2019

B.Sc.

1st Semester Examination

MATHEMATICS (Honours)

Paper - GE 1-T

(Calculus Geometry and Differential Equation)

Full Marks : 60   Time : 3 Hours

The figures in the margin indicate full marks.
Candidates are required to give their answers
in their own words as far as practicable.
Illustrate the answers wherever necessary.

Unit - I

1. Answer any *three* questions : $3 \times 2 = 6$

   (a) If $y = x^{n-1} \log x$, prove that $y_n = \dfrac{(n-1)!}{x}$.

   (b) Evaluate $\lim\limits_{x \to 0} \left(\dfrac{\sin x}{x} \right)^{\frac{1}{x^2}}$.

[Turn Over]

(c) Find the envelope of

$$y = mx + a\sqrt{1+m^2}, \text{ parameter } m.$$

(d) State Leibnitz's rule for successive differentiation. Find the n-th order derivative of $\cos(ax+b)$, where a, b are constants. $1+1$

(e) Find the points of inflexion of the curve $y = (x+1)\tan^{-1}x$.   2

2. Answer any *one* question : $1 \times 10 = 10$

   (a) (i) Trace the curve

   $$(x^2 + y^2)x - a(x^2 - y^2) = 0, (a > 0)$$ 5

   (ii) If $y^{\frac{1}{m}} + y^{-\frac{1}{m}} = 2x$, then prove that

   $$(x^2 - 1)y_{n+2} + (2n-1)xy_{n+1} + (n^2 - m^2)y_n = 0$$ 5

   (b) (i) Find the asymptotes of the curve

   $$y^3 + x^2y + 2xy^2 - y + 1 = 0$$ 5

(ii) Find the ranges of values of x in which the curve

$y = 3x^5 - 40x^3 + 3x - 20$ is concave upwards or downwards. Also find the points of inflexion.

5

Unit - II

3. Answer any *two* of the following :   2×2=4

   (a) Find the perimeter of the astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$.

   (b) Find the length of the arc of the parabola $x^2 = 4ay$ measured from the vertex to one extremity of the latus rectum.

   (c) Calculate the area bounded by the curves $y = x^2$ and $x = y^2$.

4. Answer any *two* questions :   2×5=10

   (a) Show that the volume of the solid generated by the revolution of the curve $y(x^2 + a^2) = a^3$ about the asymptote of the curve is $2\pi^2 a^3$.

[Turn Over]

(b) Find the reduction formula of $\int \sin^p x \cos^q x\, dx$, where q is positive integer and p is negative integers Hence prove that

$$\int \frac{\cos^5 x}{\sin^4 x}\, dx = -\frac{\cos^4 x}{3\sin^3 x} + \frac{4\cos^2 x}{3\sin x} + \frac{8\sin x}{3}$$

(c) Show that the are of the upper half of the cardioide $r = a(1 - \cos\theta)$ is bisected at $\theta = \frac{2\pi}{3}$. Show also that the perimeter of the curve is 8a.

5

Unit - III

5. Answer any *three* questions : $\qquad$ $3 \times 2 = 6$

(a) Find the values of 'a' for which the plane $x + y + z = a\sqrt{3}$ touches the sphere

$$x^2 + y^2 + z^2 - 2x - 2y - 6 = 0$$

(b) Write down reflection properties of parabola and hyperbola.

(c) Find the equation of the cylinder generated by straight lines parallel to z-axis and passing

through the curve of intersection of the plane

$$4x + 3y - 2z = 5$$

and the surface $3x^2 - y^2 + 2z^2 = 1$.

(d) Find the equation of the sphere which passes through the circle $x^2 + y^2 = 4, z = 0$ and is cut by the plane $x + 2y + 2z = 0,$ in a circle of radius 3.

(e) Show that the equation $4xy - 3x^2 = 1$ is transformed to $X^2 - 4Y^2 = 1$ by rotating the axes through an angle $\tan^{-1} 2$.   2

6. Answer any *one* question : $1 \times 5 = 5$

(a) The expression

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c$ changed to

$a'x'^2 + 2h'x'y' + b'y'^2 + 2g'x' + 2f'y' + c'$ when the axes are rotated through an angle θ. Show that $a + b = a' + b'$.   5

(b) Show that the sum of the reciprocals of two perpendicular focal chords of a conic is constant.   5

[Turn Over]

7. Answer any *one* question : $1 \times 10 = 10$

   (a) (i) Prove that the two conics

$$\frac{l_1}{r} = 1 + e_1 \cos\theta \quad \text{and}$$

$$\frac{l_2}{r} = 1 + e_2 \cos(\theta - \gamma)$$

will touch one another if

$$l_1^2\left(1 - e_2^2\right) + l_2^2\left(1 - e_1^2\right) = 2l_1 l_2\left(1 - e_1 e_2 \cos\gamma\right).$$

                                        5

      (ii) A sphere of constant radius r passes through origin O and meet the axes in A, B, C. Prove that the locus of the foot of perpendicular from O to the plane ABC is given by

$$\left(x^2 + y^2 + z^2\right)\left(x^{-2} + y^{-2} + z^{-2}\right) = 4r^2 \qquad 5$$

   (b) (i) Reduce the equation

$$x^2 - 5xy + y^2 + 8x - 20y + 15 = 0$$

to its canonical form and determine the nature of the conic.   5

(ii) Show that the section of the surface

$$yz + zx + xy = a^2$$

by the plane

$$lx + my + nz = p$$

will be a prabola if $\sqrt{l} + \sqrt{m} + \sqrt{n} = 0$. 5

Unit - IV

8. Answer any *two* questions :   2×2=4

(a) Find the differential equation of all parabolas having their axes parallel to y axis.

(b) Find an integrating factor of the differential equation $\left(x^2 + y^2 + 2x\right)dx + 2y\,dy = 0$.

(c) Reduce the equation

$$\sin y \cdot \frac{dy}{dx} = \cos x \left(2\cos y - \sin^2 x\right)$$

into a linear equation.   2

[Turn Over]

9. Answer any *one* question : 1×5=5

(a) Show that $y = \dfrac{Q}{P} - e^{-\int p dx}\left[\int e^{\int p dx} d\left(\dfrac{Q}{P}\right) + C\right]$ is

a solution of the differential equation

$\dfrac{dy}{dx} + py = Q$, where P and Q are functions of

x only.

(b) If $\dfrac{1}{N}\left(\dfrac{\partial M}{\partial y} - \dfrac{\partial N}{\partial x}\right)$ is function of x alone, say

$f(x)$, then prove that $e^{\int f(x)dx}$ is an integrating

factor of $Mdx + Ndy = 0$.