

M.Sc. 1st Semester Examination, 2012

PHYSICS

(Methods of Mathematical Physics)

PAPER—PHS-101 (A + B)

Full Marks : 40

Time : 2 hours

Answer Q. No. 1 and any one from the rest

The figures in the right-hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

Use separate scripts for Gr.-A & B

GROUP – A

[Marks : 20]

1. Answer any five bits : 2 × 5

(a) Show that the two functions $\sin 2x, \cos 2x$ are independent solutions of $y'' + 4y = 0$.

(b) If $f(z) = u + iv$ show that

$$|f'(z)|^2 = \left| \begin{array}{cc} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{array} \right|^2$$

(c) Evaluate :

$$\int_C \tan z \, dz$$

where $C : |z| = 2$.

(d) Show that

$$H_{2n+1}(0) = 0.$$

(e) $P_5(x) = \lambda (63x^5 - 70x^3 + 15x)$ where λ is equal to ____ (Calculate it).

(f) Prove that

$$\frac{d}{dx} [\operatorname{erf}(ax)] = \frac{2a}{\sqrt{\pi}} e^{-a^2 x^2}$$

(g) If

$$A = \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$$

Find A^8 by Cayley-Hamilton's theorem.

(h) Prove that product of two unitary matrices is a unitary matrix.

2. (a) Using residue theorem prove that

$$\int_0^{\infty} \frac{dx}{(1+x^2)^2} = \frac{\pi}{4}$$

(b) Show that :

$$J_{-3/2}(x) = -\sqrt{\frac{2}{\pi x}} \left(\sin x + \frac{\cos x}{x} \right).$$

(c) Solve :

$$xy'' + y' + \frac{1}{4}y = 0.$$

4 + 3 + 3

3. (a) Find a matrix S which transforms the matrix

$$A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$$

to the diagonal form. Hence calculate A^4 .

(b) Prove that :

$$\Gamma(m) \Gamma\left(m + \frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2m-1}} \Gamma(2m).$$

GROUP – B

[Marks : 20]

1. Answer any *five* bits :

- (a) Establish with visual observation for 1 co-relations from the first state to Fourth state and back with the different states of matter heating and cooling.
- (b) Discuss the process on which occurrence plasma in nature.
- (c) Mathematically express the Paschen's law and discuss its components Graphically.
- (d) What do you mean by Debye length ?
- (e) Draw a schematic circuit diagram of an inductively coupled R. F. toroidal discharge system for the study of the breakdown process of air with 1 torr pressure.

- (f) What are the compositions found in the fourth states of matter ? Discuss.
- (g) Graphically present the phenomena of current voltage characteristics in D.C. (10 V to 800 V) glow discharge with 1 torr pressure.
- (h) Under the concepts of plasma kinetic theory, show graphically in classical view point, the phase space and volume element.
- (i) What will be the effect on mobility when a magnetic field is applied in a direction at right angles to the direction of flow of electrons ?
2. What is thermal ionization ? Establish Saha's single ionization formula and point out its applications. 2 + 6 + 2
3. Define the mobility and diffusion coefficients of an ionized gas. Deduce Einstein's relation. 2 + 2 + 6
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