PG/IIS/PH-1201/09

2009

PHYSICS

PAPER—PH-1201

Full Marks : 40

Time : 2 hours

The figures in the right-hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

GROUP—A

[*Marks* : 20]

(*Quantum Mechanics* -II)

Answer **Q.No.1, 2** and any *one* from the rest.

1. Answer any *two* bits : 2x2

(*a*) Find the commutation relation $[J_+, J_z]$. Using this relation show that J_+ acts as a raising operator on the eigenvalues of J_z.

(*Turn Over*)

(*b*) A particle is in an eigenstate of J_z and J^2 and satisfy the eigenvalue equations

$$J_z | j, m \rangle = m\hbar | j, m \rangle$$

$$J^2 | j, m \rangle = j(j+1)\hbar^2 | j, m \rangle$$

Find the value of $\langle j, m | J_x^2 | j, m \rangle$.

(*c*) Obtain the matrix of Clebesch–Gordan coefficient for addition of angular momenta with $j_1 = 1/2$ and $j_2 = 1/2$. Write the resultant wavefunction as the linear combination of component wavefunctions.

(*d*) If a system with Hamiltonian H has energy eigenvalues, $E_1 < E_2 < E_3$, then prove that $< \psi | H | \psi > \geq E_1$ for any normalized vector $| \psi >$.

2. Answer any *two* bits : 3x2

(*a*) Optimize the trial wavefunction $e^{-\alpha r}$ and evaluate the ground state enregy for the Hamiltonian

$$H = \frac{-\hbar^2}{2m} \nabla^2 - \frac{ke^2}{r}$$

Given : In the ground state the angular momentum quantum number $l = 0$.

(*b*) Show that the three operator L_x, L_y and L_z are the generators of infinitesimal rotations about the three co-ordinate axes respectively. Determine the condition under which the angular momentum is a constant of the motion.

(*c*) Write the energy eigenvalues, degree of degeneracy and the corresponding wavefunctions for a two-dimensional harmonic oscillator for the three lowest states. Write the wavefunctions interms of the wavefunctions of the one dimensional harmonic oscillators $u_n(x)$ and $v_m(y)$. Find the first order energy shift for the second excited state of the two dimensional harmonic oscillator under the perturbing potential $c(x + y)$.

(*d*) A spin $\frac{1}{2}$ system has two spin states $|+>$ and $|->$ for $\hat{S}_z$ having eigenvalues $\hbar/2$ and $-\hbar/2$ respectively. Write the completeness condition in terms of these states and hence construct the operator $\hat{S}_z$ in terms of them.

3. (*a*) Derive expressions, for first and second order corrections to a nondegenerate energy level and first order correction to the corresponding wavefunction due to application of a stationary (time independent) perturbing potential.

(*b*) Calculate the first order correction to the ground state energy of an anharmonic oscillator of mass m and angular frequency ω subjected to a potential $V(x) = \frac{1}{2} m\omega^2 x^2 + bx^4$

Given :

$$<m|x^2|n> = \begin{cases} \frac{\hbar}{2m\omega}\sqrt{(n+2)(n+1)} & \text{for } m = n+2 \\ \frac{\hbar}{2m\omega}(2n+1) & \text{for } m = n \\ \frac{\hbar}{2m\omega}\sqrt{(n-1)n} & \text{for } m = n-2 \end{cases}$$

where $|m>$ represent the eigenfunction for m th state of the harmonic oscillator. 6 + 4

4. (*a*) Write the Hamiltonian for a charged particle in an electromagnetic field by using vector potential.

(*b*) Write (deduction not necessary) the form of the Hamiltonian keeping terms only up to first order in vector potential.

(*c*) Using time dependent perturbation theory find the transition probability per unit time for an upward transition (absorption).

(*d*) Mention the selection rules for electrical dipole transition and mention the approximations involved in it. 1 + 1 + 6 + 2

GROUP—B

[*Marks* : 20]

(*Mathematical Methods in Physics*)

Answer *all* questions.

1. Answer any *five* bits : 2 x 5

(*a*) Define $SU(2)$ group with examples.

(*b*) If

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix},\ B = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$$

Find AB.

(*c*) Define conjugate elements in a group and class with examples.

(*d*) Find Fourier transform of the function

$$f(x) = 1 \quad \text{for } |x| < a$$
$$= 0 \quad \text{for } |x| > a.$$

(*e*) Define convolution of two functions $f(x)$ and $g(x)$.

(*f*) Find the Laplace transform of the functions :

$$f(t) = \sin t \quad \text{if} \quad 0 < t < \pi$$
$$= 0 \quad \text{if} \quad t > \pi.$$

(*g*) Define Green's function.

(*h*) Prove that

$$\delta\left(\vec{r} - \vec{r}\,'\right) = \frac{\delta(r - r')}{4\pi r^2}$$

in spherical co-ordinates.

2. Answer any *two* bits : 5×2

(*a*) Use convolution theorem to find out

$$L^{-1}\left\{\frac{1}{s(s^2+1)^2}\right\}.$$ 5

(*b*) Solve by Laplace transform method

$$\frac{m\, d^2 z}{dt^2} = mg - k\frac{dz}{dt}$$

with $z(0) = 0$ and $\left.\frac{dz}{dt}\right|_{t=0} = V_0$. 5

(*c*) Consider the symmetry group of transformations of an equilateral triangle (D_3).

(*i*) Write down the symmetry operations and work out the group multiplication table.

(*ii*) Identify the classes. (Use permutation group representation). 3 + 2

(*d*) An infinite string is initially at rest and that the initial displacement is $f(x)$, $(-\infty < x < \infty)$. Determine the displacement $y(x, t)$ of the string by F.T. method of one-dimensional wave equation. 5

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