M.Sc.

2011

2nd Semester Examination

PHYSICS

PAPER—PH-201

Full Marks : 40

Time : 2 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Group—A

(*Marks : 20*)

Answer *Q. No. 1 & 2* and any *one* from the rest.

1. Answer any *two* bits : 2×2

(a) Show that the fractional change in the wave vector $\left(|\vec{k}|\right)$ over the distance $\lambda/2\pi$ must be small compared to unity for the validity of the WKB method.

(b) If σ_x, σ_y and σ_z are the Pauli spin matrices, show that they satisfy anti-commutative relations.

(Turn Over)

(c) Write down the unitary operator corresponding to an infinitesimal rotation θ about an arbitrary axis with the unit vector $\hat{n}$. Show that the conservation of total angular momentum is a consequence of the rotational invariance of the system.

(d) For an 1-D simple harmonic oscillator of mass m and angular frequency ω, use the creation (a^{+}) and annihilation operator (a) to evaluate the expectation value $< n|x^2|n >$ in terms of the energy E_n, of the $|n\rangle$ state of the oscillator.

2. Answer any *two* bits : 3×2

(a) Write down the Ritz-variation functional. Hence show that the Schrodinger equation can be deduced from this functional.

(b) Prove the relation $[J_+, J_-] = 2\hbar J_z$

where $J_\pm = J_x \pm i J_y$

(c) Show that for a normalized state $|jm\rangle$ the raising (J_+) and lowering (J_-) operators satisfy the relation.

$$J_\pm|jm\rangle = \sqrt{(j \mp m)(j \pm m + 1)}\ |j\ m \pm 1\rangle$$

(d) The Hamiltonian H and wave function ψ at t = 0 for an 1-D simple harmonic oscillator of mass m and angular frequency ω are given by.

$$H = \hbar\omega\left(a^{+}a + \tfrac{1}{2}\right)$$

$$\psi(0) = \frac{1}{\sqrt{5}}|1\rangle + \frac{2}{\sqrt{5}}|2\rangle$$

a^+ and a are the creation and annihilation operators and $|n\rangle$ denotes the n-th eigen state. Show that the expectation of the time dependent position $\langle x(t)\rangle$ oscillates sinusoidally with time.

3. (a) What do you mean by the Clebsch-Gordan co-efficients? write down and prove the selection rules for the C.G. Co-efficients. 6

(b) Write down the Hamiltonian for a Hydrogen atom placed in a weak magnetic field. Apply perturbation theory of verify the normal Zeeman effect in H-atom. 4

4. (a) Use the WKB method to estimate the ground state energy of a particle of mass m moving under the potential V(x) given by

$V(x) = +\infty, \quad x \leq 0;$

$V(x) = \lambda x, \quad x > 0;$ λ being a positive constant.

(b) Apply the time-dependent perturbation theory to obtain Fermi golden rule.

4+6

Group—B

(*Marks : 20*)

Answer *Q. No. 1* and any *one* from the rest.

1. Answer any *five* from the following :

(a) $f(x) = x - 1$ for $1 < x < 2$

$= 1$ for $x > 2$

Express f(x) in terms of unit step function.

(b) Find the Laplace transform of

$$\frac{1}{x}\delta(x-a).$$

(c) Evaluate $\frac{d^2}{dx^2}|x|$ in terms of the Dirac δ-functi

(d) $x\frac{\partial\psi}{\partial x} + y\frac{\partial\psi}{\partial y} = 3\psi$ solve it by Lagrange's method.

(e) What is Dirichet and Neuman boundary problem ?

(f) If G = {1, i, − i, 1} be a group then find the cla: G.

(g) If $A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$ and

$B = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$ be two permutation group, Find

(h) If $\hat{L}F(t) = f(s)$ then prove

that $\hat{L}\left\{\frac{F(t)}{t}\right\} = \int_s^{\infty} f(x)\,dx$.

where $\hat{L}$ represents Laplace transform operator.

2. (a) Using Fourier transform, evaluate the following integrals :

(i) $\int_0^{\infty} \frac{\cos kx}{a^2 + k^2}\,dk$

(ii) $\int_0^{\infty} \frac{k \sin kx}{a^2 + k^2}\,dk$

(b) Find the inverse Laplace transform of

$$f(s) = \frac{1}{s(s^2 + 1)^2}$$

4+6

3. (a) Construct Green's function for the following boundary value problem.

$$\frac{d^2\psi}{dx^2} - \psi = f(x); \quad \psi(0) = 0; \quad \psi(2) = 0.$$

(b) Show that the Lorentz transformation in the x-direction

$$x' = \gamma(x - vt)$$

$$t' = \gamma(t - \frac{\beta x}{c})$$ form a Lie group,

where $\gamma = \frac{1}{\sqrt{1 - \beta^2}}$; $\beta = \frac{v}{c}$

Considering infinitesimal transformation find the generator.

5+3+2
