2019

B.Sc.

1st Semester Examination

# MATHEMATICS (General)

## Paper - DSC 1A-T

## [Differential Calculus]

Full Marks : 60 Time : 3 Hours

*The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable. Illustrate the answers wherever necessary.*

1. Answer any *ten* questions. 2×10

(a) Examine the continuity of the function $f(x)$ at $x=0$ where

$$f(x)=\begin{cases} 2x+\sin\left(\frac{1}{x}\right), & x\neq 0 \\ 0, & x=0 \end{cases}$$

[ Turn Over ]

(b) If $u = f\left(\frac{y}{x}\right)$ then find the value of

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}.$$

(c) State the Taylor's theorem with Cauchy's form of remainder.

(d) Define the jump discontinuity of a function at a point with an example.

(e) If $f(x) = |x|$, show that $f(0)$ is a minimum although $f'(0)$ does not exist.

(f) At what point is the tangent to the parabola $y = x^2$ parallel to the straight line $y = 4x - 5$ ?

(g) State the geometrical interpretation of Rolle's theorem.

(h) If $V = \sqrt{x^2 + y^2 + z^2}$, show that

$$V_{xx} + V_{yy} + V_{zz} = \frac{2}{V}.$$

(i) State the Lagrange mean value theorem.

(j) Prove that the radius of curvature at any point of the catenary $y = c\cosh\left(\frac{x}{a}\right)$ varies as the square of the ordinate.

(k) Evaluate $\lim_{x \to 0} (\cos mx)^{\frac{n}{x^2}}$

(l) A function $f : [0,1] \to [0,1]$ is continuous on $[0,1]$. Prove that there exists a point $c$ in $[0,1]$ such that $f(c) = c$.

(m) If $f(a) = 2$, $f'(a) = 1$, $g(a) = -1$, $g'(a) = 2$ then find the value of

$$\lim_{x \to a} \frac{g(x) \times f(a) - g(a) \times f(x)}{x - a}.$$

(n) Verify the Rolle's theorem for the function $f(x) = x^2 - 5x + 6$ in $[1, 4]$.

(o) Sketch the curve $(x+3)(x^2 + y^2) = 4$.

*[ Turn Over ]*

2. Answer any *four* questions. 4×5=20

(a) State and prove Cauchy's mean value theorem and deduce Lagrange's mean value theorem from it. 1+3+1

(b) If $f(x,y)=\frac{x^2y^2}{x^2+y^2}$, $(x,y)\neq(0,0)$

$= 0,\qquad (x,y)=(0,0).$ 5

(c) Find the asymptotes of the cubic

$x^3-2y^3+xy(2x-y)+y(x-y)+1=0.$ 5

(d) Let. $f:\mathbb{R}\to\mathbb{R}$ be a differentiable function on $\mathbb{R}$ and $f'(x)>f(x), \forall x\in\mathbb{R}$.

If $f(0)=0$, prove that $f(x)>0,\ \forall x>0$. 5

(e) If $x\cos\alpha+y\sin\alpha=p$ touches the curve

$\frac{x^m}{a^m}+\frac{y^m}{b^m}=1$, show that

$$(a\cos\alpha)^{\frac{m}{m-1}}+(b\sin\alpha)^{\frac{m}{m-1}}=p^{\frac{m}{m-1}}$$

(f) If $u = \tan^{-1} \dfrac{x^3 + y^3}{x - y}$, prove that

$$x^2 u_{xx} + 2xy\, u_{xy} + y^2 u_{yy} = \left(1 - 4\sin^2 u\right)\sin 2u .$$

5

3. Answer any *two* questions. 2×10=20

(a) (i) If $y = a\cos(\log x) + b\sin(\log x)$, prove that

$$x^2 y_{n+2} + (2n+1)xy_{n+1} + \left(n^2 + 1\right)y_n = 0$$

6

(ii) Show that

$$1 + \frac{x}{2} - \frac{x^3}{8} < \sqrt{1+x} < 1 + \frac{x}{2},\ x > 0 .$$ 4

(b) (i) Find the condition that the curves $ax^3 + by^3 = 1$ and $a'x^3 + b'y^3 = 1$ will cut orthogonally. 5

(ii) Find the nature and position of singular points (if any) of the curve

$$x^3 - x^2 y + y^2 = 0 .$$ 5

*[ Turn Over ]*

(c) (i) Let $f(x)=\begin{cases} x^2, & x \text{ is rational} \\ 0, & x \text{ is irrational} \end{cases}$

Show that $f'(0)=0$. 4

(ii) Show that the pedal equation of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with regard to the centre is

$$\frac{a^2b^2}{p^2}=a^2+b^2-r^2.$$ 6

(d) (i) Using Maclaurin's theorem, prove that

$$\log(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\ldots,\ -1<x\leq 1.$$

5

(ii) Show that the maximum value of

$x^2\log\left(\frac{1}{x}\right)$ is $\frac{1}{2e}$. 5

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