2019

B.Sc.

1st Semester Examination

PHYSICS (Honours)

Paper - C 1-T

(Mathematical Physics)

Full Marks : 40 Time : 2 Hours

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practiable.

Group - A

Answer any ***five*** questions : 5×2=10

1. Find whether $d\varphi$ is an exact differential where $d\varphi = \left(x^2 - y\right)dx + xdy$.

2. Show that $\vec{\nabla}\cdot\left(\vec{\nabla}\times\vec{A}\right) = 0$ for any vector $\vec{A}$.

3. If the magnitude of a vector $\vec{A}$ is constant with respect to time, show that $\frac{d\vec{A}}{dt}$ is perpendicular to $\vec{A}$.

4. The random variable x1 follows a Gaussian distribution with mean μ and standard deviation σ_1. A second random variable x2 also follows a Gaussian distribution with same mean μ but different standard deviation $\sigma_2 (> \sigma_1)$. Roughly sketch the two probability density functions.

5. Show that $\delta(kx) = \frac{\delta(x)}{|k|}$, where k is any non-zero constant.

6. A bag contains 10 black balls and 10 red balls. What is the probability of drawing two balls of the same colour ?

7. Solve the equation : $\frac{dy}{dx} + \log_e x^y = 0$.

8. Derive the expression of the volume element dV in spherical polar coordinates.

Group - B

Answer any ***four*** questions. 4×5=20

9. (a) Evaluate $\oint_C \vec{F} \cdot d\vec{r}$ along a closed curve C surrounding the origin and lying in the XY plane for $\vec{F} = \dfrac{\hat{i}x + \hat{j}y}{x^2 + y^2}$.

   (b) If $\vec{r}$ be the position vector of a point on a closed contour C, prove that $\oint_C \vec{r} \cdot d\vec{r} = 0$. 3+2

10. (a) Find the order and degree of the following differential equation :

$$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{1/2} + xy = 0.$$

   (b) Solve the differential equation,

$$\frac{d^2y}{dx^2}\left(e^x + 1\right) + \frac{dy}{dx} = 0.$$ 2+3

11. (a) What do you mean by axial vector ?

(b) Solve the following vector equation for $\vec{y}$:

$K\vec{y}+\left(\vec{y}\cdot\vec{b}\right)\vec{a}=\vec{c}$; $K\neq 0$ and K is a constant scalar while $\vec{a},\vec{b}$ and $\vec{c}$ are constant vectors. 2+3

12. Find a set of vectors reciprocal to the set $\left(2\hat{i}+3\hat{j}-\hat{k}\right),\left(\hat{i}-\hat{j}-2\hat{k}\right)$ and $\left(-\hat{i}+2\hat{j}+2\hat{k}\right)$. 5

13. The probability that a pen made by a company will be defective is 1/10. If 12 such pens are manufactured, determine what will be the probability that

(a) Exactly two will be defective,

(b) At least two will be defective, and

(c) None will be defective. 2+2+1

14. (a) If $\vec{A}$ is irrotational, show that $\vec{A}\times\vec{r}$ is solenoidal.

(b) The potential energy function between two atoms in a diatomic molecule is defined for x > 0 and given by $U(x)=U_0\left[\left(\frac{a}{x}\right)^{12}-2\left(\frac{a}{x}\right)^{6}\right]$, where

U_0 and a are both positive. What will you see the nature of equilibrium during plotting of U(x) vs. x i.e., is it stable or unstable ? 2+3

Group - C

Answer any ***one*** question : 1×10=10

15. (a) State Gauss divergence theorem. 2

(b) If $\vec{A} = ax\hat{i} + by\hat{j} + cz\hat{k}$ where a, b and c are constants. Evaluate $\int_S \vec{A}.d\vec{S}$ where S is the surface of a unit sphere. 3

(c) What is the main characteristic of Poisson distribution ? Give two physical examples where this distribution is applicable. 1+1

(d) Write the expression of probability P(r) related to Poisson distribution with r-success. 1

(e) Let X follow the Poisson distribution such that $P(X=1) = P(X=2)$. Obtain the value of P(X=4). 2

[Turn Over]

16. (a) Solve the differential equation :

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = \frac{e^x}{x}; \; y(1) = 0, \; y'(1) = 1. \quad 4$$

(b) When a force is called conservative ? Is there any chance to get a corresponding potential function ? — Justify with necessary deduction. 1+1

(c) Determine whether the force field given by $\vec{F} = x^2yz\hat{i} - xyz^2\hat{k}$ is conservative or not. 2

(d) Evaluate the integral : $\int_{-1}^{5} \delta(t-2)2e^{4t}dt$. If the lower limit of integration changes to 3, what will be the value of integration ? 1+1

8. Calculate the rest mass and momentum of a photon of energy 5eV. 2

Group - B

Answer any ***four*** questions 4×5=20

9. (a) Find the position of centre of mass of a uniform solid hemisphere. 3

   (b) Show that the areal velocity of a particle moving under central force (i) is constant and (ii) is equal to half of the angular momentum per unit mass of the particle. 2

10. (a) A frame R is rotating with respect to a fixed frame F with angular velocity $\vec{\omega}$. Show that :

$$\left.\frac{d\vec{\omega}}{dt}\right|_R = \left.\frac{d\vec{\omega}}{dt}\right|_F .$$ 3

   (b) A wooden block of mass M is suspended by a string of length l. Initially the block is at rest at its equilibrium position. A bullet of mass m is fired horizontally into the block and is embedded in it. The embedded block-bullet system swings upward and rises till the string makes an angle θ with vertical. Find the velocity of the bullet. 2

11. Find the moment of inertia of a uniform solid cylinder about an axis passing through its centre of mass and perpendicular to its length. Now find the ratio of the length of the cylinder to its radius for which this moment of inertia will be maximum. 3+2

12. A small block of mass 100g is suspended from a rigid support by a massless elastic spring. The system performs damped vertical oscillation of frequency 10Hz and the amplitude reduces to half of the undamped value in one minute. Calculate (i) the resistive force per unit velocity. (ii) the quality factor and (iii) the force constant of the spring. 2+1+2

13. (a) A rod of length 60 *cm* and radius 4 *mm* is rigidly fixed at one end. A torque of 5×10^7 *dyne/cm*^{2} applied at the other end of the rod produces a twist of 4.5°. Find the rigidity modulus of the material of the rod. 2

    (b) Obtain the expression of gravitational intensity due to a uniform thin spherical shell at a point inside it. 3

14. Establish relativistic velocity addition formulae starting from Lorentz transformation equations. 5