2009

PHYSICS

PAPER—PH-1202

Full Marks : 40

Time : 2 hours

The figures in the right-hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

GROUP—A

[*Marks* : 20]

Answer *all* questions

1. Answer any *two* of the following : 2 x 2

(*a*) Prove that total energy E of a particle in the rotating frame is related to E_0 in the fixed frame by the relation $E = E_0 - \vec{\omega} \cdot \vec{L}$ where $\vec{\omega}$ is the angular velocity of the rotating frame and $\vec{L}$ is the angular momentum of the particle in the rotating frame.

(*b*) Find the Lagrange's equation of motion for an electrical circuit consisting an inductor L and a capacitor C. The capacitor is charged to q Coulomb and current flowing in the circuit is i amp.

(*c*) Derive Lagrange's equation of motion for a velocity dependent potential

$$U = U(q_1, q_2 \ldots q_k, \dot{q}_1, \dot{q}_2 \ldots \dot{q}_k).$$

2. Answer any *two* of the following : 3×2

(*a*) For the Hamiltonian

$$H = q_1 p_1 - q_2 p_2 - aq_1^2 + bq_2^2$$

Show that, $q_1 q_2 =$ constant,

$$\log q_1 = t + \text{constant and}$$

$$(p_2 - bq_2)/q_1 = \text{constant.}$$

(*b*) Consider a particle of mass m moving in a plane under central force

$$\vec{F}(r) = -\frac{k}{r^2} + \frac{k'}{r^3},\ k > 0$$

Write down the Lagrangian of the system in plane polar co-ordinate and hence obtain the equation of motion.

(*c*) Prove that

$$\frac{du}{dt}(q_k, p_k, t) = [u, H] + \frac{\partial u}{\partial t}.$$

3. Answer any *one* of the following : 10 x 1

(*a*) Find out the frequency of a linear harmonic oscillator using action-angle variable method. A particle of mass m is free to move along a line and is attached to a spring whose other end is fixed at a point. This point is at a distance l from the line. A force F is required to extend the spring to length l, find the frequency of small oscillations of the mass. 6 + 4

(*b*) (*i*) Write down Hamilton's canonical equation in terms of Poission's Bracket.

(*ii*) Show that the transformation

$Q = p^{-1}$ and $P = qp^2$ is a canonical transformation.

(*iii*) Using Hamilton-Jacobi theory, derive the equation of motion of a particle, which is falling under gravity. 2 + 2 + 6

GROUP—B

[*Marks* : 20]

Answer **Q.No.1** and any *one* from the rest

1. Answer any *four* of the following : $2\frac{1}{2} \times 4$

(*i*) Draw the graph of total polarizability (α) vs. frequency (ω) for a dipolar substance and explain.

(*ii*) Show the probability of occupancy of the states of energy E above E_F by an electron in a semiconductor is the same as the probability of occupancy of a state of energy E below E_F by a hole.

(*iii*) Define Lorentz number and give its numerical value. Is it a universal constant ? — Justify your answer.

(*iv*) An electric field of 1 kV/m is applied across a Ge sample at 300 K. Show that the carrier drift velocity is very small compared to its thermal velocity

($m_h^* = 0{\cdot}037\, m_e$; $\mu_n = 0{\cdot}39\, \text{m}^2/\text{V.s}$).

(*v*) Show the variation of effective mass with respect to wavevector in a solid. What is the physical significance of effective mass of electron near the top of the band?

(*vi*) A paramagnetic salt contains 10^{28} ions/m^3 with magnetic moment of one Bhor magneton. Find the magnetization produced in a uniform magnetic field of 10^6 A/m at room temperature.

(*vii*) Show that number of wavefunctions in an energy band is equal to number of unit cells in direct lattic.

2. (*a*) Prove that Fermi Energy decreases with increases of temperature in a metal.

(*b*) Show how the resistivity varies with temperature in case of a intrinsic semiconductor. Can you measure the band gap energy of the semiconductor from this variation? If so, how? 7 + (2 + 1)

3. (*a*) What is the physical origin of energy gap in a solid?

(*b*) Draw the first three energy bands in [100] direction under the reduced zone scheme for an f.c.c lattice using empty lattice approximation.

(*c*) The band structure of an electron in a 1D periodic potential is given by

$$E_1(k) = A(1 - \cos k) \text{ and } E_2(k) = B.$$

Find the effective mass of the electron for each band (for $k \approx 0$). Mention which branch could contribute to electrical conduction. $4 + 3 + (2 + 1)$