## Chapter 4

## Tracking Radar System

# 4.1 Analysis of Factors and their Impacts in Measurement Accuracy and Prioritisation of Radars 

## Overview

Tracking of a moving object and predicting the impact point in real time is a challenging task. This can be easily determined with the help of Radar. Radar can identify the object by using radio wave and determine the range, azimuth and elevation of that object. Many experiments have done to maximize the radar accuracy. Some factors which can affect the radar performances have been pointed out and some theories have developed to restrict those affecting factors. Here we have summarized the problem in radar measurement accuracy. We have discussed the various factors in object position measurement accuracy in radar measurement and tried to priorities on the basis of measurement accuracy.

### 4.1.1 Introduction

One of the major goals of a developing country like India is the strengthening of defence and the main focus is to place it in the rank of major superpowers. The technology is developing day by day. Various new kinds of air attack technology is also in place. So the major goal is to detect and track an incoming object while it is moving in a 3-D space and to predict its trajectory and impact point. We have to take care of the target in air, otherwise it may destroy many lives and resources.

Accurate prediction of the moving object trajectory is required to calculate the hitting point at ground as well as the origin of the moving object precisely. It is solely depends upon the Radar measurement accuracy. If we can detect the moving object more accurately in re-entry phase then the damage could be minimized.

The accuracy means the difference between measured value and the actual value. In other words radar measurement accuracy is defined as the degree of observation between the measured position and velocity at a given time with respect to the actual position and velocity. From the resolution of a radar system we can easily determine the degree of accuracy. The resolution of a radar also expressed the tracking performances of a radar. Tracking performance of a radar is defined as the ability to track two different objects in near proximity, either in angle or range.

### 4.1.2 Factors of Measurement Accuracy

Many factors combined together results into radar measurement accuracy. This factors may be noise, clutter, glint or scintillation. Radar accuracy limits due to the signal propagation, such as multi path and turbulence.

There are many other factors which also affect the accuracy of radar, these factors are signal quantization and the sampling rate. There are some factors like antenna pointing and channel to channel phase calibration, these factors also can limit the radar precision and accuracy. Any unpredictability in antenna bore sight angle can inhibit the accuracy estimation. The accuracy Estimation is shown in Fig. 4.1.


Figure 4.1: Accuracy Estimation

We can generalize errors in two types viz. Radar dependent errors and Radar independent errors.

Table 4.2 describes the system dependent errors. This also shows the affecting tracking coordinate.

## Range

The range is function of the time between transmit and receive of wave and velocity of wave in present medium. The range measurement accuracy is characterized by rms measurement error $\sigma_{R}$, which is defined as root-mean-square of three error components in equation (4.1).

$$
\begin{equation*}
\sigma_{R}=\sqrt{\left(\sigma_{R}^{2}+\sigma_{R}^{2}+\sigma_{R_{B}^{2}}^{2}\right)} \tag{4.1}
\end{equation*}
$$

Where, $\sigma_{R N}$ is signal to noise ratio dependent random range measurement error. $\sigma_{R F}$ is fixed random error including the range error from propagation due to multi path conditions. $\sigma_{R B}$ is the bias error component. Those thermal noise dependent

Table 4.1: System Dependent Errors

| ERROR | RESULTING | AFFECTED |
| :--- | :---: | :---: |
| SOURCE | TRACKING ERROR | TRACKING COORDINATE |
| Zero Range Setting | Bias Error | Range |
| Antenna Unbalance | Bias Error | Angle |
| North Alignment | Bias Error | Angle |
| Antenna Servosystem <br> Unbalance | Bias Error | Rangeand Angle |
| Receiver Thermal Noise | Noise Error | Range and Angle |
| Antenna Servosystem <br> Noise | Noise Error | Range and Angle |
| Antenna Servosystem <br> Limitations | Bias Error | Angle |

radar range error dominate the range error. The standard deviation can be defined as,

$$
\begin{equation*}
\sigma_{R N}=\frac{\Delta R}{\sqrt{S N R_{d B}}} \tag{4.2}
\end{equation*}
$$

From the equation (4.2), for a radar system with a 50 m range gate $(\Delta \mathrm{R})$, the thermal noise dependent error is 11 m at 10 dB and 5 m at 55 dB . If the SNR value is high, the fixed random error limits the Radar performances as the Radar internal noise is about 25 to 30 decibel. The bias term is highly system dependent. Range bias can be from few meters to several hundred meters as per system objective . For an air target tracking system where the absolute position of radar is important, a random error calibration is used to limit the bias errors.

## Velocity

Velocity is measured from the Doppler frequency shift of a received signal. The Doppler Effect equation represented as,

$$
\begin{equation*}
f_{d}=\frac{-2 v}{\lambda} \tag{4.3}
\end{equation*}
$$

Velocity can be expressed as, $\sigma_{V}$. which is defined as the root-mean-square of three error components are $\sigma_{V N}, \sigma_{V F}, \sigma_{V B}$.

$$
\begin{equation*}
\sigma_{V}=\sqrt{\left(\sigma_{V}^{2}+\sigma_{V}^{2}+\sigma_{V}^{2}\right)} \tag{4.4}
\end{equation*}
$$

Where, $\sigma_{V N}$ is radial velocity measurement error depends on signal to noise ratio. $\sigma_{V F}$ is fixed random error including the range error from the propagation due to multipath condition. $\sigma_{V B}$ is the radial-velocity bias error component.

Usually the thermal noise dependent error dominates the error component. Then the standard deviation can be defined as,

$$
\begin{equation*}
\sigma_{V N}=\frac{\lambda}{2 \tau \sqrt{2 S N R_{d B}}} \tag{4.5}
\end{equation*}
$$

Where $\lambda$ is pulse length.
The fixed random error limits the performance in high amplitudes and the bias terms are usually very small in absence of multi-path conditions.

## Angular

The radar angular angular measurements uses mono pulse receive antennas. Those have simultaneous receive beams offset in angle to both side of the transmit beam. Angular position can be measured from the difference pattern formed by those received beams. Measurement accuracy in each angular co-ordinate $\sigma_{A}$, is defined as the root-mean-square of three error components and it is represented as,

$$
\begin{equation*}
\sigma_{A}=\sqrt{\left(\sigma_{A N}^{2}+\sigma_{A}^{2}+\sigma_{A B}^{2}\right)} \tag{4.6}
\end{equation*}
$$

Where, $\sigma_{A N}$ is signal to noise ratio dependent random angular measurement error which is a major source of error. $\sigma_{A F}$ is fixed random error from the propagation and radar random error. It will limit the radar performance in higher signal to noise ratio values. $\sigma_{A B}$ is the angle bias error,which is small in various cases.

The thermal noise dependent angular measurement error and represented as,

$$
\begin{equation*}
\sigma_{A N}=\frac{\theta}{k_{m} \sqrt{2 S N R_{d B}}} \tag{4.7}
\end{equation*}
$$

where, $\theta$ is 3 dB beam width and $k_{m}$ is the mono pulse pattern difference slope.

## Atmospheric effect

The echo signal can be affected by the rain and other form of precipitation that mask the desired target echoes. There are various atmospheric sensation which can limit the radar performance. Due to decrease in the density of Earth's atmosphere with altitude hike, radar waves can bend when they travel through the atmosphere.

Temperature inversion can cause changes in refraction index resulting greater bending in radar waves passing through the abnormal condition. Ducting can happen which trap and guide radar energy around the curvature of Earth anf allow detection range beyond normal horizon. It happens more over water in tropical climates than in colder region. Radar energy is lost due to that atmospheric absorption.

## Radar Clutter

Radar Clutter are unwanted echoes. It can be formed by the ground, sea, rain etc. For a tracking radar system, low speed objects are seen as clutter, e.g. land, buildings, rain and birds. Two simple classifications of clutter can be made. One
is volume such as rain and chaff and another one is surface such as ground and sea.

Volume clutters are homogeneous in spatial distribution and more nearly noise like than other clutters. Surface clutter is extremely difficult to model because of in homogeneous from a radar prospective.

## Peak Power

Range capabilities of radar increases with increasing of the peak power. By doubling the peak power the range capabilities increase to 25 percent more.

## Frequency

The frequency range of radar is from about 5 MHZ to 130 GHZ . The frequencies which have higher bandwidth have more power loss as compared to lower frequency regardless of weather. The radar which has lower frequency have longer wave length for that reason it is more preferred in case of long range detection.

## Interference

Interference is a phenomenon in which two or more waves are superimpose with each other and produces an unwanted signal. Signals coming from other radars and transmitters also sometimes enter a radar receiver and produce noise. So some well mechanism is needed to recognize those and remove it enters the receiver antenna.

## Target Size

The size of a target as seen by radar is not always related to the physical size of the object. The size of the target observed by radar is known as radar cross section. For two targets with same physical cross section area to differ in radar size or may be in cross section. It varies with the target position.

## Pulse Repetition Rate

The pulse repetition rate of a regular train of pulses is defined as the number of emitted pulses per second. This explain the maximum detectable range of radar. A proper amount of time should be given between pulses for an echo to return from a target. Otherwise, echoes returning from the far target target will be blocked by succeeding transmitted pulses.

## Antenna Rotation Rate

Antenna should rotate slowly for long range detection of radar. When the antenna rotation is high, particularly for small targets the probability of detection of a target reduced.

### 4.1.3 Findings and Discussion

The various problem we are facing to track an incoming target has been studied here. The factors that affect the radar parameters are listed out and steps were taken to minimize the effect of that. Parameter sensitivity as well as the signal to
noise ratio were listed out which reduces the accuracy of the measurement. The accuracy is also degraded by the multi-path effects is a great deal, as it depends on the illumination angle and the back-scattering. Further work should be done to find out the type of the target in more accurate way, so that the trajectory of the target as well as impact point can be calculated properly. The degree of accuracy in measurement helps to prioritise radar. One radar cannot provide accurate position measurement hence, we have to use more radar for calculating accurate and reliable position measurement. If we increase the number of radar then position measurement improves. Accuracy further improves by considering priority of the radars.

# 4.2 Model-4: Multiple Radar Data Fusion to Improve the Accuracy in Position Measurement Based on Clustering Algorithm 

## Overview

The position of any moving object can be easily determined with the help of radar. It can identify the object by using radio waves and determine the range, azimuth, and elevation. To achieve reliable and accurate position measurement instead of one more number of radar should be considered. At the same time if any of the radar has some wrong measurement then combined position measurement becomes erroneous. Data fusion techniques can be applied to integrate multiple radar measurements. Data fusion is a process to solve a problem based on the idea of integrating several pieces of information to obtain more consistent, accurate and useful information. If the erroneous measurement is identified and eliminated from data fusion then final data fusion result become more accurate. Here we summarize how the k-means algorithm can be used to identify the position of any object by the process of combining data from various radars. Our main aim is to identify the erroneous radar measurements if any and establish a technique of combining the information from different radars to reach the best accurate solution.

### 4.2.1 Introduction

Multiple radar data fusion is the process to combine all the data which are given from several numbers of radar to produce the most specific information. Data fusion is an emerging technology applied to the Defence areas such as battlefield surveillance and guidance, automated target recognition, and control of autonomous vehicles. It is also used in non-Defence applications such as monitoring of medical diagnosis, complex machinery, and smart building. We are using the data fusion technique to improve the accuracy in position measurement. Radar is a detection system that can easily track a moving object and determine the range, azimuth, and elevation. Now we have to change the radar data to the Cartesian coordinate $(X, Y, Z)$ and translate to common reference point. All the radar measurement data are not equally accurate. Successful application of data fusion technique makes the result acceptable. We apply a K-means clustering algorithm to get final measurement over all the measurements. The clustering method identifies similar groups of measurement from all of the measurements. Comparatively more nearer measurements come under a cluster and other measurement belongs to the other cluster. There are several types of clustering algorithm available but here we have applied K-means algorithm because it is very time efficient. It works in an iterative manner and leads to the final local centroid in each iteration. In this way, we fuse all the data and get final data from several numbers of data and we get the more accurate and acceptable result.

### 4.2.2 Proposed Scheme

For this model, we are using radar as a sensor. Radar can track an object and determine the range, azimuth, and elevation of the object. So we have to convert the range, azimuth and elevation into the Cartesian Coordinate $(X, Y, Z)$. So first we have to convert the value of azimuth and elevation from degree to radian.

$$
\begin{equation*}
\text { Range }=r, \quad \text { Elevation }=\alpha, \quad \text { Azimuth }=\beta \tag{4.8}
\end{equation*}
$$

In Cartesian coordinate,

$$
\begin{equation*}
X=r * \cos (\alpha) * \sin (\beta), \quad Y=r * \cos (\alpha) * \cos (\beta), \quad Z=r * \sin (\alpha) \tag{4.9}
\end{equation*}
$$

So, from these formulae, we can easily convert the given Radar's data to the Cartesian Coordinate $(X, Y, Z)$.

Step-1: In this paper, We are using 15 radar measurements for experimentation. Firstly we have taken the radar measurement (Range,Azimuth,Elevation), actual location of the object, co-ordinate of radar and reference point as the inputs. Now we have to convert the radar measurement range, azimuth, and elevation to the Cartesian coordinate $(X, Y, Z)$. To convert all the data we are using the equation 4.9. Now all the data are in Cartesian form.

Step-2: In this step, we are measuring the position of the object with respect to common reference point for all radars. We have fifteen different measurements from fifteen radars for one real object due to inaccurate radar measurement. Then
we find the difference between the exact object location and the radar measured object location for all radars in $\mathrm{x}, \mathrm{y}$ and z components. And also find the range difference for all radars. So, we are finding the error of the radar measurement. Let, Actual position of the object is $(x, y, z)$. And after measuring the object's position with respect to the radar1 we get the measured position of the object $(x 1, y 1, z 1)$.
error with respect to the X coordinate $=x-x 1$
error with respect to the Y coordinate $=y-y 1$
error with respect to the Z coordinate $=z-z 1$

$$
\begin{equation*}
\text { rangeerror }=\sqrt{\left(x^{2}+y^{2}+z^{2}\right)}-\sqrt{\left(x 1^{2}+y 1^{2}+z 1^{2}\right)} \tag{4.10}
\end{equation*}
$$

Step-3: We have fifteen locations (points) of the objects as per radar measurements. Some measurements are very near to the exact location of the object and some locations are far away from the exact location of the object. So, some position measurements are within acceptable limit and some are not. Now if we integrate all the nearest points ignoring all the points beyond acceptable limit then the combined position measurement becomes acceptable. To combine all the points we are using a K-means clustering algorithm. We are considering two initial clusters. Then we calculate the initial centroid. The first centroid is chosen randomly and then calculate the Euclidean distance between the initial centroid and every point. Based on that distance we are choosing the second centroid, the furthest point from the first centroid is considered as the second centroid. Now we have to calculate the Euclidean distance between two centroids to all remaining points. Let,

Position of a point $=(x, y, z)$
The position of the centroid $=(c x, c y, c z)$
Then the Euclidean distance,

$$
\begin{equation*}
D=\sqrt{(x-c x)^{2}+(y-c y)^{2}+(z-c z)^{2}} \tag{4.11}
\end{equation*}
$$

Now we have to compare Euclidean distance between centroid two and every point and the Euclidean distance between centroid one and every point. And the centroid is updating base on the minimum distance. And the position of the updated centroid is the mean of the position of the centroid and the nearest point. Let,
$D 1=$ distance from 1st centroid to 1 st point.
$D 2=$ distance from 2 nd centroid to 1 st point.
position of the 1 st point $=(x 1, y 1, z 1)$.
position of the 1st centroid $=(c x 1, c y 1, c z 1)$.
position of the 2 nd centroid $=(c x 2, c y 2, c z 2)$.
Now, if $D 1$ is less than the $D 2$ then the position of the updated 1st centroid is updated else position of the 2 nd centroid is updated.

In this way, the centroid is updating in case of 15 points and finally, we get two centroids. Then we have to count the number of surrounding points of that centroids. Largest cluster centroid is considered as the final centroid. To specify the centroid more accurately we are using a boundary condition. If the location of any point with respect to the centroid of a cluster is within this boundary then this
point is entered into that cluster if not then we ignore that point.
Now, if D 1 is less than the D 2 then the position of the updated centroid is,

$$
\begin{equation*}
c x 1=\frac{c x 1+x 1}{2}, \quad c y 1=\frac{c y 1+y 1}{2}, \quad c z 1=\frac{c z 1+z 1}{2} \tag{4.12}
\end{equation*}
$$

And if D1 is greater than D2 then the position of the updated centroid is,

$$
\begin{equation*}
c x 2=\frac{c x 2+x 1}{2}, \quad c y 2=\frac{c y 2+y 1}{2}, \quad c z 2=\frac{c z 2+z 1}{2} \tag{4.13}
\end{equation*}
$$

In this way, the centroid is updated for all points and finally we get two centroids. Then the centroid of the largest cluster is considered as the final result. To achieve more accuracy we are using a boundary condition. If any measurement is within the boundary limit then only that is considered otherwise ignored for clustering process. We calculate the difference between the exact position of the object and the final centroid.

Let,
Object's exact position $(x, y, z)$.
Position of the final centroid ( $c x, c y, c z$ )
So,
error with respect to the X coordinate $=x-c x$
error with respect to the Y coordinate $=y-c y$
error with respect to the Z coordinate $=z-c z$


Figure 4.2: Height vs range graph

### 4.2.3 Result and Analysis

For our experimentation we have taken trajectory data of a projectile upto 80km of range and 35 km of altitude. After completing all the steps successfully we are given some plots by which we can verify and analyze our experiments and got some results. We considered many cases to analyze our results.

In ideal case when all the radars are free from noises and bias errors, in this time the total error in radar1 is varying in the maximum limit of +0.06 m . This is a very small. After applying the K-means clustering algorithm we are getting final total error is varying in the maximum limit slightly less than +0.06 m . so in ideal case no effect of clustering.

All subsequent analysis is carried out with consideration of 10 min Gaussian random noise in azimuth elevation and 25 m Gaussian random noise in range.

## Case-1: No bias, having noise in Azimuth, Elevation and Range

In this case, we are adding some noises on every radar. In the range we


Figure 4.3: Error vs range graph for radar1


Figure 4.4: Range error vs range graph for radar1
are adding 25 m noise, In azimuth and elevation, we are adding 1 -minute noise. Now for the effect of noise, the range error in radar 1 is rising up to the maximum range of +30 m approximately. After applying the K-means clustering algorithm, two clusters are produced, we are getting the final centroid, which is the final position of the object. From figures we can see that the final range error is varying and maximum limit slightly less than +30 m . So there is not much improvement occurred. In this way, we analyzed the results for every radar and we are getting


Figure 4.5: Position of all points, clusters and final centroids


Figure 4.6: Final error vs range graph
the same results as radar1.

## Case-2: Bias in Azimuth but no bias in Elevation and Range

In this case, we are adding some noises same as case-1. And we are adding some azimuth bias error in radar1, radar2, and radar3. The value of the applied azimuth bias error is 0.5 degrees. Now after adding the azimuth bias error in the three radars, we can see from the fig 4.8 that the range error is raised up to the maximum limit +650 m approximately. After applying the K-means clustering


Figure 4.7: Final range error vs range graph


Figure 4.8: Range error vs range graph for all radars
algorithm, we can see from the fig 4.10 that the range error is varying up to the maximum limit +25 m approximately. In this way, we analyzed the results in case of every combination of radars and we are getting the same result like that.

## Case-3: Bias in Elevation but no bias in Azimuth and Range

In this case, we are adding some noises same as case-1. And we are adding some elevation bias error in radar4, radar5, and radar7. The value of the applied elevation bias error is 0.5 degrees. Now after adding the elevation bias error in the


Figure 4.9: Position of all points, clusters and final centroid


Figure 4.10: Final range error vs range graph
three radars, we can see from fig 4.11 that the range error is varying maximum up to the +700 m approximately. After applying the K-means clustering algorithm, we can see the results from the fig 4.13 that the final range error is varying maximum up to +25 m approximately.In this way, we analyzed the results in case every combination of radars and we are getting the same result like that.

## Case-4: Bias in Azimuth and Elevation but no bias in Range

In this case, we are adding some noises same as case-1. And we are adding


Figure 4.11: Range error vs range graph for all radars


Figure 4.12: Position of all points, clusters and final centroids
some azimuth bias error in radar7 and radar12 and also adding some elevation error in radar11. The value of applied azimuth and elevation bias error is 0.5 degrees. After adding the noise and bias errors in the three combinations of radar we can see that the range error is varying maximum up to +700 m approximately. After applying the K-means clustering algorithm final range error is varying maximum up to +25 m approximately. The figure is quite similar as the case-3. In this way, we analyzed the results for every combination of radars and we are getting


Figure 4.13: Final range error vs range graph


Figure 4.14: Range error vs range graph for all radars
the same result like that.

## Case-5: Bias in Azimuth and Elevation but no bias in Range in other combination

In this case, we are adding some noises same as case-1. And we are adding some azimuth bias error in radar7, radar11 and radar12 and also adding some elevation error in radar7, radar11 and radar12. The value of applied azimuth and elevation bias error is 0.5 degrees. After adding the noise, azimuth bias error and


Figure 4.15: Position of all points, clusters and final centroids


Figure 4.16: Final range error vs range graph
elevation bias error, We can see that the range error is varying maximum up to +1000 m approximately. After applying the K-means clustering algorithm final range error is varying maximum up to +25 m approximately. So here the error is reduced. The figure is quite similar as the case-3. In this way, we analyzed the results for every combination of radars and we are getting the same result like that.

## Case-6: Bias in Azimuth, Elevation and Range

In this case, we are adding some noises same as case-1. And we are adding


Figure 4.17: Range error vs range graph for all radars


Figure 4.18: Position of all points, clusters and final centroids
some azimuth bias error in radar12, radar13 and radar14, adding some elevation error in radar12, radar13 and radar14 and also adding some range bias error in the radar13. The value of applied azimuth and elevation bias error is 0.5 degree and the value of applied range bias error is 10 m . After adding the noises, azimuth, elevation, and range bias error we can see that the range error is varying up to the maximum limit +1100 m . After applying the K-means clustering algorithm final range error is varying maximum up to +25 m approximately. So here the error is


Figure 4.19: Final range error vs range graph


Figure 4.20: Range error vs range graph for all radars
reduced. The figure is quite similar as the case-3. In this way, we analyzed the results in case of every combination of radars and we are getting the same result like that.

Experimental results of all cases are depicted in table 4.2. The observation is similar in all cases that maximum value of error is very high before clustering and maximum value of error is approximately 25 m after clustering.


Figure 4.21: Position of all points, clusters and final centroids


Figure 4.22: Final range error vs range graph

### 4.2.4 Finding and Discussion

Measurement of the position from a single radar suffers from an accuracy and reliability problem. The accuracy and reliability problems can be rectified by getting measurements from multiple numbers of radars. That's why in this work, We proposed a real time data fusion technique to get more accurate measurements. We are using the k-means algorithm because it is most time efficient than other

Table 4.2: Maximum error values of all cases before and after clustering

| Case | Max value of total error <br> before clustering(m) | Max value of Total error <br> after clustering(m) |
| :--- | :---: | :---: |
| Ideal | 0.06 | Less than 0.06 |
| 1st | 30 | 25 |
| 2nd | 650 | 25 |
| 3rd | 700 | 25 |
| 4th | 700 | 25 |
| 5th | 1000 | 25 |
| 6th | 1100 | 25 |

clustering algorithms. Its computational complexity $O(k * n * t)$ where k number of clusters, $n$ number of objects, t number of iteration. When we are adding some noises and bias errors then measurements are spread out. To increase the accuracy of the cluster centriod so boundary limit is established. We ignored the measurements that are beyond the boundary and able to achieve more accurate and reliable results. We added the same noises in every radar and also add the same bias errors in every combination of radars. Further experimentation can be carried out with variable bias errors and variable noise.

