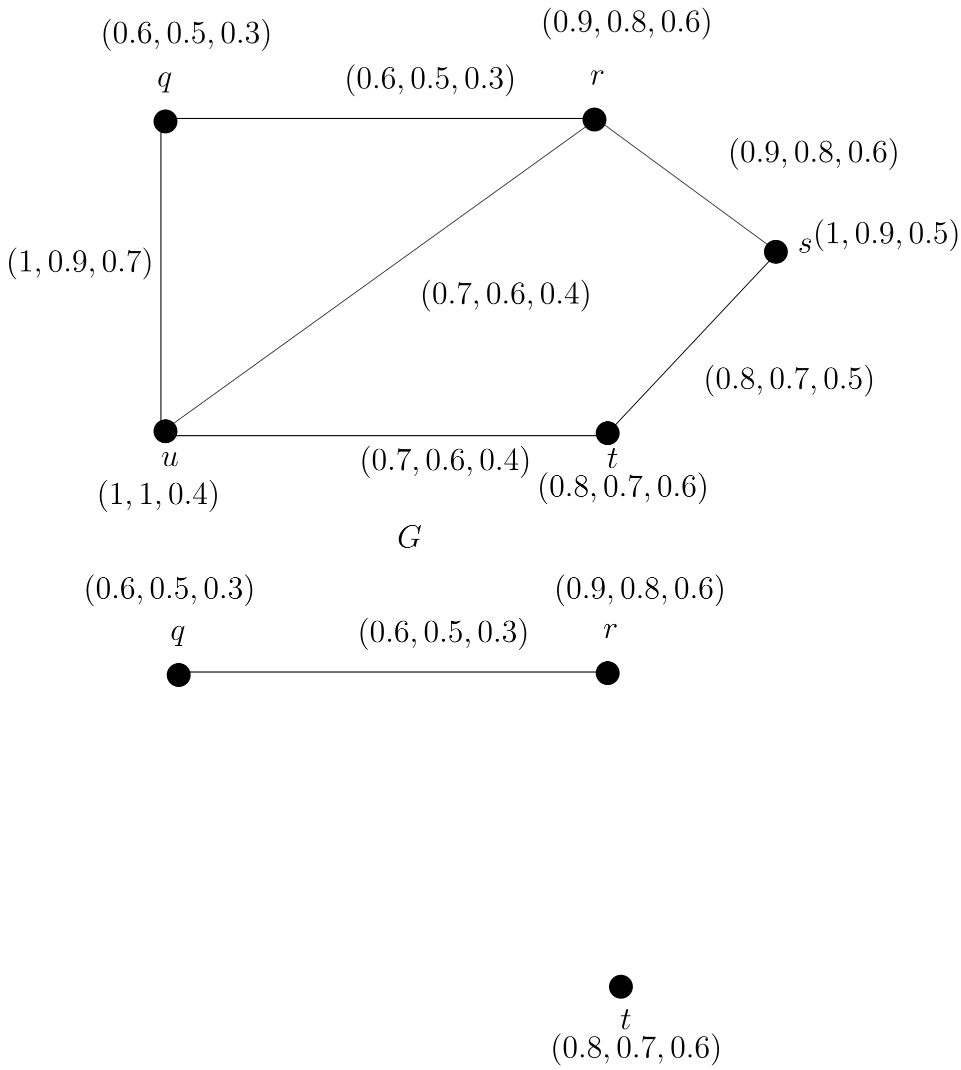

Figure 5.2: Connected 3PFG G and its $Per_{3PFD_g}(G)$.

Example 5.2.4. *From Figure 5.1, we get* $3PFD_g(q) = \{t, r\}$, $3PFD_g(r) = \{q\}$, $3PFD_g(s) = \{q, t, s\}$, $3PFD_g(t) = \{q\}$, $3PFD_g(u) = \{t, r\}$. *Its* $Ecc_{3PFD_g}(G)$ *is shown in Figure 5.2.*

Definition 5.2.6. *The mPFSG of an mPFG G is caused by the only mPF detour g central nodes is called mPF detour g centre subgraph, symbolized by* $C_{mPFD_g}(G)$. *A graph G is called mPF detour g self centered graph if all vertices of G are mPF detour g central nodes. In every mPF detour g self centered graph,* $rad_{mPFD_g}(G) = diam_{mPFD_g}(G)$.

Theorem 5.2.1. *Each node of an mPFG G is an mPF detour g eccentric iff G is an mPF detour g self centered.*

Proof. Let, every vertex is an mPF detour g eccentric node in G. Here we think that

G is not an mPF detour g self centrad graph. So $rad_{mPFD_g}(G) \neq diam_{mPFD_g}(G)$ and then $\exists$ a vertex $l \in G$ s.t. $e_{mPFD_g}(l) = diam_{mPFD_g}(G)$. Also let $r \in mPFD_g(l)$. Let B be a $l - r$ mPF detour in G. Then a vertex k on B must exist for which the vertex k is not an mPF detour g eccentric node of B. Also, k cannot be an mPF detour g eccentric node for the other node. Again if k is an mPF detour g eccentric node of a node a (say), means $k \in mPFD_g(a)$. Then $\exists$ an extension of $a - k$ mPF $g-$detour up to l or up to r. But, there is a contradiction in the facts that $k \in mPFD_g(a)$. So $rad_{mPFD_g}(G) = diam_{mPFD_g}(G)$. Hence G is an mPF detour g self centered graph.

Conversely, let us consider G be an mPF detour g self centrad graph and $x \in V$. Let $a \in mPFD_g(x)$. So this implies $e_{mPFD_g}(x) = mPFD_g(a, x)$. Again we know every vertex of G is mPF detour g central node i.e. $e_{mPFD_g}(y) = rad_{mPFD_g}(G)\ \forall\ y \in G$ because G is an mPF detour g self centrad graph, which means. So we have, $e_{mPFD_g}(a) = e_{mPFD_g}(x) = mPFD_g(a, x)$ and which implies that $x \in mPFD_g(a)$. Hence x is an mPF detour g eccentric node. □

Theorem 5.2.2. *If G is an mPF detour g self centrad graph with n number of nodes, then $rad_{mPFD_g}(G) = diam_{mPFD_g}(G) = n - 1$.*

Proof. Suppose G be an mPF detour g self centrad graph. If possible, let $diam_{mPFD_g}$ $(G) = l < n - 1$.

Suppose B_1 and B_2 are two distinct mPF detour g peripheral paths. Let $a \in B_1, b \in B_2$. So a strong mPF path exists in between a and b, because of connectedness of G. Then there exist nodes on B_1 and B_2, whose eccentricity $> l$, but which is impossible, because $diam_{mPFD_g}(G) = l$. Hence B_1 and B_2 are not distinct. Since B_1 and B_2 are arbitrary, then there exists a node x in G which x present in each mPF detour g peripheral paths. So, $e_{B.F.D_g}(x) < l$, which is also impossible, because G is an mPF detour g self centrad. Hence, $diam_{mPFD_g}(G) = n - 1 = rad_{mPFD_g}(G)$. □

Corollary 5.2.1. *Let G be a connected mPFG with the n number of nodes. Then $Per_{mPFD_g}(G) = G$ iff the mPF detour g eccentricity of every node of G is $n - 1$.*

Proof. Let $Per_{mPFD_g}(G) = G$. Then $e_{mPFD_g}(a) = diam_{mPFD_g}(G)$, $\forall\ a \in G$. So every node of G is an mPF detour g periphery node. Therefore, G is an mPF detour g self centrad graph and $rad_{mPFD_g}(G) = diam_{mPFD_g}(G) = n - 1$. So, $n - 1$ is the mPF detour g eccentricity of each node of G.

Conversely, suppose the mPF detour g eccentricity of each node of G is $n-1$. So $rad_{mPFD_g}(G) = diam_{mPFD_g}(G) = n-1$. All nodes of G are mPF detour g peripheral nodes and hence $Per_{mPFD_g}(G) = G$. □

Corollary 5.2.2. *For a connected mPFG G, $Ecc_{mPFD_g}(G) = G$ iff the mPF detour g eccentricity of every node of G is $n-1$.*

Proof. Suppose $Ecc_{mPFD_g}(G) = G$. So every node of G is mPF detour g eccentric node. Therefore G is mPF detour g self centrad graph and $red_{mPFD_g}(G) = diam_{mPFD_g}(G) = n-1$. Hence, $n-1$ is the mPF detour g eccentricity of each node of G.

Conversely, suppose the mPF detour g eccentricity of each node of G is $n-1$. So $rad_{mPFD_g}(G) = diam_{mPFD_g}(G) = n-1$. All nodes of G are mPF detour g peripheral nodes as well as mPF detour g eccentric node. Hence, $Ecc_{mPFD_g}(G) = G$. □

Theorem 5.2.3. *In a connected mPFG G, a node s is an mPF detour g peripheral node if and only if s is an mPF detour g eccentric node.*

Proof. Let us assume that $t \in Per_{mPFD_g}(G)$. So there exists an mPF detour g peripheral node, say t (distinct from a). Therefore, s is an mPF detour g eccentric node of s.

Conversely, let us say that s be an mPF detour g eccentric node of G and let $s \in mPFD_g(b)$. Let q and r be two mPF detour g peripheral nodes, then $mPFD_g(q,r) = diam_{mPFD_g}(G) = k(say)$. Let B_1 and B_2 be any $q-r$ and $t-s$ mPF g detour in G respectively. Then two cases will arise.

Case 1: When a is not an internal node in G i.e, there is only one node, say u which is adjacent to a. So $u \in B_2$. Since G is connected, u is connected to a node of B_1, say u'. So either $u' \in B_2$ or $c' \in (B_1 \cap B_2)$. Thus in any case the path from t to m or t to n through u and u' is longer than B_2. But it is impossible, since s is an mPF detour g eccentric node of t. Hence $e_{mPFD_g}(t) = diam_{mPFD_g}(G)$ i.e, $s \in Per_{mPFD_g}(G)$.

Case 2: When s is an internal node in G, then there exists a connection between s to m and a to n, because of connectedness of G. Then $t-s$ mPF g detour can be extended to m or n. This is impossible, because s is an mPF detour g eccentric node of t. Hence $e_{mPFD_g}(t) = diam_{mPFD_g}(G)$ i.e, s is an mPF detour g peripheral node of G. □

5.3 mPF detour g boundary node and mPF detour g interior node of an mPFG

In this section we defined mPF detour g boundary node and mPF detour g interior node of an mPFG G and discussed some results on these nodes.

Definition 5.3.1. *A vertex k in a connected mPFG G is an mPF detour g boundary node of a node l if $mPFD_g(l,k) \geq mPFD_g(l,j)$ for each j in G, where j is a neighbor of k. The set of every mPF detours g boundary nodes of l symbolized as $mPFD_gB(l)$. The set of every mPF detour g boundary nodes of G, Symbolized as $mPFD_gB(G)$.*

Example 5.3.1. *For the connected mPFG G shown in Figure 5.3, $mPFD_gB(q) = \{s,w\}$, $mPFD_gB(r) = \{q,v,w\}$, $mPFD_gB(s) = \{q,w,v\}$, $mPFD_gB(t) = \{q,s,w,v\}$, $mPFD_gB(u) = \{q,v,w\}$, $mPFD_gB(w) = \{q,s,v\}$,$mPFD_gB(v) = \{q,w\}$. Here q,s,v,w are the m-polar detour g- fuzzy boundary nodes of G.*

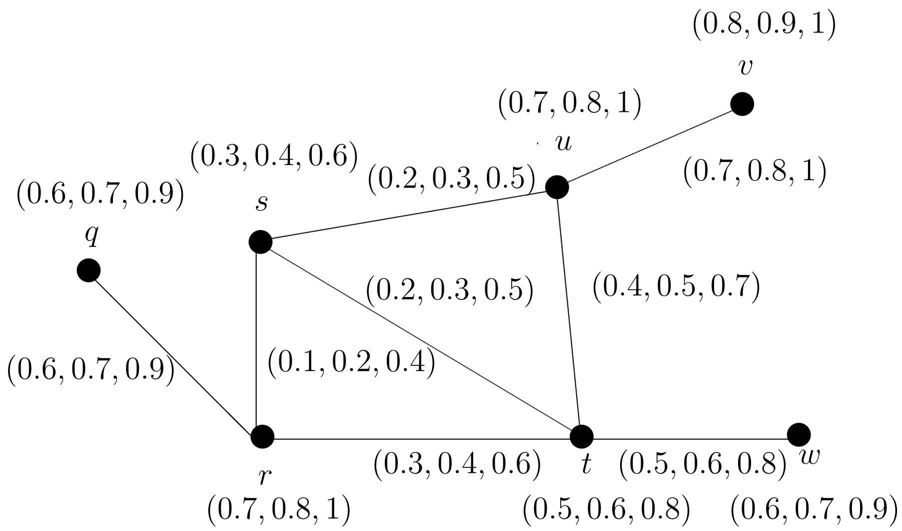

Figure 5.3: Connected 3PF graph G with boundary nodes $\{a, c, f, g\}$.

Definition 5.3.2. *The set of every neighbors of s is symbolized as $N_{mPF}(s)$ and the set of every strong mPF neighbors of u is symbolized as $N_{mPFS}(s)$.*

Definition 5.3.3. *If an mPF subgraph formed by strong m-polar neighbor of a node a in an mPFG G, form a complete mPFG then the node b is said to be a complete node of G.*

Theorem 5.3.1. *A node in a complete mPFG is mPF detour g boundary node of every other node if and only if the node is complete.*

Proof. Let a node l be a node in a connected mPFG G and l is complete node. Let k be another node of G. Each arc in G is strong, because of completeness of G. So $mPFD_g(k,l) = n-1 = mPFD_g(k,s), \forall s \in N(l)$. Then $l \in mPFD_G B(a)$.

Conversely, let l be an mPF detour g boundary node of every other node. Then each arc in G is strong, because of completeness of G. Then $mPFD_g(k,l) = n-1, \forall k \in G$. So all neighbors of l are strong. Hence by Definition 4.3, the node l is complete. □

Theorem 5.3.2. *If a vertex in a connected mPFG G is a complete vertex, then the vertex is an mPF detour g boundary node of every other node.*

Proof. Here a vertex l is a complete vertex in a connected mPFG G. If k is another vertex of G. Assume that $k = l_0, l_1, \ldots, l_{k-1}, l_k = l$ be a $k-l$ mPF g-detour and $c \in N_m PFS(b)$. Here two cases will aries

Case 1: If $c = l_{k-1}$, then $mPFD_g(k,c) \leq mPFD_g(k,l)$. Hence, l be a m-polar detour g-fuzzy boundary node of k.

Case 2: If $c \neq l_{k-1}$, since c is a strong neighbor of l, so a arc (c, l_{k-1}) is a strong mPF arc and also $c \neq l_{k-1}$. So the length of a path $k = l_0, l_1, \ldots, l_{k-1}, c, l_k = l$ is greater than the length of a path $k = l_0, l_1, \ldots l_{k-1}, l_k = l$. That is $mPFD_g(k,c) \leq mPFD_g(k,l)$. Hence, $l \in mPFD_g B(k)$. □

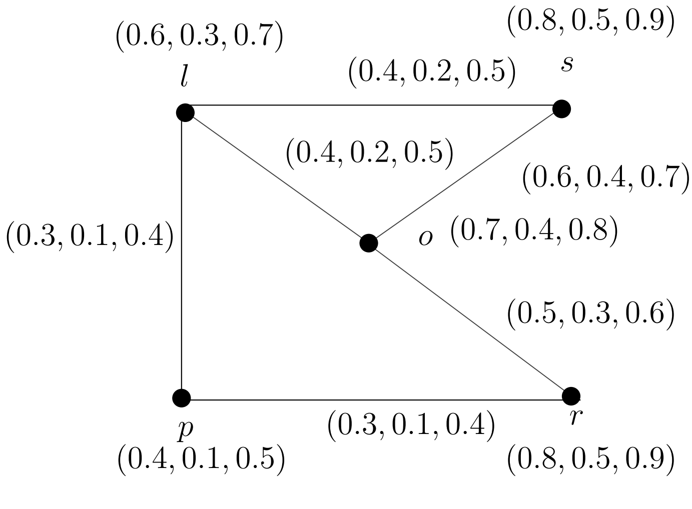

Figure 5.4: Connected mPFG G.

Remark 5.3.1. *It may not be possible to converse the above Theorem. For example, let the $mPFG$ of Figure 5.4. We see that s is an mPF detour g boundary node of every another nodes, but s is not a complete node.*

Theorem 5.3.3. *A connected $mPFG$ G is an $mPFT$ iff G is mPF detour g graph.*

Proof. Between any two vertices in mPFT G, there is exactly one strong mPFP. So $mPFD_g(l,k) = mPFD_g(l,k)$ for any two nodes l,k in G. Hence, G is mPF g-detour graph.

Conversely, let G be an mPF g-detour graph, which has n nodes. Then $mPFD_g(l,k) = mPFD_g(l,k)$ for any two nodes l,k in G. If $n = 2$ then G is an mPFT.

Let $n \geq 3$. If G is not an mPFT. So two nodes a, b exist in G for which there are at least two strong mPFPs between a and b. Let B_1 and B_2 be two $a - b$ strong mPFPs. So, $B_1 \cup B_2$ has a cycle $C(say)$ in G. If the nodes p and q are adjacent nodes in G, then we have $mPFD_g(q,p) = 1$ and $mPFD_g(q,p) > 1$. This contradicts the fact that $mPFD_g(q,p) = mPFD_g(q,p)$. So, G is an mPFT. □

Theorem 5.3.4. *In an $mPFT$ G, a vertex l is an mPF detour g boundary node of G iff l cannot be an $mPFCN$ of G.*

Proof. Suppose a node l in mPFT G is an mPF detour g boundary node of a node g in G. If l is an mPFCN of G.

Let E be an mPF maximum spanning tree ($MST_{mPF}(G)$) in G and this tree is unique in G. Again since l is an mPFCN that means l cannot be an internal node of E. Let $p \in N_{mP.F.S}(l)$ s.t. p does not lie on the mPF detour in E. Therefore, $mPFD_g(q,z)$ is the same when q,z be any two nodes of E. But $mPFD_g(g,p) = mPFD_g(g,l) + mPFD_g(l,p) > mPFD_g(g,l)$. This contradicts the fact that $l \in mPFD_gB(G)$. Therefore the node l cannot be an mPFCN of G.

Conversely, suppose l be not an mPFCN of the mPFG G. So l is the end vertex of $MST_{mPF}(G)$, which is unique. Then l has a strong neighbor which is also unique [130]. So there does not exist any extension of any mPF g-detour for a node p to l. Hence, $l \in mPFD_g(G)$. □

Definition 5.3.4. *A node l in an $mPFG$ G is an $mPFEN$ of G if h is only strong mPF neighbor of l, where $h \in G$.*

Example 5.3.2. *For the mPFG G in Fig. 5.3, the nodes q, v, w are mPFEN of G.*

Theorem 5.3.5. *A vertex a in an mPFT G is an mPF detour g boundary node then b is an mPFEN. Again if b is an mPFEN then a is an mPF detour g boundary node*

Proof. Suppose a is an belonging in $mPFD_gB(b)$ in an mPFT G. Let E be a $MST_{mPF}(G)$ in G, which is unique in G [130]. By Theorem 4.9, each node of G is an mPFCN of G or an mPFEN of G. So by Theorem 4.9, a must be an mPFEN of G.

Conversely, let a be an mPFEN of an mPFT G. Let E be the $MST_{mPF}(G)$ of G. Then a is an mPFEN of E. Hence, a is not an mPFCN. Therefore, by Theorem 4.9, $a \in mPFD_gB(G)$.

□

In a connected mPFG G, a node b lie between the nodes a and c in the sense of mPF detour g-distance if $mPFD_g(a, c) = mPFD_g(a, b) + mPFD_g(b, c)$.

Definition 5.3.5. *In a connected mPFG G, a vertex b is an m-polar detour $g-$fuzzy interior nodes if for each node a in G different from b, there is a node c in G for which $mPFD_g(a, c) = mPFD_g(a, b) + mPFD_g(b, c)$.*

Definition 5.3.6. *The set of all mPF detour g-interior node of G, Symbolized as $Int_{mPFD_g}(G)$, form an mPFSG of G.*

Example 5.3.3. *For the mPFG in Figure 5.3, $Int_{mP.F.D_g}(G) = \{r, u, t\}$.*

Theorem 5.3.6. *A vertex in a connected mPFG G is an mPF detour g boundary node of G iff the node cannot be an mPF detour g interior node of G.*

Proof. Let $b \in mPFD_gB(a)$ in a connected mPFG G. If possible, let $b \in Int_{mPFD_g}(G)$. So there a node c exists different from a and b s.t. b lies between a and c. Let $U : a = b_1, b_2, \ldots, b = b_k, b_{k+1}, \ldots, b_l = c$ be a $a - c$ mPF $g-$detour and $1 < k < l$. Then $b_{k+1} \in N_{mP.F.S}(b)$, and this implies $mPFD_g(a, b_{k+1}) > mPFD_g(a, b)$, so contradiction arise. Hence $b \notin G$.

Conversely, let the node $b \notin Int_{mPFD_g}(G)$. Then a node a exist in G for which any node c different from b and a, $mPFD_g(a, c) \neq mPFD_g(a, b) + mPFD_g(b, c)$. Therefore, $mPFD_g(a, q) \leq mPFD_g(a, b)$ where $q \in N_{mP.F.S}(b)$. This implies that b is a m-polar detour g-fuzzy boundary node of a. □

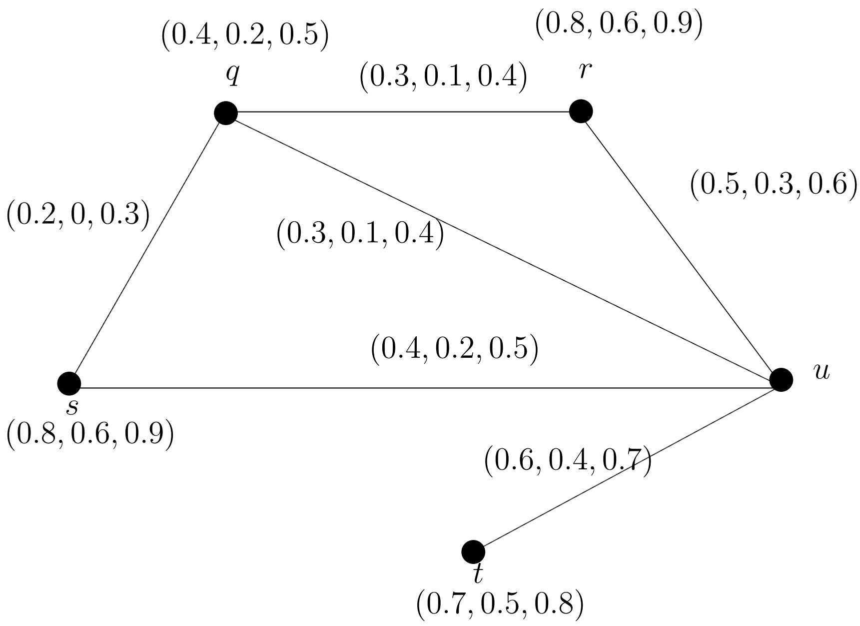

Figure 5.5: Connected mPFG G.

Example 5.3.4. *For the Connected mPFG G shown in Figure 5.5,* $mPFD_gB(s) = \{t\}$, $mPFD_gB(q) = \{t\}$, $mPFD_gB(r) = \{t\}$, $mPFD_gB(u) = \{q, t\}$, $mPFD_gB(t) = \{q\}$. *Here* q, t *are the m-polar detour g- fuzzy boundary nodes of* G, *but* q, t *are not m-polar detour g- fuzzy interior nodes of* G. *Again* s, u, r *are m-polar detour g- fuzzy interior nodes of* G *but they are not m-polar detour g- fuzzy boundary nodes of* G. *So if we consider any Connected mPFG then we can easily show that the above Theorem is true.*

Theorem 5.3.7. *A mPFEN of a connected mPFG G cannot be an mPF detour g interior node.*

Proof. Let q be an mPFEN of an mPFG G. Then there is only one mPF strong neighbor of q. So there is no strong mPF g-detour for which b lies between a and c, where a and c be two nodes of G and also different from b. Hence, $b \notin Int_{mPFD_g}(G)$. □

5.4 Application

Many problems in the real world involve multipolar information or multi-agents or multi-objects. Compared to a fuzzy graph, mPFG gives more accurate and exact results for real problems. Here, we present an application of mPFG about how a person can reach his destination in a short time using a strong path. In modern days, if we go from one town to another town then we usually use car, train, bus, etc. The availability of buses or trains are not the same everywhere. When a person travels to work or school every day, this form of journey is commonly known as commuting. Some people visit other states, towns or countries during their holidays. If the communication system is good then the journey will be good. Again, if the economic system of a city is good then the condition of the road is generally good. This communication system depends not only on the economic condition but also on many other things such as for example infrastructure, environment, fire safety, security, etc.

Here, we present a model of 3PFG which is used to find the shortest strong path between two cities. Fig. 5.6 shows a model of the road network which is represented by a 3PFG $G = (V, A, B)$. Here the vertices stand for cities and each edge of G stands for the roads between two cities. Here six cities are considered and they are denoted as $V = \{V_6, V_5, V_4, V_3, V_2, V_1\}$. Then the membership value of every vertices depended on three criterion namely {environment, economic system, infrastructure } and the membership value of each road depended on three criterion namely {Transportation availability, traffic, road length } and these characteristics are uncertain. Using the relation $B(u, v) \leq min\{A(u), A(v)\}$ for all $(u, v) \in E$, we calculated Edge membership value and edge membership value represent the relation between two cities.

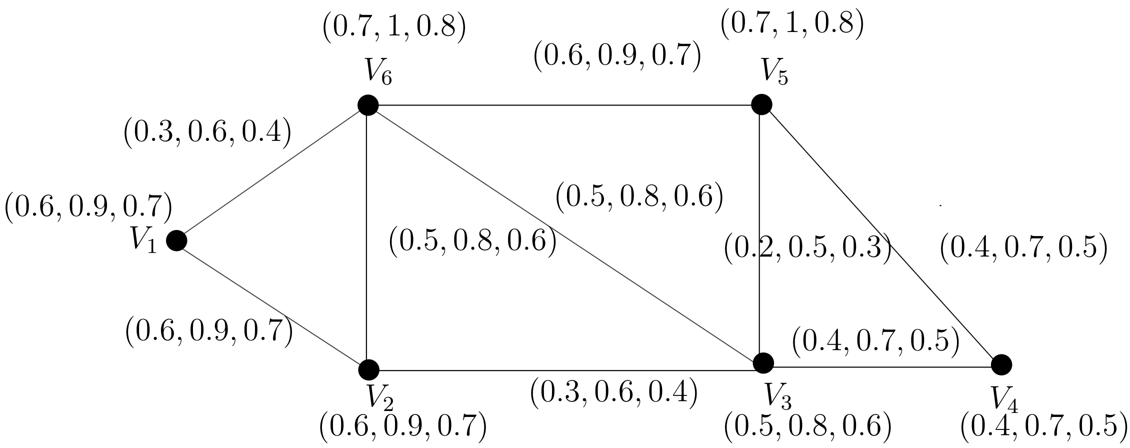

Figure 5.6: 3PFG G corresponding to the communication between some towns.

Suppose a person has started his/her journey from V_1 and he/she wants to go to the place V_5. Then his first goal is to find the strong path between V_1 and V_5. And then he/she wants to find out the shortest path between those strong paths. So, he try to find out shortest strong path between V_1 and V_5 for his safe journey. For the 3PFG G in Figure 5.6, the arcs $(V_5, V_4), (V_4, V_3), (V_3, V_6), (V_5, V_6), (V_6, V_2)$, (V_2, V_1) are strong arcs. The paths $V_1 - V_2 - V_6 - V_3 - V_4 - V_5$ and $V_1 - V_2 - V_6 - V_5$ are only two strong paths from V_1 to V_5. So $mPFD_g(V_1, V_5) = 5$ and $B.F.d_g(V_1, V_5) = 3$. So the path $V_1 - V_2 - V_6 - V_5$ is the shortest strong path from V_1 to V_5. If a person wants to go from V_1 to V_5 in the shortest path with the best communication system, then for him the path $V_1 - V_2 - V_6 - V_5$ will be the best route to go for his safe journey.

5.5 Summary

In this article, we have introduced mPF detour g-distance, mPF detour g-boundary nodes, mPF detour g-interior nodes in mPFGs and properties of these. We initiated theorems on mPF detour g-interior node, mPF detour g-boundary node, mPF cut node in mPFG, using maximum mPF spanning tree. We are extending our research work to define connectivity index on mPFG and its properties and its applications on real life problems etc.