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2019

B.Sc. (General)

2nd Semester Examination

MATHEMATICS

Paper - DSC 1BT

Full Marks : 60 Time : 3 Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

1. Answer any *ten* questions : 10×2=20

(a) Determine the degree and order of the differential equation $r\,\frac{d^2y}{dx^2}=\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}$

(b) Find h so that $(ax+hy+g)\,dx+(2x+by+f)\,dy=0$ becomes an exact differential equation.

(c) If $f'(x)+f(x)=0$ and $f(0)=2$ then find $f(x)$.

(d) Find the integrating factor of :

$$sec^2 y\frac{dy}{dx}+2x\tan y=x^3$$

[*Turn Over*]

(e) If $y_1(x)=sin3x$, $y_2(x)=cos3x$ are two solutions of the differential equation $\frac{d^2y}{dx^2}+9y=0$ show that $y_1(x)$ and $y_2(x)$ are linearly independent solutions.

(f) Classify the partial differential equation :

$$\frac{\partial^2 z}{\partial x^2}+2\frac{\partial^2 z}{\partial x\partial y}+5\frac{\partial^2 z}{\partial y^2}+\frac{\partial z}{\partial x}-2\frac{\partial z}{\partial y}-3z=0$$

(g) Verify that $\frac{1}{y^2}$ is an integrating factor of $y(1+xy)dx-xdy=0$.

(h) Reduce the differential equation $(px-y)\ (x-py)=2p$ to clairaut's form by substitution $x^2=u$, $y^2=v$.

(i) Find the particular integral of the differential equation $(D+2)y=x^3e^{-2x}$.

(j) If $\frac{d}{dx}u(x)=v(x), \frac{d}{dx}v(x)=-u(x)$ and $u(0)=1$ and $v(0)=1$ then find $u(x)$.

(k) What curve through (1,1) has at every point $\frac{dy}{dx}=\frac{x-y}{x+y}$?

(l) State the condition under which a 1st order differential equation of the form $M(x,y)\,dx + N(x,y)\,dy = 0$ is said to be exact.

(m) Solve : $xp + yq = z$ where $p = \dfrac{\partial z}{\partial x}$, $q = \dfrac{\partial z}{\partial y}$

(n) Find the complete primitive of the differential equation : $y = px + \sqrt{1 + p^2}$

(o) Find the value of : $\dfrac{1}{D+1}(1 + x^2)$

2. Answer any *four* questions : 4×5=20

(a) Find the complete integral $px + qy = pq$.

(b) Solve the simultaneous equations :

$(D+4)x + 3y = t, \quad (D+5)y + 2x = e^t.$

(c) Solve : $x^2 \dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} + y = \log x, x > 0$

(d) Solve by the method of variation of parameters

$$\frac{d^2y}{dx^2} + a^2 y = \tan ax.$$

(e) Solve the differential equation

$$\sin\left(x\frac{dy}{dx}\right)\cos = \cos\left(x\frac{dy}{dx}\right)\sin y + \frac{dy}{dx}.$$

Also find its singular solution.

(f) Show that the equation

$(x^3-3x^2y+2xy^2)\,dx-(x^3-2x^2y+y^3)dy=0$ is exact and find the solution if $y=1$ when $x=1$.

3. Answer any *two* questions : 2×10=20

(a) (i) Solve : $x(y^2-z^2)p+y(z^2-x^2)q=z(x^2-y^2)$

(ii) Find partial differential equation by eliminating arbitrary function f from the relation :

$f(x^2+y^2+z^2,\ z^2-2xy)=0$ 5+5

(b) (i) Solve : $y(2xy+e^x)\,dx-e^x dy=0$

(ii) Obtain the complete primitive of $(px-y)\,(px+x)=h^2p$. Also find singular solution of the differential. equation if any.

5+5

(c) (i) Solve : $(D^2+5D+6)y=e^{-2x}\sin 2x$

(ii) Verify that $y=x$ is a solution of the reduced equation of $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y=x^2$. Solve the equation after reducing it to a first order linear equation. 5+5