UG/2nd Sem/Stat/H/19

2019

B.Sc. (Hons.)

2nd Semester Examination

STATISTICS

Paper—C3T

Full Marks : 60 Time : 3 Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Mathematical Analysis

1. Answer any *ten* questions : 10×2

   (a) Prove that the sequence $\{1 + (-1)^n\}$ is neither convergent nor divergent.

   (b) Show that the set of all integers Z is countable.

   (c) Show that the sequence $\left\{\frac{1}{n+1}+\frac{1}{n+2}+\ldots+\frac{1}{n+n}\right\}$ is strictly monotonic increasing and bounded.

   (d) If $f'(x) > 0$ in [a, b], prove that $f(x)$ is increasing in [a, b].

[*Turn Over*]

(e) Prove that for $0 < x < \frac{\pi}{2}$, $\frac{\sin x}{x}$ decreases.

(f) Given $f(x)=\begin{cases}0, & \text{if } x \geq 0 \\ x^n, & \text{if } x<0,\end{cases}$ where n is a +ve integer. For what values of n, f(x) is differentiable for all values of x ?

(g) Give geometric interpretation of Lagrange's MVT.

(h) Show that the set Q of rational numbers is not order complete.

(i) State the order Axioms of the set R of real numbers.

(j) Define the least upper bound of a bounded set and obtain it for the set

$$A = \left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots\ldots, \frac{n}{n+1}, \ldots\ldots\ldots\right\}.$$

(k) Define an absolutely convergent series and a conditionally convergent series. Give one example for each.

(l) Prove that $\lim_{x \to a} f(x)$, when exists, is unique.

(m) Give an example to show that a function which is continuous in only an open interval, may not be bounded there.

(n) A function f is defined in (0, 2) by $f(x) = x - [x]$, prove that f is not continuous at $x = 1$.

(o) If $\phi(x)$ be a polynomial in x and λ is a real number, then prove that $\exists$ a root of $\phi'(x) + \lambda\phi(x) = 0$ between any pair of roots of $\phi(x) = 0$.

2. Answer any *four* questions : $4\times5=20$

(a) (i) If S be a bounded set of real numbers, then prove that the set $T = \{-x : x \in S\}$ is also bounded and $\text{Sup } T = -\inf S$ and $\inf T = -\text{Sup } S$.

(ii) Verify that the harmonic sequence $\left\{\frac{1}{n}\right\}$ converges to 0. (4+1)

(b) State Cauchy's Root test for convergence or divergence of a series of positive terms. Use it to prove that $\sum_{n=1}^{\infty} \frac{1}{2^{n+(-1)^n}}$ is convergent.

(1+4)

(c) State Leibnitz's theorem on successive derivatives. Use it to show that if $y = \tan^{-1}x$, then $(1 + x^2)\, y_{n+2} + 2(n + 1)\, xy_{n+1} + n\,(n + 1)y_n = 0$. (1+4)

(d) (i) Find a, b such that

$$\lim_{x \to 0} \frac{x(1 + a\cos x) - b\sin x}{x^3} = 1$$

(ii) If $u = \log r$ and $r^2 = x^2 + y^2 + x^2$, prove that $r^2 (u_{xx} + u_{yy} + u_{zz}) = 1$. (3+2)

(e) Examine the convergence of $\int_0^1 \frac{x^{n-1}}{1-x} dx$. 5

(f) (i) Express $\int_a^b (x-a)^m (b-x)^n dx$ in terms of Beta function.

(ii) Find the value of $\int_0^\infty e^{-x^2} dx$. (3+2)

3. Answer any *two* questions : 2×10

(a) (i) If $f(h) = f(0) + hf'(0) + \frac{h^2}{2!} f''(\theta h)$, $0 < \theta < 1$, find θ, when $h = 1$ and $f(x) = (1-x)^{5/2}$.

(ii) Consider the function f defined by

$$f(x, y) = \begin{cases} \frac{x^2 y^2}{x^2 + y^2}, & \text{when } x^2 + y^2 \neq 0 \\ 0 & , \text{when } x = 0, y = 0 \end{cases}$$

Show that $f_{xy}(0, 0) \neq f_{yx}(0, 0)$. Also show that $f_{xy}(x, y)$ and $f_{yx}(x, y)$ are not continuous at (0, 0). (4+6)

(b) (i) Examine for extreme values for the function $f(x, y) = x^3 + 3x^2 + y^2 + 4xy$.

(ii) Examine the existence of maxima, or minima of the function $f(x, y) = xy$ subject to the condition $5x + y = 13$. (5+5)

(c) (i) Show that $\iint_R \sqrt{4a^2 - x^2 - y^2}\, dx\, dy$

$= \frac{4}{9}(3\pi - 4)a^3$, where R is the upper half of the circle $x^2 + y^2 - 2ax = 0$.

(ii) Prove that $\int_0^1 \frac{\log(1+x)}{1+x^2} dx = \frac{\pi}{8}\log 2$.

(6+4)
