

2019

B.Sc.

2nd Semester Examination

**MATHEMATICS (Honours)**

Paper - C4T

Full Marks : 60

Time : 3 Hours

*The figures in the margin indicate full marks.  
Candidates are required to give their answers  
in their own words as far as practicable.*

**Unit - I****[Marks - 22]**1. Answer any *one* question : 1×2

(a) Show that  $\frac{dy}{dx} = 3y^{\frac{2}{3}}$ ,  $y(0) = 0$  has more than one solution and indicate the possible reason.

(b) Let  $W(y_1, y_2)$  be the Wronskian of two linearly independent solutions  $y_1$  and  $y_2$  of the

*[ Turn Over ]*

equation  $\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = 0$  then  
 prove that

$$W(y_1, y_2)P(x) = y_2 \frac{d^2y_1}{dx^2} - y_1 \frac{d^2y_2}{dx^2}.$$

2. Answer any *two* questions : 5×2

(a) Let  $r_1, r_2$  be the roots of the indicial polynomial for the equation

$$\frac{d^2y}{dx^2} + \frac{ady}{dx} + by = 0 \text{ where } a, b \text{ are constants.}$$

If  $r_1 \neq r_2$  then show that two independent solutions are  $e^{r_1x}$  and  $e^{r_2x}$  respectively. 5

(b) Solve the differential equation

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} = x + e^x \sin x$$

by the method of undertermine coefficient. 5

(c) Solve the differential equation

$$(2+3x)\frac{d^2y}{dx^2} + 5(2+3x)\frac{dy}{dx} - 3y = x^2 + x + 1,$$

$$-\frac{2}{3} < x < \alpha.$$

5

3. Answer any *one* equation :

10×1

(a) (i) A 2nd order linear differential equation of

the form  $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = F(x)$ , where

$P$ ,  $Q$  and  $F$  are continuous functions of  $x$  on  $[a, b]$ . Then by using the method of variation of parameters prove that the general solution of given ODE is given by

$$y(x) = Au(x) + Bv(x) \left\{ -\int \frac{V(t)F(t)}{W(u, v)} dt \right\} u(x) +$$

$$\left\{ \int \frac{u(t)F(t)}{W(u, v)} dt \right\} V(x), \quad x \in [a, b]$$

[ Turn Over ]

where  $A$  and  $B$  are two arbitrary constants  
and  $W(u, v) = uv' - vu'$ . 5

(ii) Solve the differential equation

$\frac{d^2y}{dx^2} + a^2y = \sec(ax)$  by the method of  
variation of parameters. 5

(b) (i) State and prove the super position principle  
for homogeneous linear differential equation. 4

(ii) Using the fact that  $y = x^2$  is a solution of

the equation  $x^2 \frac{d^2y}{dx^2} - 3x \frac{dy}{dx} + 4y = 0$ ,

$0 < x < \infty$  then find another independent  
solution. 4

(iii) If  $y = x \cos x$  is a solution of an  $n$ -th order  
linear differential equation

$$\frac{d^n y}{dx^n} + a_1 \frac{d^{n-1} y}{dx^{n-1}} + \dots + a_{n-1} \frac{dy}{dx} + a_n y = 0$$

with real constant coefficients, then find the  
least possible value of  $n$ . 2

## Unit - II

[Marks - 13]

4. Answer any four questions :

2×4

(a) Show that

$$(yz + xyz)dx + (xy + xyz)dz + (zx + xyz)dy = 0$$

is integrable.

(b) Let  $y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$  be a solution of the system of

$$\text{equations } \begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ a & b \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, \text{ where } a, b \in \mathbb{R}$$

Then prove that every solution  $y(x) \rightarrow 0$  as  $x \rightarrow \infty$  if  $a < 0$  and  $b < 0$ .(c) Show that  $f(t, x) = (3t + 2x_1, x_1 - x_2)$  on  $S: \{|t| < \infty, \|x\| < \infty\}$  satisfying a Lipschitz condition where  $x = (x_1, x_2) \in \mathbb{R}^2$ .

[ Turn Over ]

- (d) Find the first order simultaneous differential equations for the third order differential equation

$$\frac{d^3x}{dt^3} - 6\frac{d^2x}{dt^2} + 12\frac{dx}{dt} - 8x = 18e^{2t}.$$

- (e) Solve :  $\frac{dx}{3y-2z} = \frac{dy}{z-3x} = \frac{dz}{2x-y}$ .

- (f) Write fundamental matrix for homogeneous system of linear equation given below

$$\dot{X}(t) = A(t)X(t) \text{ where}$$

$$X(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{pmatrix}, \quad t \in [a, b].$$

5. Answer any *one* question :

5×1=5

- (a) Find the fundamental matrix and the solution

$$X(t) \text{ such that } X(0) = \begin{pmatrix} 1 \\ 6 \end{pmatrix} \text{ for the system}$$

$$\begin{pmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 0 & -4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad 5$$

(b) (i) Solve :

$$\frac{dx}{x^2(y-z)} = \frac{dy}{y^2(z-x)} = \frac{dz}{z^2(x-y)} \quad 2\frac{1}{2}$$

(ii) Solve :

$$(yz + 2x)dx + (zx - 2y)dy + (xy - 2z)dz = 0 \quad 2\frac{1}{2}$$

### Unit - III

#### [Marks - 9]

6. Answer any *two* questions : 2×2

(a) Find the equilibrium point of the system of differential equations

$$\dot{x} = e^{x-1} - 1 \text{ and } \dot{y} = ye^x.$$

[ Turn Over ]

- (b) Show that  $x = 0$  is the regular singular point of the differential equation

$$2x^2 \frac{d^2 y}{dx^2} + 7x(x+1) \frac{dy}{dx} - 3y = 0$$

- (c) If  $\sum_{m=0}^{\infty} c_m x^{r+m}$  is assumed to be a solution of

$$x^2 y'' - xy' - 3(1+x^2)y = 0 \text{ then find the values of } r.$$

7. Answer any *one* question : 5×1

- (a) Find a power series solution of the equation

$$(x^2 - 1) \frac{d^2 y}{dx^2} + 3x \frac{dy}{dx} + xy = 0.$$

Given that  $y(0) = 4$  and  $\left. \frac{dy}{dx} \right|_{x=0} = 6.$  5

- (b) (i) Find the phase curve of the dynamical system of equations  $\dot{x} = 2x - y$  and  $\dot{y} = -4y$ . Also describe the nature of stationary point.



(ii) Determine the steady state and their stability

of the differential equation  $\frac{dx}{dt} = x^2 - 5x + 6$ .

$$3+2=5$$

### Unit - IV

[Marks - 16]

8. Answer any *three* questions : 2×3

(a) If the vector  $\vec{a}$  and  $\vec{c}$  are perpendicular to each other then show that the vectors

$\vec{a} \times (\vec{b} \times \vec{c})$  and  $(\vec{a} \times \vec{b}) \times \vec{c}$  are perpendicular to each other.

(b) Find the value of  $x$  such that the vectors  $2\hat{i} - \hat{j} + \hat{k}$ ,  $\hat{i} + 2\hat{j} - \hat{k}$  and  $x\hat{i} - 4\hat{j} + 5\hat{k}$  are Coplanar.

(c) If  $\vec{a} = t^2\hat{i} - t\hat{j} + (2t+1)\hat{k}$  and

$\vec{b} = (2t-3)\hat{i} + \hat{j} - t\hat{k}$  then show that

$$\frac{d}{dt}(\vec{a} \cdot \vec{b}) = -6 \text{ at } t = 1.$$

[ Turn Over ]

(d) Evaluate

$$\int_0^1 \left( \vec{r} \times \frac{d^2 \vec{r}}{dt^2} \right) dt \quad \text{where } \vec{r} = t^3 \hat{i} + 2t^2 \hat{j} + 3t \hat{k}.$$

(e) A necessary condition for the vector  $\vec{c}(t)$  to be

constant is  $\frac{d\vec{c}}{dt} = \vec{0}$ . Prove it.

9. Answer any *one* question :

10×1

(a) (i) A necessary and sufficient condition that a proper vector  $\vec{u}$  has a constant length is

$$\text{that } \vec{u} \cdot \frac{d\vec{u}}{dt} = 0. \quad 5$$

(ii) If  $\frac{d\vec{a}}{dt} = \vec{r} \times \vec{a}$  and  $\frac{d\vec{b}}{dt} = \vec{r} \times \vec{b}$  then show

$$\text{that } \frac{d}{dt}(\vec{a} \times \vec{b}) = \vec{r} \times (\vec{a} \times \vec{b}) \quad \text{where } \vec{r} \text{ is a}$$

constant vector and  $\vec{a}, \vec{b}$  are vector functions of a scalar variable  $t$ . 5

(b) (i) Four points whose position vectors are  $\vec{a}, \vec{b}, \vec{c}, \vec{d}$  are Coplanar if and only if

$$[\vec{a} \vec{b} \vec{c}] = [\vec{b} \vec{c} \vec{d}] + [\vec{c} \vec{a} \vec{d}] + [\vec{a} \vec{b} \vec{d}] \quad 5$$

(ii) If  $\frac{d^2\vec{r}}{dt^2} = 6t\hat{i} - 24t^2\hat{j} + 4\sin t\hat{k}$  and if

$$\vec{r} = 2\hat{i} + \hat{j} \quad \text{and} \quad \frac{d\vec{r}}{dt} = -\hat{i} - 3\hat{k} \quad \text{when } t=0,$$

then show that

$$\vec{r} = (t^3 - t + 2)\hat{i} + (1 - 2t^4)\hat{j} + (t - 4\sin t)\hat{k}.$$