

**M.Sc. 1st Semester Examination, 2012****PHYSICS***( Quantum Mechanics )***PAPER—PHS-102 (A + B)****Full Marks : 40****Time : 2 hours**

*The figures in the right-hand margin indicate marks  
Candidates are required to give their answers in their  
own words as far as practicable*

*Illustrate the answers wherever necessary*

**Use separate scripts for Gr. -A & B****GROUP — A****[ Marks : 20 ]****Answer Q. No. 1 and any one from the rest****1. Answer any five questions : 2 × 5**

(a) A particle in the harmonic oscillator potential starts out in the state,

$$\psi(x) = A [3\psi_0(x) + 4\psi_1(x)]$$

**Find A.**

- (b) Find  $|\psi(x, t)|^2$  in prob. (a).
- (c) Find  $\langle x \rangle$  in prob. (a).
- (d) Find  $\langle p \rangle$  in prob. (a).
- (e) Check that Ehrenfest's theorem holds for this wave function as described in prob. (a).
- (f) Is the ground state of the infinite square well an eigen function of momentum? If so, what is its momentum? If not, why not?
- (g) The Hamiltonian for a certain two-level system is

$$\hat{H} = \epsilon \left[ |1\rangle\langle 1| - |2\rangle\langle 2| + |1\rangle\langle 2| + |2\rangle\langle 1| \right]$$

where  $|1\rangle, |2\rangle$  is an orthonormal basis and  $\epsilon$  is a number with the dimensions of energy. Find its eigen values.

(h) For hydrogen atom

$$\psi_{100} = \frac{1}{\sqrt{\pi a_B^3}} e^{-r/a}$$

Find  $\langle x^2 \rangle$ .

2. (a) A particle of mass  $m$  is in the potential

$$V(x) = \infty \text{ for } x < 0$$

$$= -32 \hbar^2 / ma^2 \text{ for } 0 \leq x \leq a$$

$$= 0 \text{ for } x > a.$$

How many bound states are there? Answer with necessary deduction.

5

(b) If

$$[\hat{A}, \hat{B}] = i\hat{C}$$

then prove the generalized uncertainty principle. i.e.

$$\sigma_A^2 \sigma_B^2 \geq \frac{1}{4} \langle \hat{C} \rangle^2.$$

5

3. (a) For harmonic oscillator, prove that

$$[\hat{a}, \hat{a}^+] = 1.$$

$$\text{Where } a^+ = \frac{1}{\sqrt{2\hbar m\omega}} (-ip + m\omega x)$$

and its hermitian conjugate is  $a$ .

- (b) If  $V(r) = 0$  for  $r < a$   
 $= \infty$  for  $r > a$

Find the wave functions and the allowed energies.

- (c) Show that the commutator of two Hermitian operators is anti-Hermitian.

GROUP – B

[ Marks : 20 ]

Answer Q. No. 1 & 2 and any **one** from the rest

1. Answer any *two* bits :

- (a) Find the packing fraction of Diamond structure

- (b) Show that face centered tetragonal lattice does not exist.
- (c) Draw the variation of  $B$  and  $M$  with  $H$  for a type-1 superconductor.

2. Answer any *two* bits : 3 x 2

- (a) Explain with neat diagram the symmetry elements screw and glide.
- (b) Find the structure factor in terms of fractional coordinate.
- (c) Show that the inclusion of anharmonic interaction is necessary to understand the phenomenon of thermal expansion.

3. Derive Laue equation assuming scattering of X-ray by a crystal. Prove the equivalence of Laue equation and Bragg Diffraction condition. 8 + 2

- 4. (a) Derive London's equation and explain how its solution explain Meissner's Effect.
- (b) Prove that the monoatomic chain acts as a low pass mechanical filter. (4 + 3)+ 3