Chapter 3

Single-item Supply Chain Management models

3.1 Introduction

The classical inventory models on deteriorating items are normally developed with the common assumption that the capacity of the retailer's outlet is sufficient, i.e., the outlet has sufficient space to store the order quantity [12, 50, 130, 152, 202, 206]. However, in several real-life problems, this assumption may not appropriate. There are a number of factors which influence the marketing decisions in different ways. Sometimes these factors may force the retailer to buy more than his/her own warehouse (OW) capacity. The retailer may overcome the situation using an additional rented warehouse (RW), having sufficient capacity, normally with higher rent relative to the OW [58, 117].

Influence of displayed inventory level on the demand of any item is a well established phenomenon [65, 66]. Due to this reason, a retailer normally uses a decorated outlet at the market place to attract the customers and uses another storehouse near the outlet to stock the excess order quantity [113, 117, 127, 141]. Also inventory modelings of the deteriorating items draw significant attention by the researchers [13, 34, 50, 100, 125, 130, 201, 206]. During last two decades, several researchers on inventory control problems developed their models incorporating the above mentioned two important phenomenon, i.e., inventory models of deteriorating items with displayed inventory dependent demand under retailer's two warehouse facility [38, 56, 58, 124, 152, 183].

From the perspective of the customers' buying behavior, it is noticed that besides stock of the products other factors such as promotional cost through advertising, free gift coupon etc., also influence customers' preferences and their purchasing decisions and hence the market demand. Many researchers also have considered promotional effort dependent demand in their research [2, 23, 59, 118, 136, 193]. None of these investigations gives attention to study the joint effect of the stock level and the promotional effort on the demand of a deteriorating item specially in a SC under coordination mechanism. For some products when a retailer is out of stock, the demand is lost which means that the customer finds the item or a similar one in another store. Yang et al. [202] developed an inventory model under inflation for deteriorating items with stock-dependent consumption rate and partial backlogging shortages. Sarkar and Sarkar [165] proposed an improved inventory model with partial backlogging, time varying deterioration and stock-dependent demand. But, none of these investigations on deteriorating items, studied the effects of shortages when stock and promotional efforts jointly influences the demand of the item specially in a SC under promotional cost sharing strategy.

Due to the rapid increasing complexities of the environment it is difficult to define different inventory parameters precisely. As a result, it may not be possible to define the different inventory costs as well as the constraints precisely. For example, production of an item in any manufacturing organization deeply depends on efficiency, effectiveness of the system, i.e., quality of the process output, inventory turnover ratio and so many factors related to the production process, which leads to uncertainty/impreciseness in any production process. Impreciseness can be modelled using fuzzy, stochastic and rough variables. Kao and Hsu [88] discussed the inventory problem with fuzzy demands where back-orders are permitted. Maiti and Maiti [113] developed an inventory model under fuzzy constraint, where purchase cost, investment amount and storehouse capacity are imprecise in nature. On the other hand, Mondal et al. [126] and Guchhait et al. [69] use rough variables to represent imprecise inventory parameters. There are also some other inventory/SC models with fuzzy/rough inventory parameters in the literature [10, 98, 99, 150, 186]. Moreover, presence of imprecise parameters leads to imprecise optimisation problem and till now no proper guideline is available in the literature to find optimal solution of such problems. Different heuristic approaches establish their ability to solve different real life problems in science and technology [21, 28, 61, 133]. Among different heuristic approaches, Particle Swarm Optimisation (PSO) [92] draws more attention for continuous optimisation problems due to its easy implementation and less computational time [55, 62, 84, 122].

To overcome the above stated lacunas of the SC models on deteriorating items, in the first model (Model 3.1), a wholesaler-retailer-customer SC model of a deteriorating item is considered where the retailer runs his/her business with a warehouse having limited capacity. Due to limited capacity of own warehouse (OW), the retailer rents another warehouse (named RW) to store the excess order quantity of the item (if required). Units are sold from the OW and are continuously replenished from the RW. The item has a base demand d and another portion of the demand is linearly influenced by the stocks level at OW. Retailer invests some promotional cost to improve the base demand of the item. The product is deteriorated with a constant rate. Shortages are also considered and backlogged partially. As the retailer introduces the promotional cost and there is no bonding of the retailer with the wholesaler, in the first phase of the study, it is assumed that the retailer is the leader and the wholesaler is the follower, i.e., the retailer determines his/her marketing strategy according to his/her interest and accordingly the wholesaler fixes his/her marketing decision. This situation is named as non-coordination scenario (NCS). In the second phase of the study, it is assumed that the wholesaler will to share a compromise portion of the promotional cost spends by the retailer to take part in the joint marketing decision with the retailer and this situation is named as coordination scenario (CS). It is established that the profits of both the parties improves in CS. The crisp (precise) model is solved following GRG method using LINGO 14.0 software. Model is analyzed in imprecise environment also, when different inventory costs like set-up cost, holding cost and the promotional cost function are fuzzy/rough in nature. As optimization of fuzzy/rough objective is not well defined, following credibility/trust measure of fuzzy/rough event, an approach is followed for the comparison of fuzzy/rough objectives and a PSO is implemented and used to determine the marketing decisions of the model in imprecise environment. Proper parameter setting of PSO for solving the model is made following Taguchi approach [204]. The crisp models are also solved using PSO and compared the results with those obtained using LINGO. Another heuristic, Artificial Bee Colony (ABC) [96] is also implemented to establish the uses of PSO in solving the models. Models are illustrated with numerical examples and some managerial insights are outlined. The existence of the joint marketing decision is established analytically and numerically in crisp as well as in imprecise environments.

Customers' demand is the most important factor in any supply chain, as total revenue from the chain mostly depends on it. For seasonal product, demand normally depends on time. At the beginning of the season, the demand normally increases with time and reaches a maximum limit at peak season time. Then, it gradually decreases and at the end of the season, demand comes down to off-season level. So, the seasonal demand curve of such an item is parabolic in type. A considerable number of research papers have been published on time dependent demand, but none has considered this phenomenon specially for a supply chain model. Most of the authors considered time dependent increasing demand [17, 39, 73, 94, 167]. Few authors considered ramp type demand rate [121, 142, 175, 195]. Very few authors considered time dependent decreasing demand [32, 98]. Again dependency of demand on selling price is a well established phenomenon [68, 110, 112, 113, 138].

Promotional effort strategy is essential policy to boost the demand of an item in the oligopoly marketing system [136, 153, 193]. This strategy is utilised by both large and small business houses to inform, persuade and remind customers about the products and services they have to offer. As a result of promotional efforts, customers are informed about new products and are also reminded about existing products. For a seasonal product, dependency of demand includes time, price as well as promotional cost. Study on inventory control problem incorporating the effect of these factors on demand are studied separately by several authors, but none has studied the combined effect of these factors on demand. Again all these studies are normally made only from retailers point of view. Deterioration is an important factor for inventory decisions of deteriorating items [33, 95, 125, 130, 179, 201]. Though considerable amount of research has been done on supply chain management, not much attention has been made for seasonal products which are normally deteriorating in nature. Shortages is a normal phenomenon in most of the inventory management system [1, 22, 165, 196, 202]. In the classical economic order quantity model, it is often assumed that the shortages are either completely backlogged or completely lost. In reality, often some customers are willing to wait until replenishment, especially if the waiting time is short, while others are more impatient and go elsewhere.

In the second model (Model 3.2), a two level supply chain consisting of a wholesaler and a retailer for seasonal deteriorating item in a finite planning horizon (seasonal time) is considered, where the demand of a seasonal product is time, price and promotional cost dependent. Thus, the present investigation is more general. It can be used in the business of seasonal products as a retailer brings the product from the wholesaler in bulk for several times during the season period. This type of business is seen in most of the district/sub-divisional towns of the developing countries like India, Nepal etc. For example, for the business of the winter/warm garments, a district businessman (retailer) brings the item in bulk from a merchant (wholesaler) at the capital of the state and do the business for a time period. The retailer repeats this process several times during the winter/summer season which is finite in nature. Nowadays, due to the better communication system and availability of several merchants at the capital of the state, the said retailer just travels to the capital and do the purchase in no time (instantaneously) and transports the purchased goods through the developed transport system. For this reason, the lead time for the order is nowadays not much observed as it was in earlier days. Moreover, due to stiff competitive market, merchants (wholesalers) do always maintain sufficient stocks. The new ideas incorporated in this investigation are as follows:

- The joint effect of time, price and promotional cost on demand is considered.
- Dependency of demand on time is modelled as parabolic with a pre-estimated maximum limit.
- The supply chain models with above demand are formulated and solved in fuzzy environment.
- PSO is appropriately developed and used for near-optimum results.

It is established that the price discount policy and the promotional cost sharing policy play significant role in improving channel performance in any SC. A considerable number of studies has been made on price discount policy to establish it as an essential part of coordination mechanism of any SC [135, 154, 161, 187, 193, 207]. Other group of authors emphasis on promotional effort strategy to enhance the individual goals as well as the channel goal [78, 150, 151, 184, 185].

In any SC different promotional efforts like – advertisement, free gifts, credit period, price discount etc., are introduced to improve the performance of the chain. Total cost involves in such efforts is called promotional cost and is mainly introduced to improve the base demand of an item. It is already established that if different partners of the chain share this cost, then channel performance as well as individual performance improved. On the other hand, the price discount policy is a specified direction of the introduction of the promotional cost. In all these studies, it is considered that the promotional effort ρ magnify the base demand of the considered item and the corresponding promotional cost is a function of ρ. But in reality, when some promotional cost like price discount is introduced, then during price discount period, due to low price the demand increases. After that specified period of time, the discount is withdrawan, but, by this process the base demand increases due to the fact that some of the customers have already accustomed with the product during the price discount period and do not switch over to other products though price discount is withdrawn. Some research works have already been done in this direction [99, 138].

But those studies does not focus how this promotional cost could be shared to increase the channel performance as well as individual performances. So in a nutshell, the lacunas of the above mentioned studies are:

- Though considerable amount of research works have been done to improve channel performance of a SC, none have outlined the actual process of utilization of promotional cost and its sharing in the chain.
- None of the studies focuses on actual way of increase of the demand of an item in the chain.
- In any production system, the production rate of an item normally imprecise in nature. But none of the manufacturer-supplier SC model considered fuzzy production rate.
- Also a little attention has been paid to consider the demand of an item in a SC as imprecise in nature.
- Moreover, none of these studies considered the influence of inflation and time value of money in the SC, specially when price discount is introduced as promotional effort.

In the third model (Model 3.3), an attempt has been made to develop a supplier-manufacturer SC model in a fuzzy planning horizon, which is free from the above mentioned lacunas. Here it is considered that a supplier supplies raw materials to the manufacturer. The production rate of the manufacturer is constant but fuzzy in nature. Demand of the finished good is price sensitive and imprecise in nature. A price discount is offered by the manufacturer at the beginning of each of his/her cycle to enhance the demand of the item. It is established that if the supplier shares some portion of the amount of money required to reduce the selling price then profit of both the supplier as well as the manufacturer enhanced. The study is made incorporating the inflation and time value of money. Following Kundu et al. [99], FDE [19] and FRI [194] is used to find the α-cut of the channel profit as well as individual profits from the whole planning horizon. Considering the α-cut of the objectives as interval numbers and using fuzzy preference ordering of intervals [170] for the comparison of intervals, a soft computing technique, multi-choice artificial bee colony (MCABC) algorithm (cf. § 2.2.2.3) is proposed and used to find marketing decision of the model. The algorithm is capable of solving any single objective optimization problem in crisp as well as in imprecise environment. Efficiency of the proposed MCABC algorithm is tested against a set of benchmark test problems in crisp environment available in the literature. Moreover, the algorithm is capable of solving optimization problem in imprecise environment. Model is illustrated with numerical examples and some managerial insights are drawn.

3.2 Model 3.1: Two-Level Supply Chain for a Deteriorating Item with Stock and Promotional Cost Dependent Demand Under Shortages

3.2.1 Assumptions and Notations

The following assumptions and notations are used in this study:

Assumptions:

- It is an infinite time horizon EOQ model for a constantly deteriorating item.
- Lead time is zero.
- Demand is stock and promotional effort dependent.
- Shortages are allowed and partially backlogged.
- Rate of replenishment is infinite.
- Two warehouses – OW and RW are considered. Sales are performed from OW and units are transferred from RW to OW in continuous release pattern.
- The promotional cost to boost the demand is shared by both the wholesaler and the retailer.

Notation	**Meaning**
W	capacity of the OW.
$I_o(t)/I_r(t)$	inventory level at the OW/RW at time t.
$S(t)$	shortage level of the retailer.
$I_W(t)$	inventory level of the wholesaler.
$I_W^i(t)$	inventory level of the wholesaler during i-th interval of its inventory period.
α	deterioration rate at the OW.
β	deterioration rate at the RW ($\beta > \alpha$).
γ	deterioration rate at the wholesaler.
δ	percentage of demand which is backlogged during shortage time.

Notation	**Meaning**
A_R/A_W	replenishment fixed cost per order of the retailer/wholesaler.
p_R/p_W	purchase price of the retailer/wholesaler per unit.
s_R/s_W	selling price of the retailer/wholesaler per unit ($s_W = p_R$).
h_o/h_r	unit holding cost per unit time at the OW/RW.
h_W	unit holding cost per unit time of the wholesaler.
d_R/d_W	deterioration cost per unit per unit time of the retailer/wholesaler.
c_{sf}	unit lost sale cost.
c_{sv}	unit shortage cost per unit time for backlogged demand.
t_o/t_r	time at which the inventory level at the OW/RW reaches zero.
t_s	time during which the retailer is out of stock and the demand is partially backlogged.
T_R/T_W	inventory period of the retailer/wholesaler.
Q_R/Q_W	order quantity of the retailer/wholesaler.
D_R/D_W	quantity of deterioration of the retailer/wholesaler.
ρ	retailer's promotional effort, $\rho \geq 1$.
F	fraction of the retailer's promotional cost shared by the wholesaler.
PrC	promotional effort cost per unit time for the item: $PrC = g(\rho - 1)^2 d^m$; where, g, m are the parameters so chosen to best fit the promotional cost and d is the base demand of the item.
$D(t)$	demand rate at time t, the demand is assumed to be deterministic, stock-dependent and promotional cost dependent: $D(t) = cI_o(t) + d\rho$; where c is a parameter so chosen to best fit the demand function and d is the base demand of the item.

Symbols ˜ and ˘ are used on the top of some of the above parameters/variables to indicate fuzzy and rough variable respectively.

3.2.2 Mathematical Formulation of the Model

3.2.2.1 Retailer's Inventory Level (OW and RW)

It is assumed that at the beginning of each cycle, the retailer orders an amount of Q_R units of the item to the wholesaler. Among Q_R units, W units are stored at OW and remaining units are stored at RW. The units are sold from OW and continuously replenished from RW (cf. Figure 3.1). Let $I_o(t)$ and $I_r(t)$ be the

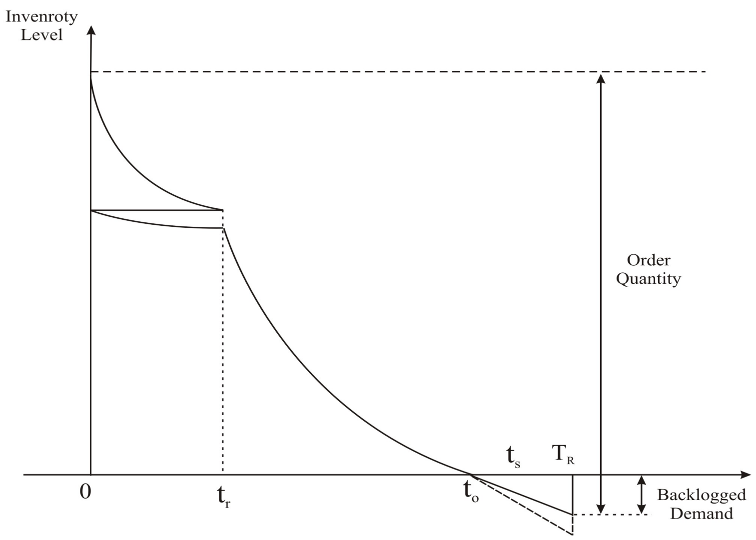

FIGURE 3.1: Inventory level at the RW and the OW of the retailer

inventory level at the OW and the RW respectively. At the RW, the inventory is depleted by a demand which is connected to the inventory level at the OW. Therefore, the changes of inventory level at the RW between the start of the inventory period and t_r can be presented by the following differential equation:

$$\frac{dI_r(t)}{dt} = -cI_o(t) - d\rho - \beta I_r(t), \text{ for } 0 \le t \le t_r \tag{3.1}$$

While the retailer is using the inventory at the RW to meet the demand, the inventory level at the OW goes down by a constant rate of the inventory level due to deterioration as follows:

$$\frac{dI_o(t)}{dt} = -\alpha I_o(t), \text{ for } 0 \le t \le t_r \tag{3.2}$$

At time t_r, the inventory at the RW is depleted completely and the inventory at the OW is used. The inventory level at the OW decreases due to the demand and deterioration until it reaches zero at t_o. This changes of inventory level at the OW is presented by the following differential equation:

$$\frac{dI_o(t)}{dt} = -cI_o(t) - d\rho - \alpha I_o(t), \text{ for } t_r \le t \le t_o \tag{3.3}$$

From t_o to T_R the system is out of stock and unmet demand is partially backlogged.

$$\frac{dI_o(t)}{dt} = -\delta d\rho, \text{ for } t_o \le t \le T_R \tag{3.4}$$

In order to solve the presented differential equations, the following boundary conditions should be considered:

$$I_o(0) = W, \;\; I_o(t_o) = 0, \;\; I_r(t_r) = 0$$

By solving the differential equations in (3.1)-(3.4), the inventory levels at the OW and RW are obtained:

$$I_r(t) = \frac{cWe^{-\alpha t}}{\beta - \alpha}\{e^{(\beta-\alpha)(t_r - t)} - 1\} + \frac{d\rho}{\beta}\{e^{\beta(t_r - t)} - 1\}, \text{ for } 0 \le t \le t_r \quad (3.5)$$

$$I_o(t) = We^{-\alpha t}, \text{ for } 0 \le t \le t_r \quad (3.6)$$

$$I_o(t) = \frac{d\rho}{c+\alpha}\{e^{(c+\alpha)(t_o - t)} - 1\}, \text{ for } t_r \le t \le t_o \quad (3.7)$$

$$S(t) = -I_o(t) = \delta d\rho(t - t_o), \text{ for } t_o \le t \le T_R \quad (3.8)$$

Equating the inventory level of OW at $t = t_r$ from (3.6) and (3.7), the following result is derived.

$$t_o = t_r + \frac{1}{c+\alpha} ln\Big(1 + \frac{c+\alpha}{d\rho}We^{-\alpha t_r}\Big) \quad (3.9)$$

which shows that t_o is a function of t_r and ρ.
The order quantity for the retailer is the sum of the initial inventory level at the RW and the OW and the total backlogged demand during one inventory period.

$$\begin{aligned} Q_R &= I_r(0) + I_o(0) + S(T_R) \\ &= \frac{cW}{\beta-\alpha}\{e^{(\beta-\alpha)t_r} - 1\} + \frac{d\rho}{\beta}\{e^{\beta t_r} - 1\} + W + \delta d\rho t_s \end{aligned} \quad (3.10)$$

which shows that Q_R is a function of t_r, t_s and ρ.
The length of the inventory period of the retailer is the sum of t_o and t_s.

$$\begin{aligned} T_R &= t_o + t_s \\ &= t_r + \frac{1}{c+\alpha} ln\Big(1 + \frac{c+\alpha}{d\rho}We^{-\alpha t_r}\Big) + t_s \end{aligned} \quad (3.11)$$

This shows that T_R is a function of t_r, t_s and ρ.

3.2.2.2 Wholesaler's Inventory Level

The order quantity and the length of the inventory period of the wholesaler are Q_W and T_W respectively. Here it is assumed that T_W is a multiple of T_R (i.e., $T_W = kT_R$, where k is an integer).

The order quantity of the wholesaler is equal to the inventory needed for k periods of the retailer, plus the amount of deterioration during the wholesaler's inventory cycle. It should be noted that during the k-th interval, there is no inventory at the wholesaler and after receiving Q_W units of the item at the end of T_W, Q_R is again sent to the retailer. Therefore there is no deterioration during this interval at the wholesaler. The order quantity of the wholesaler can be calculated as follows:

$$Q_W = kQ_R + D_W \tag{3.12}$$

where, D_W is the deterioration during the wholesaler's inventory cycle.

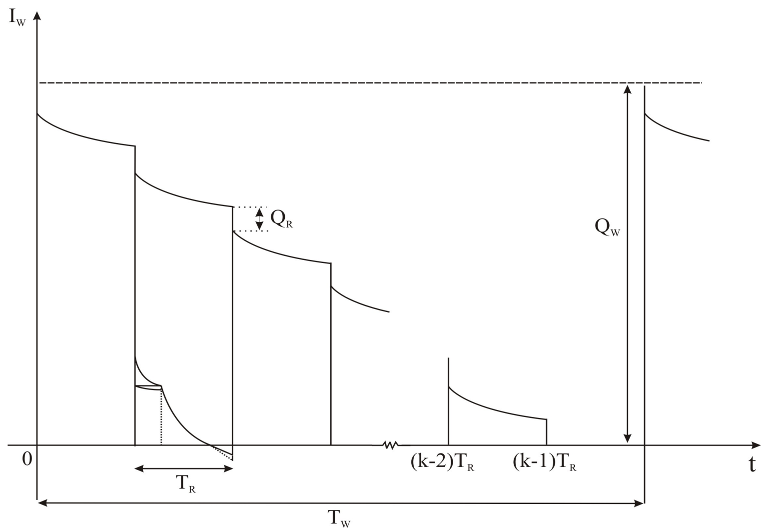

FIGURE 3.2: Inventory level of the wholesaler

Figure 3.2 illustrates the inventory level of the wholesaler. Here, one inventory period of the wholesaler consists of k retailer's inventory periods. At the time $(k-2)T_R$ and $(k-1)T_R$, the inventory level of the wholesaler drops by Q_R and a constant rate of the inventory is deteriorated during the interval $[(k-2)T_R, (k-1)T_R]$. The change in inventory level of the wholesaler during this interval can be presented by the following differential equation:

$$\frac{dI_W(t)}{dt} = -\gamma I_W(t) \tag{3.13}$$

Considering the inventory level of the wholesaler at $(k-1)T_R$ which is Q_R, the inventory level for the specific period will be:

$$I_W(t) = Q_R e^{\gamma[(k-1)T_R - t]} \text{ for } (k-2)T_R \leq t \leq (k-1)T_R \tag{3.14}$$

In a similar way, the inventory level of the wholesaler can be obtained for the period starts at $(k-3)T_R$ using (3.13), considering the boundary condition derived from (3.14) at $t = (k-2)T_R$:

$$I_W(t) = Q_R(1 + e^{\gamma T_R})e^{\gamma[(k-2)T_R - t]} \text{ for } (k-3)T_R \leq t \leq (k-2)T_R \tag{3.15}$$

In this way, the inventory level of the wholesaler during i-th interval can be calculated as follows:

$$I_W^i(t) = Q_R\Big\{\sum_{m=i+1}^{k} e^{(k-m)\gamma T_R}\Big\}e^{\gamma[iT_R - t]} \text{ for } i = 1, 2, ..., k-1. \tag{3.16}$$

3.2.2.3 Retailer's Profit

Total deteriorated units (D_R) during the retailer's inventory cycle is the sum of the deteriorated units at the RW (D_{RW}) and at the OW (D_{OW}):

$$D_R = D_{RW} + D_{OW} \tag{3.17}$$

$$\text{Deteriorated units at the RW} = D_{RW} = \int_0^{t_r} \beta I_r(t)dt$$

$$= (e^{\beta t_r} - 1)\Big\{\frac{cWe^{-\alpha t_r}}{\beta - \alpha} + \frac{d\rho}{\beta}\Big\} + \frac{\beta cW}{\alpha(\beta - \alpha)}(e^{-\alpha t_r} - 1) - d\rho t_r$$

$$\text{Deteriorated units at the OW} = D_{OW} = \int_0^{t_r} \alpha I_o(t)dt + \int_{t_r}^{t_o} \alpha I_o(t)dt$$

$$= W\Big\{1 - \frac{ce^{-\alpha t_r}}{c + \alpha}\Big\} - \frac{\alpha d\rho}{c + \alpha}(t_o - t_r)$$

Hence, the total selling price per unit time of the retailer:

$$TS_R = \frac{(Q_R - D_R)s_R}{T_R} \tag{3.18}$$

The retailer has different types of costs: ordering cost (A_R), purchase, carrying, deterioration and shortage costs. The purchase cost of the retailer is

$$PC_R = p_R Q_R \tag{3.19}$$

Total inventory carrying cost (ICC_R) during the retailer's inventory period is the sum of the carrying cost at the RW (ICC_{RW}) and at the OW (ICC_{OW}):

$$ICC_R = ICC_{RW} + ICC_{OW} \tag{3.20}$$

$$\begin{aligned}
\text{Carrying Cost at the RW} &= ICC_{RW} = h_r \int_0^{t_r} I_r(t)dt = \frac{h_r}{\beta} D_{RW} \\
\text{Carrying Cost at the OW} &= ICC_{OW} = h_o \int_0^{t_r} I_o(t)dt + h_o \int_{t_r}^{t_o} I_o(t)dt = \frac{h_o}{\alpha} D_{OW}
\end{aligned}$$

Deterioration cost (DC_R) of the retailer includes both the deterioration at the RW and the OW:

$$DC_R = DC_{RW} + DC_{OW} \tag{3.21}$$

$$\begin{aligned}
\text{Deterioration cost at the RW} = DC_{RW} = d_R D_{RW} \\
\text{Deterioration cost at the OW} = DC_{OW} = d_R D_{OW}
\end{aligned}$$

During the shortage period, the demand is partially backlogged. There are two different types of shortage cost; one is based on per unit for the lost sale and the second is for the backlogged demand which is per unit per unit of time.

$$\begin{aligned}
SC_R &= c_{sf} \int_{t_o}^{T_R} \{(1-\delta)d\rho\}dt + c_{sv} \int_{t_o}^{T_R} \delta d\rho(t - t_o)dt \\
&= c_{sf}(1-\delta)d\rho t_s + \frac{1}{2} c_{sv} \delta d\rho t_s^2 \qquad (3.22)
\end{aligned}$$

Hence, the total cost per unit time of the retailer:

$$TC_R = \frac{1}{T_R}(A_R + PC_R + ICC_R + DC_R + SC_R) \tag{3.23}$$

With the above costs, the retailer spends some promotional cost (PrC) to increase the demand as follows:

$$PrC = g(\rho - 1)^2 d^m \tag{3.24}$$

where, g and m are the parameters so chosen to best fit the promotional cost. Using (3.18), (3.23) and (3.24), the total profit per unit time of the retailer (TP_R) is

$$TP_R = TS_R - TC_R - PrC \tag{3.25}$$

3.2.2.4 Wholesaler's Profit

Based on (3.16), the amount of the deterioration in each interval can be calculated as follows:

Time Period	Deterioration
$[(k-1)T_R, kT_R]$	0
$[(k-2)T_R, (k-1)T_R]$	$Q_R(e^{\gamma T_R} - 1)$
$[(k-3)T_R, (k-2)T_R]$	$Q_R(e^{\gamma T_R} - 1)(1 + e^{\gamma T_R})$
$[(k-4)T_R, (k-3)T_R]$	$Q_R(e^{\gamma T_R} - 1)(1 + e^{\gamma T_R} + e^{2\gamma T_R})$
...	...
$[(i-1)T_R, iT_R]$ (i-th interval)	$Q_R(e^{\gamma T_R} - 1) \sum_{m=i+1}^{k} e^{(k-m)\gamma T_R}$

Therefore, the total deteriorated units of the wholesaler (D_W) during T_W can be obtained as:

$$D_W = Q_R(e^{\gamma T_R} - 1)\sum_{i=1}^{k-1}\sum_{m=i+1}^{k} e^{(k-m)\gamma T_R} = Q_R\left(\frac{e^{k\gamma T_R} - 1}{e^{\gamma T_R} - 1} - k\right) \tag{3.26}$$

Using (3.12) and (3.26), the wholesaler's order quantity can be calculated as follows:

$$Q_W = kQ_R + Q_R\left(\frac{e^{k\gamma T_R} - 1}{e^{\gamma T_R} - 1} - k\right) = Q_R\left(\frac{e^{k\gamma T_R} - 1}{e^{\gamma T_R} - 1}\right) \tag{3.27}$$

Hence, the total selling price per unit time of the wholesaler:

$$TS_W = \frac{(Q_W - D_W)s_W}{T_W} \tag{3.28}$$

The wholesaler has the following costs: ordering cost (A_W), purchase, carrying and deterioration costs. The purchase cost of the wholesaler is

$$PC_W = p_W Q_W \tag{3.29}$$

Inventory carrying cost of the wholesaler during the i-th interval is

$$\int_{(i-1)T_R}^{iT_R} h_W I_W^i(t)dt = \frac{h_W Q_R}{\gamma}(e^{\gamma T_R} - 1) \sum_{m=i+1}^{k} e^{(k-m)\gamma T_R}$$

Hence, the inventory carrying cost of the wholesaler (ICC_W) during one inventory period (consider that there is no carrying cost during k-th interval) is

$$\begin{aligned} ICC_W &= \frac{h_W Q_R}{\gamma}(e^{\gamma T_R} - 1) \sum_{i=1}^{k-1} \sum_{m=i+1}^{k} e^{(k-m)\gamma T_R} \\ &= \frac{h_W Q_R}{\gamma}\Big(\frac{e^{k\gamma T_R} - 1}{e^{\gamma T_R} - 1} - k\Big) = \frac{h_W}{\gamma} D_W \end{aligned} \tag{3.30}$$

The total deterioration cost of the wholesaler (DC_W) during T_W is

$$DC_W = d_W D_W \tag{3.31}$$

Hence, the total cost per unit time of the wholesaler:

$$TC_W = \frac{1}{T_W}(A_W + PC_W + ICC_W + DC_W) \tag{3.32}$$

Using (3.28) and (3.32), the total profit per unit time of the wholesaler (TP_W) is

$$TP_W = TS_W - TC_W \tag{3.33}$$

According to the above discussion, two cases may arise:

- The wholesaler does not share any part of the promotional cost. In this case, retailer is the primary decision maker. So inventory decisions are made by the retailer first, i.e., the retailer will fix his/her marketing decision to maximise his/her profit only. Depending upon the retailer's decision, the wholesaler will fix his/her marketing decision, i.e., here the retailer is the leader and the wholesaler is the follower.

- The wholesaler shares a compromise part F of the promotional cost; i.e., the wholesaler pays $Fg(\rho-1)^2d^m$ and the retailer pays the remaining part $(1-F)g(\rho-1)^2d^m$ of the promotional cost. In this case, inventory decisions are made jointly by the retailer and the wholesaler, i.e., joint profit of the retailer and the wholesaler is maximized to find marketing decision.

These phenomena are termed as *non-coordination scenario* and *coordination scenario* respectively. These two scenarios are discussed separately.

3.2.2.5 Non-Coordination Scenario (NCS)

In this scenario the retailer spends the total amount of the promotional cost and hence he/she is the primary decision maker. Goal of the retailer is to maximize the profit function TP_R, which is a function of t_r, t_s and ρ. So the problem of the retailer mathematically takes the following form:

$$\left.\begin{array}{l}\text{Determine } t_r\,, t_s \text{ and } \rho\\ \text{to maximize } \;\; TP_R(t_r, t_s, \rho)\\ \text{subject to } \; t_r, t_s \geq 0;\;\; \rho \geq 1\end{array}\right\} \tag{3.34}$$

Depending upon the decision of the retailer, the wholesaler tries to improve his/her profit. So in this scenario, the problem of the wholesaler mathematically takes the following form:

$$\left.\begin{array}{l}\text{Determine } k\\ \text{to maximize } \;\; TP_W(t_r, t_s, \rho, k)\\ \text{where, } t_r\,, t_s \text{ and } \rho \text{ are determined by the retailer}\\ \qquad\quad \text{and } k > 0 \text{ is an integer.}\end{array}\right\} \tag{3.35}$$

To solve these single objective optimisation problems, here a heuristic algorithm PSO is implemented and tested (cf. § 2.2.2.1). Problems are solved using this PSO. To check the validity of the results obtained using PSO, the problems are also solved following GRG approach using LINGO 14.0 software.

3.2.2.6 Coordination Scenario (CS)

In this scenario, the wholesaler likes to take part in the joint marketing decision with the retailer to improve his/her profit. For this purpose he/she agrees to pay a compromise part (F) of the promotional cost spend by the retailer. For this contribution of the promotional cost by the wholesaler, the contribution of the retailer towards the promotional cost reduces with the same amount. Then the retailer's profit becomes

$$TP_R^F = TS_R - TC_R - (1-F)g(\rho-1)^2 d^m \tag{3.36}$$

The wholesaler's profit reduces to

$$TP_W^F = TS_W - TC_W - Fg(\rho-1)^2 d^m \tag{3.37}$$

As the retailer and the wholesaler both have the same power to take part in the joint marketing decision, here the joint profit of the retailer and the wholesaler (TP) is to be maximized and TP is a function of t_r, t_s, ρ, k and is given by

$$TP = TP_R^F + TP_W^F \tag{3.38}$$

The joint optimal decision will be acceptable to the retailer as well as the wholesaler if the decision improves their individual profits, i.e., the retailer's and the wholesaler's profits under the NCS are viewed as the lower bounds for the model under the coordination scenario. Let $TP_{R'}$ and $TP_{W'}$ be the profits of the retailer and the wholesaler respectively in NCS. So the profit of the retailer in this scenario, i.e., in CS, will be better than the NCS, if

$$\begin{aligned}
& \text{(Retailer's profit in CS)} - \text{(Retailer's profit in NCS)} \geq 0 \\
\text{i.e., if} \quad & TP_R^F(t_r, t_s, \rho) - TP_{R'} \geq 0 \\
\text{i.e., if} \quad & \{TS_R - TC_R - (1-F)PrC\} - TP_{R'} \geq 0 \\
\text{i.e., if} \quad & F.PrC \geq TP_{R'} - (TS_R - TC_R - PrC) \\
\text{i.e., if} \quad & F \geq [TP_{R'} - (TS_R - TC_R - PrC)]/PrC = F_{min} \text{ (say)}
\end{aligned}$$

Again, the profit of the wholesaler in this scenario, i.e., in CS, will be better than the NCS, if

$$\begin{aligned}
&\text{(Wholesaler's profit in CS)} - \text{(Wholesaler's profit in NCS)} \geq 0\\
\text{i.e., if}\quad & TP_W^F(t_r, t_s, \rho, k) - TP_{W'} \geq 0\\
\text{i.e., if}\quad & \{TS_W - TC_W - FPrC\} - TP_{W'} \geq 0\\
\text{i.e., if}\quad & F \leq [(TS_W - TC_W) - TP_{W'}]/PrC = F_{max} \text{ (say)}
\end{aligned}$$

From the above discussion, the following proposition follows.

Proposition 3.1. If $F_{min} < F < F_{max}$, then the profits for both the parties (the retailer and the wholesaler) increase in the CS than the NCS, where

$$\begin{aligned}
F_{min} &= [TP_{R'} - (TS_R - TC_R - PrC)]/PrC\\
\text{and } F_{max} &= [(TS_W - TC_W) - TP_{W'}]/PrC
\end{aligned}$$

So for a compromise value of $F \in (F_{min}\,, F_{max})$, the problem in this scenario mathematically takes the following form:

$$\left.\begin{array}{l}
\text{Determine } t_r\,, t_s, \rho \text{ and } k\\
\text{to maximize }\; TP(t_r, t_s, \rho, k)\\
\text{subject to } t_r, t_s \geq 0;\; \rho \geq 1;\; k \text{ is an integer.}
\end{array}\right\} \tag{3.39}$$

3.2.2.7 Fuzzy Model

It is discussed in the introduction section that in real life most of the inventory parameters are imprecise in nature and can be represented by fuzzy numbers. When some of the inventory parameters are fuzzy in nature, the model reduces to a fuzzy model. Normally set up cost, holding cost etc., are imprecise in nature. In this model, the set up costs A_R, A_W, holding costs h_r, h_o, h_W and the constant g of the promotional cost are considered as fuzzy numbers $\tilde{A}_R$, $\tilde{A}_W$, $\tilde{h}_r$, $\tilde{h}_o$, $\tilde{h}_W$, $\tilde{g}$ respectively, and hence the profits in both the scenarios become imprecise in nature and are presented below.

Fuzzy Model in Non-Coordination Scenario: According to the above assumptions in this case individual profits of the retailer, the wholesaler and their joint profit are transformed to the fuzzy numbers $\widetilde{TP}_R$, $\widetilde{TP}_W$, $\widetilde{TP}$ respectively and are represented by

$$\begin{aligned}
\widetilde{TP}_R &= TS_R - \widetilde{TC}_R - \widetilde{PrC} \\
\widetilde{TP}_W &= TS_W - \widetilde{TC}_W \\
\widetilde{TP} &= \widetilde{TP}_R + \widetilde{TP}_W
\end{aligned}$$

$$\begin{aligned}
\text{where,} \qquad \widetilde{TC}_R &= \frac{1}{T_R}(\widetilde{A}_R + PC_R + \widetilde{ICC}_R + DC_R + SC_R) \\
\widetilde{PrC} &= \tilde{g}(\rho-1)^2 d^m \\
\widetilde{TC}_W &= \frac{1}{T_W}(\widetilde{A}_W + PC_W + \widetilde{ICC}_W + DC_W) \\
\text{where,} \qquad \widetilde{ICC}_R &= \frac{\tilde{h}_r}{\beta} D_{RW} + \frac{\tilde{h}_o}{\alpha} D_{OW} \\
\widetilde{ICC}_W &= \frac{\tilde{h}_W}{\gamma} D_W
\end{aligned}$$

Considering the fuzzy numbers $\tilde{A}_R$, $\tilde{A}_W$, $\tilde{h}_r$, $\tilde{h}_o$, $\tilde{h}_W$, $\tilde{g}$ as triangular fuzzy numbers (TFNs) $(A_{R1}, A_{R2},\ A_{R3})$, (A_{W1}, A_{W2}, A_{W3}), (h_{r1}, h_{r2}, h_{r3}), (h_{o1}, h_{o2}, h_{o3}), (h_{W1}, h_{W2}, h_{W3}), (g_1, g_2, g_3) respectively, the fuzzy numbers $\widetilde{TP}_R$, $\widetilde{TP}_W$, $\widetilde{TP}$ becomes $(TP_{R1}, TP_{R2}, TP_{R3})$, $(TP_{W1}, TP_{W2}, TP_{W3})$, (TP_1, TP_2, TP_3) respectively, where for $i = 1, 2, 3$

$$\begin{aligned}
TP_{Ri} &= TS_R - TC_{R(4-i)} - PrC_{4-i} \\
TP_{Wi} &= TS_W - TC_{W(4-i)} \\
TP_i &= TP_{Ri} + TP_{Wi}
\end{aligned}$$

$$\begin{aligned}
\text{where,} \qquad TC_{Ri} &= \frac{1}{T_R}(A_{Ri} + PC_R + ICC_{Ri} + DC_R + SC_R) \\
PrC_i &= g_i(\rho-1)^2 d^m \\
TC_{Wi} &= \frac{1}{T_W}(A_{Wi} + PC_W + ICC_{Wi} + DC_W) \\
\text{where,} \qquad ICC_{Ri} &= \frac{h_{ri}}{\beta} D_{RW} + \frac{h_{oi}}{\alpha} D_{OW} \\
ICC_{Wi} &= \frac{h_{Wi}}{\gamma} D_W
\end{aligned}$$

In this scenario, the retailer spends the total amount of promotional cost and hence he/she is the primary decision maker. Goal of the retailer is to maximize the profit function $\widetilde{TP}_R$, which is a function of t_r, t_s and ρ. So the problem of the retailer mathematically takes the following form:

$$\left.\begin{array}{l}\text{Maximize } \widetilde{TP}_R = (TP_{R1}, TP_{R2}, TP_{R3}) \\ \text{subject to } t_r, t_s \geq 0;\ \rho \geq 1\end{array}\right\} \tag{3.40}$$

Depending upon the decision of the retailer, the wholesaler likes to improve his/her profit. So in this scenario, the problem of the wholesaler mathematically takes the following form:

$$\left.\begin{array}{l}\text{Maximize } \widetilde{TP}_W = (TP_{W1}, TP_{W2}, TP_{W3}) \\ \text{where, } t_r,\ t_s \text{ and } \rho \text{ are determined by the retailer} \\ \text{and } k > 0 \text{ is an integer.}\end{array}\right\} \tag{3.41}$$

The problems are solved using proposed PSO (cf. § 2.2.2.1) where comparisons of the objectives are made by the credibility measure approach of fuzzy events. Let $\widetilde{TP}_{Ra}$, $\widetilde{TP}_{Rb}$ be the two objectives corresponding to two solutions X_a, X_b respectively. Then the credibility measure approach of comparison of two solutions is given below:

- According to this approach X_a dominates X_b if the credibility measure (Cr) of the fuzzy event $(\widetilde{TP}_{Ra} > \widetilde{TP}_{Rb})$ is grater than 0.5, i.e., if $\text{Cr}(\widetilde{TP}_{Ra} > \widetilde{TP}_{Rb}) > 0.5$ (cf. Lemma 2.4). In this approach, no crisp equivalent of the fuzzy numbers are used to find marketing decisions. This is a valid fuzzy comparison operation as $Cr(\tilde{A} > \tilde{B}) + Cr(\tilde{A} \leq \tilde{B}) = 1$ [107].

Fuzzy Model in Coordination Scenario: For the coordination scenario, the individual profits and the total profit as fuzzy numbers are represented by

$$\begin{aligned}\widetilde{TP}_R^F &= TS_R - \widetilde{TC}_R - (1-F).\widetilde{PrC} \\ \widetilde{TP}_W^F &= TS_W - \widetilde{TC}_W - F.\widetilde{PrC} \\ \widetilde{TP} &= \widetilde{TP}_R^F + \widetilde{TP}_W^F\end{aligned}$$

As discussed in the CS of crisp model, the wholesaler bears a compromise part (F) of the promotional cost spend by the retailer to take joint marketing decision with the retailer. The joint optimal decision will be acceptable to the retailer as well as the wholesaler if the decision improves their individual profits, i.e., the retailer's and the wholesaler's profits under the NCS are viewed as the lower bounds for the model under the coordination scenario. Let $\widetilde{TP}_{R'}$ and $\widetilde{TP}_{W'}$ be the profits of the retailer and the wholesaler respectively in NCS. If a proper value of F is chosen which improves the profits of both the parties, then for that chosen value of F, they will take joint marketing decision for the benefit of them. Now according to Liu [107], the profit of the retailer in this scenario, i.e., in CS, will be better than the NCS, if

$$\begin{aligned}
& Cr(\text{Retailer's Profit in CS} \geq \text{Retailer's Profit in NCS}) > 0.5 \\
\text{i.e., if} \quad & Cr(\widetilde{TP}^F_R(t_r, t_s, \rho) \geq \widetilde{TP}_{R'}) > 0.5 \\
& \text{where, } \widetilde{TP}^F_R = (TP^F_{R1}, TP^F_{R2}, TP^F_{R3}) \text{ and } \widetilde{TP}_{R'} = (TP_{R'1}, TP_{R'2}, TP_{R'3}) \\
\text{i.e., if} \quad & TP^F_{R2} \geq TP_{R'2} \text{ [cf. Lemma 2.5]} \\
\text{i.e., if} \quad & \{TS_R - TC_{R2} - (1-F)PrC_2\} - TP_{R'2} \geq 0 \\
\text{i.e., if} \quad & F \geq [TP_{R'2} - \{TS_R - TC_{R2} - PrC_2\}]/PrC_2 = F_{min} \text{ (say)}
\end{aligned}$$

Also, the profit of the wholesaler in this scenario, i.e., in CS, will be better than the NCS, if

$$\begin{aligned}
& Cr(\text{Wholesaler's Profit in CS} \geq \text{Wholesaler's Profit in NCS}) > 0.5 \\
\text{i.e., if} \quad & Cr(\widetilde{TP}^F_W(t_r, t_s, \rho, k) \geq \widetilde{TP}_{W'}) > 0.5 \\
\text{where,} \quad & \widetilde{TP}^F_W = (TP^F_{W1}, TP^F_{W2}, TP^F_{W3}) \text{ and } \widetilde{TP}_{W'} = (TP_{W'1}, TP_{W'2}, TP_{W'3}) \\
\text{i.e., if} \quad & TP^F_{W2} \geq TP_{W'2} \text{ [cf. Lemma 2.5]} \\
\text{i.e., if} \quad & \{TS_W - TC_{W2} - FPrC_2\} - TP_{W'2} \geq 0 \\
\text{i.e., if} \quad & F \leq [\{TS_W - TC_{W2}\} - TP_{W'2}]/PrC_2 = F_{max} \text{ (say)}
\end{aligned}$$

From the above discussion, the following proposition follows.

Proposition 3.2. In fuzzy environment, if $F_{min} < F < F_{max}$, then the profits for both the parties (the retailer and the wholesaler) increase in the CS than the NCS, where

$$\begin{aligned}
F_{min} &= [TP_{R'2} - (TS_R - TC_{R2} - PrC_2)]/PrC_2 \\
\text{and } F_{max} &= [(TS_W - TC_{W2}) - TP_{W'2}]/PrC_2
\end{aligned}$$

When the assumed fuzzy parameters reduces to the crisp parameters then $TP_{R1} = TP_{R2} = TP_{R3} = TP_R$, $TP_{W1} = TP_{W2} = TP_{W3} = TP_W$, $TP_{R'1} = TP_{R'2} =$

$TP_{R'3} = TP_{R'}$, $TP_{W'1} = TP_{W'2} = TP_{W'3} = TP_{W'}$, and then clearly the Proposition 3.2 reduces to the Proposition 3.1, i.e., Proposition 3.1 is a special case of Proposition 3.2.

So for a compromise value of $F \in (F_{min}, F_{max})$, the problem in this scenario mathematically takes the following form:

$$\left.\begin{array}{l} \text{Maximize } \widetilde{TP} = (TP_1, TP_2, TP_3) \\ \text{subject to } t_r, t_s \geq 0;\ \rho \geq 1;\ k \text{ is an integer} \end{array}\right\} \tag{3.42}$$

The problem is solved using proposed PSO (cf. § 2.2.2.1), where comparisons of the objectives are made using Credibility Measure approach, which is discussed earlier.

3.2.2.8 Rough Model

It is discussed in the introduction section that another tool to represent the imprecise parameters is rough set. In this model, set up costs A_R, A_W, holding costs h_r, h_o, h_W and the constant g of the promotional cost are considered as rough variables $\check{A}_R$, $\check{A}_W$, $\check{h}_r$, $\check{h}_o$, $\check{h}_W$, $\check{g}$ respectively. Then profits in both the scenario become rough in nature.

Rough Model in Non-Coordination Scenario: According to the above assumptions in this case, the individual profits and the total profit of the retailer and the wholesaler are reduces to rough variables and are represented by

$$\begin{aligned} T\check{P}_R &= TS_R - T\check{C}_R - P\check{r}C \\ T\check{P}_W &= TS_W - T\check{C}_W \\ T\check{P} &= T\check{P}_R + T\check{P}_W \end{aligned}$$

$$\begin{aligned} \text{where,} \quad T\check{C}_R &= \frac{1}{T_R}(\check{A}_R + PC_R + I\check{C}C_R + DC_R + SC_R) \\ P\check{r}C &= \check{g}(\rho - 1)^2 d^m \\ T\check{C}_W &= \frac{1}{T_W}(\check{A}_W + PC_W + I\check{C}C_W + DC_W) \\ \text{where,} \quad I\check{C}C_R &= \frac{\check{h}_r}{\beta} D_{RW} + \frac{\check{h}_o}{\alpha} D_{OW} \\ I\check{C}C_W &= \frac{\check{h}_W}{\gamma} D_W \end{aligned}$$

Considering the rough variables $\check{A}_R$, $\check{A}_W$, $\check{h}_r$, $\check{h}_o$, $\check{h}_W$, $\check{g}$ as $([A_{R1}, A_{R2}][A_{R3}, A_{R4}])$, where $A_{R3} \leq A_{R1} \leq A_{R2} \leq A_{R4}$; $([A_{W1}, A_{W2}][A_{W3}, A_{W4}])$, where $A_{W3} \leq A_{W1} \leq A_{W2} \leq A_{W4}$; $([h_{r1}, h_{r2}][h_{r3}, h_{r4}])$, where $h_{r3} \leq h_{r1} \leq h_{r2} \leq h_{r4}$; $([h_{o1}, h_{o2}][h_{o3}, h_{o4}])$, where $h_{o3} \leq h_{o1} \leq h_{o2} \leq h_{o4}$; $([h_{W1}, h_{W2}][h_{W3},\ h_{W4}])$, where $h_{W3} \leq h_{W1} \leq h_{W2} \leq h_{W4}$; $([g_1, g_2][g_3, g_4])$, where $g_3 \leq g_1 \leq g_2 \leq g_4$ respectively, the rough variables $\check{TP}_R$, $\check{TP}_W$, $\check{TP}$ becomes $([TP_{R1}, TP_{R2}][TP_{R3}, TP_{R4}])$, $([TP_{W1}, TP_{W2}][TP_{W3}, TP_{W4}])$, $([TP_1,\ TP_2][TP_3, TP_4])$ respectively, where

$$\begin{aligned}
\text{For } i = 1,2 \qquad TP_{Ri} &= TS_R - TC_{R(3-i)} - PrC_{3-i} \\
TP_{Wi} &= TS_W - TC_{W(3-i)} \\
TP_i &= TP_{Ri} + TP_{Wi} \\
\text{For } i = 3,4 \qquad TP_{Ri} &= TS_R - TC_{R(7-i)} - PrC_{7-i} \\
TP_{Wi} &= TS_W - TC_{W(7-i)} \\
TP_i &= TP_{Ri} + TP_{Wi}
\end{aligned}$$

and for $i = 1,2,3,4$ the following relations hold

$$\begin{aligned}
TC_{Ri} &= \frac{1}{T_R}(A_{Ri} + PC_R + ICC_{Ri} + DC_R + SC_R) \\
PrC_i &= g_i(\rho - 1)^2 d^m \\
TC_{Wi} &= \frac{1}{T_W}(A_{Wi} + PC_W + ICC_{Wi} + DC_W) \\
\text{where,} \qquad ICC_{Ri} &= \frac{h_{ri}}{\beta} D_{RW} + \frac{h_{oi}}{\alpha} D_{OW} \\
ICC_{Wi} &= \frac{h_{Wi}}{\gamma} D_W
\end{aligned}$$

As the retailer is the leader decision maker and the supplier is the follower in NCS, so in this case the problem reduces to

$$\left.\begin{array}{l}
\text{Maximize } \ \check{TP}_R = ([TP_{R1}, TP_{R2}][TP_{R3}, TP_{R4}]) \\
\text{subject to } \ t_r, t_s \geq 0;\ \rho \geq 1
\end{array}\right\} \tag{3.43}$$

Depending upon the decision of the retailer, the wholesaler likes to improve his/her profit. So in this scenario, the problem of the wholesaler mathematically takes the following form:

$$\left.\begin{array}{l}\text{Maximize} \quad \check{TP}_W = ([TP_{W1}, TP_{W2}][TP_{W3}, TP_{W4}]) \\ \text{where, } t_r,\, t_s \text{ and } \rho \text{ are determined by the retailer} \\ \text{and } k > 0 \text{ is an integer.}\end{array}\right\} \tag{3.44}$$

The problems are solved using proposed PSO (cf. § 2.2.2.1), where comparisons of the objectives are made by the trust measure approach of rough events. Let $\check{TP}_{Ra}$, $\check{TP}_{Rb}$ be the two objectives corresponding to two solutions X_a, X_b respectively. Then the approach of comparison of two solutions using trust measure approach is given below:

- According to this approach X_a dominates X_b, if the trust measure (Tr) of the rough event ($\check{TP}_{Ra} > \check{TP}_{Rb}$) greater than 0.5, i.e., if $Tr(\check{TP}_{Ra} > \check{TP}_{Rb}) > 0.5$ (cf. Lemma 2.8). In this approach, no crisp equivalent of rough variables are used to find the marketing decisions. This is a valid rough comparison operation as $Tr(\check{A} > \check{B}) + Tr(\check{A} \leq \check{B}) = 1$ [107].

Rough Model in Coordination Scenario: For the coordination scenario, the individual profits and the total profit as rough variables are represented by

$$\begin{aligned}\check{TP}_R^F &= TS_R - \check{TC}_R - (1-F).\check{PrC}\\ \check{TP}_W^F &= TS_W - \check{TC}_W - F.\check{PrC}\\ \check{TP} &= \check{TP}_R^F + \check{TP}_W^F\end{aligned}$$

In the crisp and fuzzy model, it is established that a compromise value of F can be chosen which improves the gain of both the parties in the CS, i.e., using joint decision. In this case also, the following proposition ensures the existence of a feasible region of F for which profit of both the parties improves in CS than NCS.

Let $\check{TP}_{R'}$ and $\check{TP}_{W'}$ be the profits of the retailer and the wholesaler respectively in NCS. Now according to Liu [107] and Pramanik et al. [150], the profit of the retailer in this scenario, i.e., in CS, will be better than the NCS, if

$$
\begin{aligned}
& Tr(\text{Retailer's Profit in CS} \geq \text{Retailer's Profit in NCS}) > 0.5 \\
\text{i.e., if} \quad & Tr(\check{TP}_R^F(t_r, t_s, \rho) \geq \check{TP}_{R'}) > 0.5 \\
& \text{where, } \check{TP}_R^F = ([TP_{R1}^F, TP_{R2}^F][TP_{R3}^F, TP_{R4}^F]) \\
& \text{ and } \check{TP}_{R'} = ([TP_{R'1}, TP_{R'2}][TP_{R'3}, TP_{R'4}]) \\
\text{i.e., if} \quad & TP_{Ri}^F \geq TP_{R'i} \text{ for } i = 1,2,3,4 \text{ [cf. Lemma 2.9]} \\
\text{i.e., if} \quad & \{TS_R - TC_{R(m-i)} - (1-F)PrC_{(m-i)}\} - TP_{R'i} \geq 0 \\
& \text{where, } m = 3 \text{ for } i = 1,2 \text{ and } m = 7 \text{ for } i = 3,4 \\
\text{i.e., if} \quad & F \geq [TP_{R'i} - \{TS_R - TC_{R(m-i)} - PrC_{(m-i)}\}]/PrC_{(m-i)} = F_{Ri} \text{ for } i = 1,2,3,4 \\
\text{i.e., if} \quad & F \geq Max\{F_{R1}, F_{R2}, F_{R3}, F_{R4}\} = F_{min} \text{ (say)}
\end{aligned}
$$

Also, the profit of the wholesaler in this scenario, i.e., in CS, will be better than the NCS, if

$$
\begin{aligned}
& Tr(\text{Wholesaler's Profit in CS} \geq \text{Wholesaler's Profit in NCS}) > 0.5 \\
\text{i.e., if} \quad & Tr(\check{TP}_W^F(t_r, t_s, \rho, k) \geq \check{TP}_{W'}) > 0.5 \\
& \text{where, } \check{TP}_W^F = ([TP_{W1}^F, TP_{W2}^F][TP_{W3}^F, TP_{W4}^F]) \\
& \text{ and } \check{TP}_{W'} = ([TP_{W'1}, TP_{W'2}][TP_{W'3}, TP_{W'4}]) \\
\text{i.e., if} \quad & TP_{Wi}^F \geq TP_{W'i} \text{ for } i = 1,2,3,4 \text{ [cf. Lemma 2.9]} \\
\text{i.e., if} \quad & \{TS_W - TC_{W(m-i)} - FPrC_{(m-i)}\} - TP_{W'i} \geq 0 \\
& \text{where, } m = 3 \text{ for } i = 1,2 \text{ and } m = 7 \text{ for } i = 3,4 \\
\text{i.e., if} \quad & F \leq [\{TS_W - TC_{W(m-i)}\} - TP_{W'i}]/PrC_{(m-i)} = F_{Wi} \text{ for } i = 1,2,3,4 \\
\text{i.e., if} \quad & F \leq Min\{F_{W1}, F_{W2}, F_{W3}, F_{W4}\} = F_{max} \text{ (say)}
\end{aligned}
$$

From the above discussion, the following proposition follows.

Proposition 3.3. In rough environment, if $F_{min} < F < F_{max}$, then the profits for both the parties (the retailer and the wholesaler) increase in the CS than the NCS, where

$$
\begin{aligned}
F_{min} &= Max\{F_{R1}, F_{R2}, F_{R3}, F_{R4}\} \\
\text{and } F_{max} &= Min\{F_{W1}, F_{W2}, F_{W3}, F_{W4}\}
\end{aligned}
$$

$$\begin{aligned} \text{where, } F_{Ri} &= [TP_{R'i} - \{TS_R - TC_{R(m-i)} - PrC_{(m-i)}\}]/PrC_{(4-i)} \\ \text{and } F_{Wi} &= [\{TS_W - TC_{W(m-i)}\} - TP_{W'i}]/PrC_{(m-i)} \\ & \text{where, } m = 3 \text{ for } i = 1,2 \text{ and } m = 7 \text{ for } i = 3,4 \end{aligned}$$

When the assumed rough parameters reduces to the crisp parameters, then $TP_{R1} = TP_{R2} = TP_{R3} = TP_{R4} = TP_R$, $TP_{W1} = TP_{W2} = TP_{W3} = TP_{W4} = TP_W$, $TP_{R'1} = TP_{R'2} = TP_{R'3} = TP_{R'4} = TP_{R'}$, $TP_{W'1} = TP_{W'2} = TP_{W'3} = TP_{W'4} = TP_{W'}$, and then clearly the Proposition 3.3 reduces to the Proposition 3.1, i.e., Proposition 3.1 is a special case of Proposition 3.3.

So for a compromise value of $F \in (F_{min}, F_{max})$, the problem in this scenario mathematically takes the following form:

$$\left.\begin{array}{l} \text{Maximize } \ \check{TP} = ([TP_1, TP_2][TP_3, TP_4]) \\ \text{subject to } t_r, t_s \geq 0;\ \rho \geq 1;\ k \text{ is an integer} \end{array}\right\} \tag{3.45}$$

The problem is solved using proposed PSO (cf. § 2.2.2.1), where comparisons of the objectives are made using Trust Measure approach, which is discussed earlier.

3.2.3 Numerical Illustration and Discussion

Real-life Problem-1: Nowadays, in the developing countries, like India, Bangladesh etc., number of middle income group is increasing very fast. Instead of traditional raw spices, they prefer ready-made crashed spices (in packets) for preparation of food as it saves labour and time for the family. In Kharagpur, West Bengal, India, there is a small wholesaler (may be considered as a retailer) who sales cookme crashed spices only in the market. The retailer purchases from big wholesaler in Kolkata, capital of West Bengal. In the market, there are a lot of competitors of cookme such as '*Sunrise*', '*Data*', '*Rupa*', '*Patanjali*', etc. Thus, to capture the demand of Kharagpur town, the retailer gives advertisement in electronic media, hoardings in important places etc. From his last few years' experiences, he observed that the demand increases with the promotional effort and amount of displayed units. The retailer fixes his/her marketing decision to increase the annual profit. The big wholesaler also desires to share a part of the promotional cost to make joint marketing decision with the retailer to increase his/her profit. Now, the retailer is in a dilemma whether to share the promotional cost with the wholesaler or not, though it is the fact that the profit increases with the promotional effort. On the part of the wholesaler also, whether it will be profitable to take part in the promotional effort from the business point of view. This model answers to these questions. The following example consists of the data set in appropriate units for one of the such spice and is used to illustrate the model.

Example 3.1. The data sets for the model in different environments are presented below:

Crisp model: $c = 0.2$, $d = 200$, $g = 1.1$, $m = 1.2$, $W = 200$, $\alpha = 0.05$, $\beta = 0.08$, $\gamma = 0.03$, $\delta = 0.85$, $A_R = 1500$, $A_W = 2500$, $p_W = 5$, $s_W = p_R = 8$, $s_R = 14$, $h_r = 1.2$, $h_o = 0.8$, $h_W = 0.3$, $d_R = 1$, $d_W = 1$, $c_{sf} = 15$, $c_{sv} = 0.8$.

Fuzzy model: $(A_{R1}, A_{R2}, A_{R3}) = (1400, 1500, 1600)$, $(A_{W1}, A_{W2}, A_{W3}) = (2400, 2500, 2600)$, $(h_{r1}, h_{r2}, h_{r3}) = (1.1, 1.2, 1.3)$, $(h_{o1}, h_{o2}, h_{o3}) = (0.7, 0.8, 0.9)$, $(h_{W1}, h_{W2}, h_{W3}) = (0.25, 0.3, 0.35)$, $(g_1, g_2, g_3) = (1.0, 1.1, 1.2)$. All other parametric values are same as the crisp model.

Rough model: $([A_{R1}, A_{R2}][A_{R3}, A_{R4}]) = ([1400, 1500][1350, 1600])$, $([A_{W1}, A_{W2}]\ [A_{W3}, A_{W4}]) = ([2500, 2600][2400, 2650])$, $([h_{r1}, h_{r2}][h_{r3}, h_{R4}]) = ([1.1, 1.2]\ [1.05, 1.25])$, $([h_{o1}, h_{o2}][h_{o3}, h_{o4}]) = ([0.8, 0.9][0.75, 0.95])$, $([h_{W1}, h_{W2}]\ [h_{W3}, h_{W4}]) = ([0.25, 0.3][0.2, 0.35])$, $([g_1, g_2][g_3, g_4]) = ([1.05, 1.15][1.0, 1.2])$. All other parametric values are same as the crisp model.

For the above set of parametric values, for crisp model, in NCS, initially TP_R is optimized to find optimum decision for the retailer and the optimum values of t_r, t_s, ρ are determined. For these values of t_r, t_s, ρ; TP_W is optimized to find optimum k for the wholesaler. Again in CS, the optimum results are obtained by optimizing TP. The value of F is taken as $F = \frac{1}{2}(F_{max} + F_{min})$. Results are obtained using both LINGO 14.0 software and PSO algorithm developed for this purpose and almost same results are found which are presented in Table 3.1. In PSO, the parametric study is made on k to optimize TP_W for NCS and to optimize TP for CS and these results are presented in Table 3.2. It is observed from Table 3.1 that the results of PSO are at least as good as the results of LINGO software. But LINGO software is not capable of solving fuzzy/rough model. Moreover, efficiency of the implemented proposed PSO in solving continuous optimisation problems is rigorously tested (cf. § 2.2.2.1). Due to this reason, in further study, PSO is only used to find results in different cases.

For Example 3.1, in crisp model, the total profit is optimized in CS due to

Table 3.1: Optimum Results for Example 3.1

Technique	Scenario	TP_R	TP_W	TP	t_r	t_s	ρ	k	T_R	T_W	F_{min}	F_{max}	F
GRG	NCS	568.8537	348.2102	917.0639	1.49	0.54	1.59	2	2.58	5.15	-	-	-
(LINGO)	CS	628.0926	407.4526	1035.5452	2.53	1.94	1.90	1	4.91	4.91	0.475	0.705	0.590
PSO	NCS	568.8541	348.1272	916.9813	1.49	0.54	1.59	2	2.58	5.15	-	-	-
	CS	628.0778	407.4680	1035.5459	2.53	1.95	1.90	1	4.91	4.91	0.475	0.705	0.590
ABC	NCS	568.8541	348.1582	917.0123	1.49	0.54	1.59	2	2.57	5.15	-	-	-
	CS	628.1179	407.4279	1035.5459	2.52	1.94	1.90	1	4.91	4.91	0.475	0.705	0.590

Table 3.2: Parametric study of k for Example 3.1 using PSO technique

Scenario	k	TP_R	TP_W	TP	t_r	t_s	ρ	T_R	T_W
NCS	1	568.85	89.70	658.55	1.49	0.54	1.59	2.57	2.57
	2	568.85	**348.13**	**916.98**	1.49	0.54	1.59	2.57	5.15
	3	568.85	270.69	839.55	1.49	0.54	1.59	2.57	7.72
	4	568.85	99.44	668.30	1.49	0.54	1.59	2.57	10.30
	5	568.85	-117.85	451.00	1.49	0.54	1.59	2.57	12.87
CS	**1**	-	-	**1035.55**	2.52	1.94	1.90	4.91	4.91
	2	-	-	1034.21	1.84	0.05	2.00	2.32	4.64
	3	-	-	983.30	1.46	0	2.00	1.90	5.71
	4	-	-	883.15	1.23	0	1.99	1.67	6.69
	5	-	-	767.30	1.06	0	1.97	1.51	7.57

Table 3.3: Values of TP_R, TP_W due to different F in CS for Example 3.1 using PSO technique

F	TP_R	TP_W	TP
0.47	**566.30**	469.25	1035.55
0.48	571.51	464.03	1035.55
0.54	602.42	433.12	1035.55
0.59	628.08	407.47	1035.55
0.64	653.91	381.63	1035.55
0.70	684.70	350.85	1035.55
0.71	689.73	**345.81**	1035.55

sharing of different portion (F) of promotional cost between F_{min} and F_{max} by the wholesaler and the profits of both the parties are tabulated in Table 3.3. For NCS, $TP_R = 568.85$ and $TP_W = 348.13$. Here it is observed that if $F = 0.47 < F_{min}$, where $F_{min} = 0.475$ then the retailer's profit in the CS (566.30) is less than that in the NCS (568.85). Also if $F = 0.71 > F_{max}$, where $F_{max} = 0.705$ then the wholesaler's profit in the CS (345.81) is less than that in the NCS (348.13). So it is found that for $F_{min} < F < F_{max}$ profit of both the parties increase to some extent. All these calculations are done using PSO technique and the value of k is determined by parametric study on k.

For Example 3.1, in crisp model, a sensitivity analysis of c and d are presented

TABLE 3.4: Sensitivity Analysis of c and d for Example 3.1 using PSO technique

			TP_R	TP_W	TP	t_r	t_s	ρ	k	F^*
Sensitivity Analysis of c	c=0.16	NCS	546.99	333.95	880.94	1.49	0.65	1.58	2	-
		CS	607.74	400.61	1008.35	2.52	2.06	1.89	1	0.59
	c=0.17	NCS	552.36	337.31	889.67	1.49	0.62	1.58	2	-
		CS	612.83	402.26	1015.09	2.52	2.03	1.90	1	0.59
	c=0.18	NCS	557.80	341.01	898.80	1.49	0.60	1.58	2	-
		CS	617.99	403.88	1021.87	2.52	2.00	1.90	1	0.59
	c=0.19	NCS	563.29	344.53	907.82	1.49	0.57	1.59	2	-
		CS	622.98	405.70	1028.69	2.52	1.97	1.90	1	0.59
	c=0.20	NCS	568.85	348.13	916.98	1.49	0.54	1.59	2	-
		CS	628.08	407.47	1035.55	2.53	1.94	1.90	1	0.59
	c=0.21	NCS	574.48	351.86	926.34	1.49	0.51	1.59	2	-
		CS	635.41	409.98	1045.39	1.84	0.02	2.01	2	0.30
	c=0.22	NCS	580.18	355.54	935.72	1.49	0.48	1.60	2	-
		CS	641.56	415.17	1056.73	1.84	0	2.01	2	0.30
Sensitivity Analysis of d	d=190	NCS	523.05	314.81	837.86	1.51	0.64	1.58	2	-
		CS	575.64	382.64	958.28	2.56	2.07	1.90	1	0.59
	d=195	NCS	545.83	331.45	877.27	1.50	0.59	1.59	2	-
		CS	601.76	395.04	996.80	2.54	2.00	1.90	1	0.59
	d=200	NCS	568.85	348.13	916.98	1.49	0.54	1.59	2	-
		CS	628.08	407.47	1035.55	2.53	1.94	1.90	1	0.59
	d=205	NCS	592.14	365.08	957.21	1.48	0.49	1.59	2	-
		CS	655.69	423.48	1079.17	1.83	0.01	2.01	2	0.30
	d=210	NCS	615.69	382.04	997.72	1.47	0.44	1.60	2	-
		CS	683.42	441.11	1124.53	1.82	0	2.01	2	0.30
	d=215	NCS	639.50	398.89	1038.39	1.47	0.39	1.60	2	-
		CS	712.05	458.02	1170.08	1.80	0	2.01	2	0.30
	d=220	NCS	663.59	416.16	1079.74	1.46	0.34	1.60	2	-
		CS	740.66	475.14	1215.80	1.79	0	2.01	2	0.30

* Values of F are supplied

in Table 3.4 following PSO technique. Here it is noticed that if c or d increases then in both the scenarios (NCS and CS) the individual profits as well as the total profit of the retailer and the wholesaler increase. All these observations agrees with reality. A graphical representation of the results obtained in these parametric studies are also presented in Figure 3.3 and Figure 3.4.

For fuzzy and rough models, the Example 3.1 is made using PSO technique following credibility measure and trust measure approach respectively and the results are presented in Table 3.5. In coordination scenario, the value of F is taken as 0.59 and in both the scenarios optimum k are obtained by parametric studies on k.

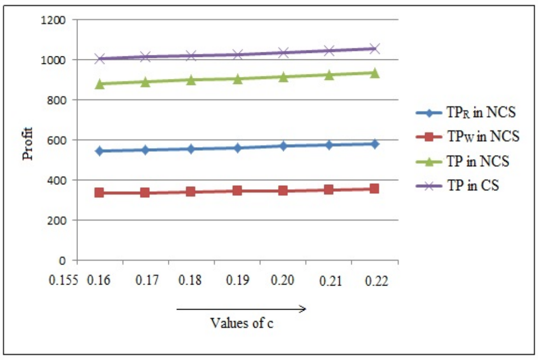

FIGURE 3.3: Graphical representation of the results of Example 3.1 of the parametric studies on c

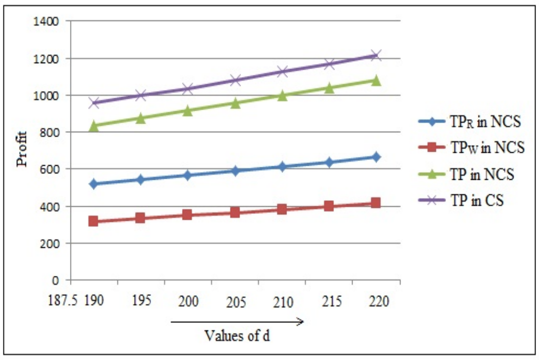

FIGURE 3.4: Graphical representation of the results of Example 3.1 of the parametric studies on d

TABLE 3.5: Results of Fuzzy and Rough model following PSO for Example 3.1

		NCS	CS
Fuzzy	$\widetilde{TP}_R$	(480.80, 568.85, 656.91)	(549.10, 628.08, 707.06)
	$\widetilde{TP}_W$	(305.05, 348.13, 391.21)	(359.52, 407.47, 455.41)
	$\widetilde{TP}$	(785.85, 916.98, 1048.12)	(908.63, 1035.55, 1162.46)
	t_r	1.49	2.53
	t_s	0.54	1.94
	ρ	1.59	1.90
	k	2	1
	F	-	0.59
Rough	$\check{TP}_R$	([544.25, 636.32][477.96, 682.35])	([602.66, 683.31][552.15, 723.63])
	$\check{TP}_W$	([331.87, 375.19][298.69, 418.50])	([377.92, 426.28][353.74, 460.65])
	$\check{TP}$	([876.12, 1011.50][776.65, 1100.85])	([980.58, 1109.59][905.88, 1184.28])
	t_r	1.49	2.57
	t_s	0.44	1.90
	ρ	1.60	1.91
	k	2	1
	F	-	0.59

Real-life Problem-2: In Haldia, West Bengal, India, there is a retailer (Sampa Fish Center) who sells fresh fishes by collecting fish from a supplier (Digha Fish Supplier), who supplies fish from Shankarpur fishing harbour, Digha, West Bengal, India. As availabilities of different types of fishes vary in different seasons, to keep the demand of his fish high, the retailer gives advertisements in different ways in a regular interval in the local area. The following example consists of the data set in appropriate units for one type of such fish and is used to illustrate the model.

Example 3.2. The data sets for the model in different environments are presented below:

TABLE 3.6: Optimum Results for Example 3.2

Technique	Scenario	TP_R	TP_W	TP	t_r	t_s	ρ	k	T_R	T_W	F_{min}	F_{max}	F
GRG	NCS	1228.2540	548.7954	1777.0494	1.42	0.78	1.80	3	2.64	7.91	-	-	-
(LINGO)	CS	1319.4982	640.0478	1959.5460	1.96	0.66	2.23	2	2.97	5.95	0.174	0.340	0.257
PSO	NCS	1228.2546	548.6689	1776.9236	1.42	0.78	1.80	3	2.64	7.91	-	-	-
	CS	1319.5983	639.9483	1959.5466	1.96	0.66	2.23	2	2.97	5.94	0.174	0.340	0.257
ABC	NCS	1228.2546	548.7019	1776.9565	1.42	0.78	1.80	3	2.64	7.91	-	-	-
	CS	1319.5483	639.9984	1959.5468	1.97	0.66	2.23	2	2.98	5.95	0.174	0.340	0.257

Crisp model: $c = 0.18$, $d = 200$, $g = 1.27$, $m = 1.2$, $W = 180$, $\alpha = 0.05$, $\beta = 0.07$, $\gamma = 0.02$, $\delta = 0.83$, $A_R = 1500$, $A_W = 4000$, $p_W = 6$, $s_W = p_R = 10$, $s_R = 18$, $h_r = 1.2$, $h_o = 1.0$, $h_W = 0.3$, $d_R = 0.9$, $d_W = 0.9$, $c_{sf} = 6$, $c_{sv} = 1.4$.
Fuzzy model: $(A_{R1}, A_{R2}, A_{R3}) = (1400, 1500, 1600)$, $(A_{W1}, A_{W2}, A_{W3}) = (3900, 4000, 4100)$, $(h_{r1}, h_{r2}, h_{r3}) = (1.1, 1.2, 1.3)$, $(h_{o1}, h_{o2}, h_{o3}) = (0.9, 1.0, 1.1)$, $(h_{W1}, h_{W2}, h_{W3}) = (0.25, 0.3, 0.35)$, $(g_1, g_2, g_3) = (1.25, 1.27, 1.3)$. All other parametric values are same as crisp model.
Rough model: $([A_{R1}, A_{R2}][A_{R3}, A_{R4}]) = ([1400, 1500][1350, 1600])$, $([A_{W1}, A_{W2}][A_{W3}, A_{W4}]) = ([4000, 4100][3900, 4150])$, $([h_{r1}, h_{r2}][h_{r3}, h_{R4}]) = ([1.1,\ 1.2]\ [1.05, 1.25])$, $([h_{o1}, h_{o2}][h_{o3}, h_{o4}]) = ([1.0,\ 1.1][0.95, 1.15])$, $([h_{W1}, h_{W2}]\ [h_{W3},\ h_{W4}]) = ([0.25, 0.3][0.2,\ 0.35])$, $([g_1, g_2][g_3, g_4]) = ([1.27, 1.3][1.25,\ 1.32])$. All other parametric values are same as crisp model.

For the above set of assumed parametric values, TP_R and TP are optimized for NCS and CS respectively and the optimum results obtained using LINGO 14.0 software and PSO technique are presented in Table 3.6. It is found that the results obtained following both the techniques are almost same. All other results of Example 3.2 are almost same as Example 3.1. Results of fuzzy and rough model are computed following PSO technique for Example 3.2 and are presented in Table 3.7.

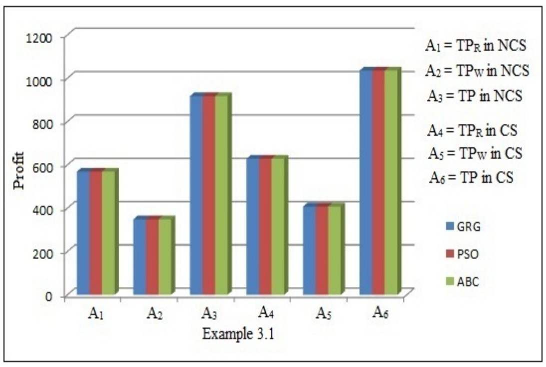

FIGURE 3.5: Comparison of the results obtained in Example 3.1 following different approaches

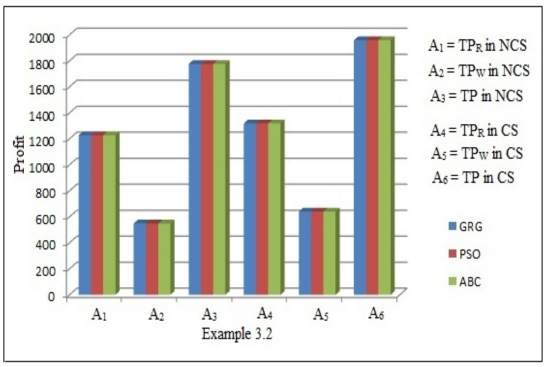

FIGURE 3.6: Comparison of the results obtained in Example 3.2 following different approaches

TABLE 3.7: Results of Fuzzy and Rough model following PSO for Example 3.2

		NCS	CS
Fuzzy	$\widetilde{TP}_R$	(1153.13, 1228.25, 1299.68)	(1225.31, 1322.81, 1413.89)
	$\widetilde{TP}_W$	(484.00, 548.67, 613.34)	(576.45, 636.74, 694.77)
	$\widetilde{TP}$	(1637.13, 1776.92, 1913.01)	(1801.75, 1959.55, 2108.67)
	t_r	1.42	1.96
	t_s	0.78	0.66
	ρ	1.80	2.23
	k	3	2
	F	-	0.26
Rough	$\check{TP}_R$	([1206.16, 1282.24][1147.04, 1322.12])	([1283.74, 1381.82][1214.89, 1434.02])
	$\check{TP}_W$	([537.57, 601.75][479.81, 665.93])	([620.69, 681.14][570.80, 739.35])
	$\check{TP}$	([1743.73, 1883.99][1626.85, 1988.04])	([1904.43, 2062.95][1785.69, 2173.38])
	t_r	1.42	2.01
	t_s	0.74	0.64
	ρ	1.80	2.22
	k	3	2
	F	-	0.26

3.2.3.1 ANOVA Test

To check the efficiency of the PSO algorithm for solving the model, here along with LINGO software, another heuristic algorithm, ABC is also used to solve the crisp models for both the examples. Results of different models using ABC are presented in Table 3.1 and Table 3.6 for Example 3.1 and Example 3.2 respectively. A pictorial representation of the results of both the examples following three different approaches (GRG, PSO, ABC) are presented in Figure 3.5 and Figure 3.6. From these figures, it is clear that the performances of three approaches are almost same for solving the model. A statistical test ANOVA [85] is also performed on the obtained results following the three approaches. To perform this test, here, the results obtained following these three approaches for six models $M1$, $M2$, $M3$, $M4$, $M5$, $M6$ are considered, where $M1$ = Values of TP_R in NCS for Example 3.1, $M2$ = Values of TP_W in NCS for Example 3.1, $M3$ = Values of TP in CS for Example 3.1, $M4$ = Values of TP_R in NCS for Example 3.2, $M5$ = Values of TP_W in NCS for Example 3.2, $M6$ = Values of TP in CS for Example 3.2 respectively, which are presented in Table 3.8. These three sets of results are considered as three samples ($J = 3$). Clearly size of each sample is $I = 6$. Critical value of the F-ratio is $F(J-1, J(I-1)) = F(2, 15) = 3.68$, for significance level 0.05. As three samples are almost same, calculated value of the F-ratio is 0 which is less than the critical

TABLE 3.8: Values for ANOVA test

Approach	Obtained optimum value					
	$M1$	$M2$	$M3$	$M4$	$M5$	$M6$
PSO	568.8541	348.1272	1035.5459	1228.2546	548.6689	1959.5466
Lingo	568.8537	348.2102	1035.5452	1228.2540	548.7954	1959.5460
ABC	568.8541	348.1582	1035.5459	1228.2546	548.7019	1959.5468

TABLE 3.9: Computational time and number of function evaluation in different approaches

Approach	[Computational Time (in second), No of function evolution]			
	To find $M1$, $M2$	To find $M3$	To find $M4$, $M5$	To find $M6$
PSO	[3.261, 1440]	[3.314, 1290]	[3.494, 1140]	[3.352, 1530]
Lingo	[< 1, -]	[< 1, -]	[< 1, -]	[< 1, -]
ABC	[3.709, 1660]	[3.626, 1340]	[4.013, 2140]	[3.702, 1940]

value $F(2, 15) = 3.68$. So there is no significant difference between these samples, i.e., performances of these approaches are almost same for solving the proposed model. But here PSO is used to solve the models as it takes less computational time as well as less function evaluations to find the marketing decision (cf. Table 3.9) relative to ABC.

3.3 Model 3.2: Two-Level Supply Chain of a Seasonal Deteriorating Item with Time, Price and Promotional Cost Dependent Demand Under Finite Time Horizon [1]

3.3.1 Assumptions and Notations

The following assumptions and notations are used in this study:

Assumptions:

- It is a finite time horizon EOQ model for constantly deteriorating items.
- Lead time is zero.
- Demand is time, price and promotional cost dependent.
- Shortages are allowed and partially backlogged.
- Lost sale cost is considered.
- Rate of replenishment is infinite.
- The promotional cost to boost demand is shared by both the wholesaler and the retailer.

Notation	**Meaning**
T_2	total time horizon.
T_1	time where the time-dependent demand is maximum.
n	number of cycles in retailer's inventory period.
m	number of retailer-cycles in one cycle of wholesaler's inventory period.
k	number of full cycles of wholesaler's inventory period.
m_1	number of retailer-cycles excess after completing k full cycles in wholesaler's inventory period.
t_1	length of each retailer-cycle (i.e., $t_1 = \frac{T_2}{n}$).

[1] This model has been published in **American Journal of Mathematical and Management Sciences, 2017, 36(4), 292-315**, *Taylor & Francis*, with title "*Two-Level Supply Chain of a Seasonal Deteriorating Item with Time, Price, and Promotional Cost Dependent Demand Under Finite Time Horizon*"

Notation	**Meaning**
s	duration of retailer-cycle for which inventory level is positive; i.e., $s = \lambda t_1$, where, $\lambda \in (0, 1)$.
t_s	duration of shortage period in a retailer-cycle; i.e., $t_s = t_1 - s$.
s_i	time at which retailer's inventory level of i-th cycle reaches to zero; i.e., $s_i = (i-1)t_1 + s$.
M	maximum time-dependent demand at time T_1.
ρ	retailer's promotional effort, $\rho \geq 1$.
α	deterioration rate at the retailer.
γ	deterioration rate at the wholesaler.
A_R/A_W	replenishment or ordering cost per order for the retailer/wholesaler.
m_k	mark-up of selling price for the retailer ($m_k \geq 1$).
p_R/p_W	purchasing price for the retailer/wholesaler per unit item.
s_R/s_W	selling price for the retailer/wholesaler per unit item ($p_R = s_W$ and $s_R = m_k p_R$).
h_R/h_W	unit holding cost per unit of time of the retailer/wholesaler.
c_s	unit shortage cost per unit of time for backlogged demand.
c_l	unit lost sale cost per unit of time.
$I_{R,i}(t)$	inventory level of the retailer during i-th cycle at time t of retailer's inventory period $[(i-1)t_1 \leq t \leq s_i]$.
$I_{S,i}(t)$	inventory level of the retailer during i-th cycle at the time of shortage $[s_i \leq t \leq it_1]$.
$I^r_{W,j}(t)$	inventory level of the wholesaler during j-th retailer-cycle of r-th wholesaler-cycle.
Q_R/Q_W	total order quantity for the retailer/wholesaler.
D_R/D_W	total deterioration of the retailer/wholesaler.
$D(t)$	demand rate at time t. The demand is assumed to be deterministic, time, price and promotional cost dependent and is of the form: $D(t) = \frac{A+Bt-Ct^2}{(m_k p_R)^\delta} = a + bt - ct^2$; where, $a = \frac{A}{(m_k p_R)^\delta}$, $b = \frac{B}{(m_k p_R)^\delta}$, $c = \frac{C}{(m_k p_R)^\delta}$ and $A > 0$ is constant, $B = CT_2$, $C = \frac{M\rho - A}{T_1(T_2 - T_1)}$ and δ is a parameter so chosen to best fit the demand function.
$B_i(t)$	backlogging rate for i-th retailer-cycle: $B_i(t) = \frac{1}{1+\beta(it_1 - t)}$; where $\beta > 0$ is constant.

Symbols ˜ is used on the top of some notations to indicate the fuzzy variable.

3.3.2 Mathematical Formulation of the Model

Here, a wholesaler-retailer-customer supply chain of a deteriorating seasonal item is considered where demand of the item, $D(t)$, increases with time at the beginning of the season, attains its maximum level, M, at a time T_1 and then it gradually decreases to off-season level, i.e., $D(0) = D(T_2)$, where, T_2 is the length of the season, which is also planning horizon. It is assumed that $D(t)$ is a parabolic function of time t $(0 \le t \le T_2)$. Retailer introduces a promotional cost $PrC = g(\rho - 1)^2 M^{\xi}$ to enhance the maximum demand, M, of the item to $M\rho$, where $\rho(\ge 1)$ is promotional effort [97]. Demand of the item also depends on the unit selling price $s_R = m_k p_R$, where p_R is unit purchase cost and $m_k(\ge 1)$ is a mark-up which is a decision variable. It is assumed that demand of the item at time t, is of the form $D(t) = \dfrac{A + Bt - Ct^2}{s_R^{\delta}}$, where δ is a parameter so chosen to best fit the demand function. According to the assumptions $D(t)$ takes the form $D(t) = a + bt - ct^2$, where $a = \dfrac{A}{(m_k p_R)^{\delta}}$, $b = \dfrac{B}{(m_k p_R)^{\delta}}$, $c = \dfrac{C}{(m_k p_R)^{\delta}}$ and $A > 0$ is constant, $B = CT_2$, $C = \dfrac{M\rho - A}{T_1(T_2 - T_1)}$.

3.3.2.1 Retailer's Inventory Level

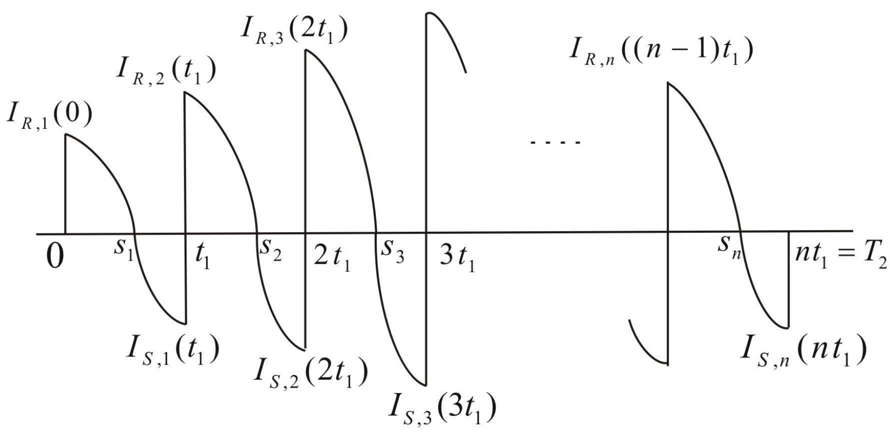

FIGURE 3.7: Inventory level of the retailer

Figure 3.7 illustrates the inventory level of the retailer. T_2 is the planning horizon and the demand of the item reaches maximum level at time T_1. There are n equal cycles of retailer and t_1 is the length of each cycle (i.e., $nt_1 = T_2$). For i-th cycle, let the inventory level reaches to zero at time s_i; where $s_i = (i-1)t_1 + s$ and $s = \lambda t_1$, $\lambda \in (0,1)$. In each cycle the system is out of stock during $[s_i, it_1]$ and the duration of shortage period is $t_s = t_1 - s$. Customers are assumed to be impatient and a fraction $B_i(t) = \dfrac{1}{1+\beta(it_1 - t)}$; of the demand during the stock-out period is backlogged and the remaining fraction $(1 - B_i(t))$ is lost (cf. Figure 3.8), where $\beta > 0$ is constant. The item deteriorates at a rate α at retailer's outlet. Rate of replenishment is assumed to be infinite, but, replenishment amount (order quantity) is finite.

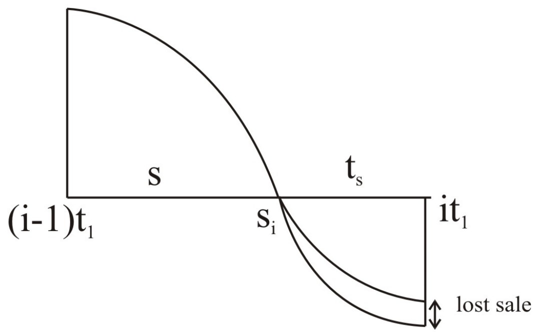

FIGURE 3.8: Retailer's i-th cycle

So the changes of inventory level of i-th cycle of the retailer $[I_{R,i}(t)]$ between $(i-1)t_1$ and s_i can be presented by the following differential equations:

$$\frac{dI_{R,i}(t)}{dt} = -D(t) - \alpha I_{R,i}(t) \tag{3.46}$$

for $(i-1)t_1 \le t \le s_i$; where, $s_i = (i-1)t_1 + s$, $s = \lambda t_1$ and $i = 1, 2, \ldots, n$.

From s_i to it_1 the system is out of stock and the shortage level $[I_{S,i}(t)]$ can be presented by

$$\frac{dI_{S,i}(t)}{dt} = -B_i(t)D(t) \tag{3.47}$$

for $s_i \le t \le it_1$ and $i = 1, 2, \ldots, n$, with boundary conditions

$$I_{R,i}(t) = I_{S,i}(t) = 0, \text{ at } t = s_i$$

Solving the differential equations (3.46) and (3.47), the inventory and shortage levels of the item in i-th cycle of the retailer are obtained as

$$\begin{aligned} I_{R,i}(t) = {} & \left(-\frac{a}{\alpha} + \frac{b}{\alpha^2} + \frac{2c}{\alpha^3}\right) - \left(\frac{b}{\alpha} + \frac{2c}{\alpha^2}\right)t + \frac{c}{\alpha}t^2 \\ & + \left[\frac{a + bs_i - cs_i^2}{\alpha} - \frac{b - 2cs_i}{\alpha^2} - \frac{2c}{\alpha^3}\right]e^{\alpha(s_i - t)} \end{aligned} \tag{3.48}$$

$$I_{S,i}(t) = -\frac{c}{2\beta}(t^2 - s_i^2) + \frac{1}{\beta}\left\{b - \frac{c(1+i\beta t_1)}{\beta}\right\}(t - s_i) + \frac{1}{\beta^2}\left\{a\beta + b(1+i\beta t_1) - \frac{c}{\beta}(1+i\beta t_1)^2\right\}log\left\{\frac{1+\beta(it_1 - t)}{1+\beta t_s}\right\} \quad (3.49)$$

The order quantity $Q_{R,i}$ of the retailer in i-th cycle is

$$Q_{R,i} = I_{R,i}((i-1)t_1) - I_{S,i-1}((i-1)t_1) \quad (3.50)$$

where, $I_{S,0}(0) = 0$ and $I_{R,n+1}(nt_1) = 0$; because for the 1st retailer's cycle, it is assumed that there is no shortages for previous cycle and similarly at the end of last retailer's cycle, only shortage quantity is ordered.

Hence, the total order quantity of the retailer is

$$Q_R = \sum_{i=1}^{n+1} Q_{R,i} \quad (3.51)$$

3.3.2.2 Wholesaler's Inventory Level

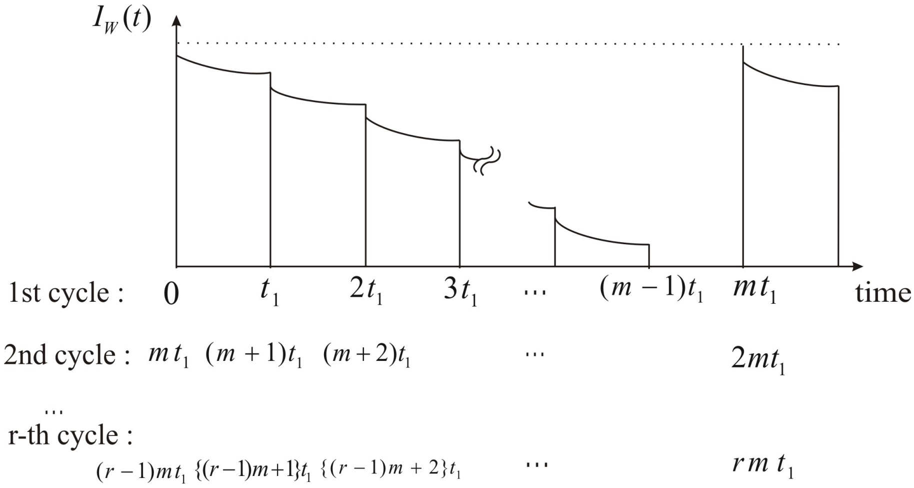

FIGURE 3.9: Inventory level of the wholesaler (excluding the last cycle)

Figure 3.9 illustrates the inventory level, $I_W(t)$, of the wholesaler. There are m retailer-cycles ($m < n$) in one cycle of the wholesaler. Wholesaler has k full cycles and one partial cycle during planning horizon $[0, T_2]$, where $k = [\frac{n}{m}]$. After completing k full cycles, the remaining number of retailer-cycles in the last wholesaler-cycle is m_1 (i.e., $m_1 = n - km$). γ is the rate of deterioration at the wholesaler's warehouse. The change in inventory level of the wholesaler during

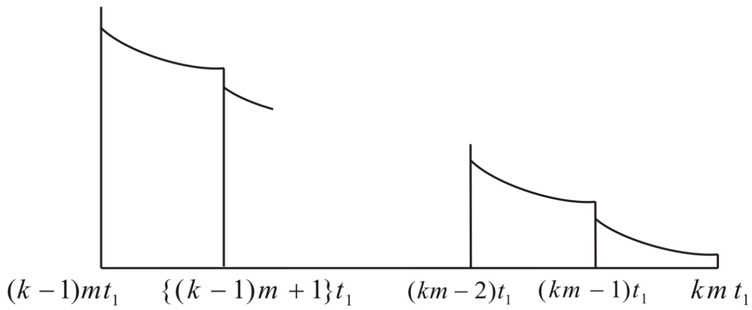

FIGURE 3.10: Inventory level of the last wholesaler's cycle (when n is divisible by m)

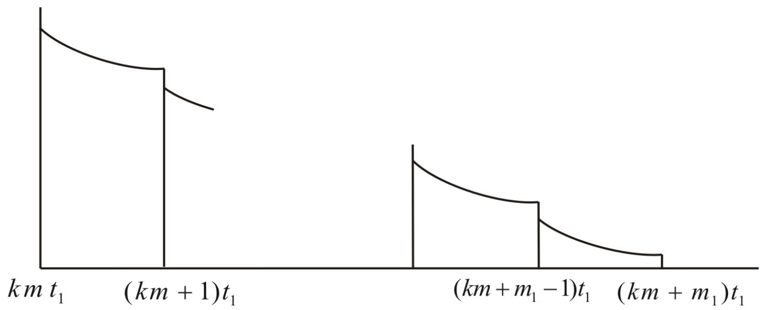

FIGURE 3.11: Inventory level of the last wholesaler's cycle (when n is not divisible by m)

j-th retailer-cycle $[(j-1)t_1, jt_1]$ can be represented by the following differential equation:

$$\frac{dI_W(t)}{dt} = -\gamma I_W(t), \text{ for } (j-1)t_1 < t \leq jt_1 \tag{3.52}$$

For m-th cycle of r-th wholesaler-cycle, i.e., the inventory level of the wholesaler during the interval $[(rm-1)t_1, rmt_1]$ is zero (i.e., $I_W(t) = 0$). Considering the inventory level of the wholesaler at $(rm-1)t_1$ which is $Q_{R,rm}$; the inventory level for the period $[(rm-2)t_1, (rm-1)t_1]$ will be:

$$I_W(t) = e^{\gamma[(rm-1)t_1-t]}Q_{R,rm}, \text{ for } (rm-2)t_1 < t \leq (rm-1)t_1 \tag{3.53}$$

In a similar way, the inventory level of the wholesaler can be obtained for the period starts at $(rm-3)t_1$ using (3.52), considering the boundary condition derived from (3.53) at $t = (rm-2)t_1$:

$$I_W(t) = e^{\gamma[(rm-2)t_1-t]}\{Q_{R,rm-1} + Q_{R,rm}e^{\gamma t_1}\}, \quad \text{for } (rm-3)t_1 < t \leq (rm-2)t_1 \tag{3.54}$$

Similarly, the inventory level of the wholesaler can be obtained for the period starts at $(rm-4)t_1$ will be:

$$\begin{aligned} I_W(t) &= e^{\gamma[(rm-3)t_1-t]}\{Q_{R,rm-2}+Q_{R,rm-1}e^{\gamma t_1}+Q_{R,rm}e^{2\gamma t_1}\}, \\ &\quad \text{for } (rm-4)t_1 < t \le (rm-3)t_1 \end{aligned} \tag{3.55}$$

In this way, the inventory level during j-th retailer's cycle of r-th wholesaler's cycle (excluding the last wholesaler's cycle) can be calculated as follows:

$$\begin{aligned} I^r_{W,j}(t) &= e^{\gamma[jt_1-t]}\sum_{u=j+1}^{rm} Q_{R,u}e^{(u-\overline{j+1})\gamma t_1}, \\ &\quad \text{for } (r-1)m+1 \le j \le rm-1 \end{aligned} \tag{3.56}$$

$$\text{and } I^r_{W,j}(t) = 0, \text{ for } j = rm \tag{3.57}$$

For the last wholesaler's cycle: If $m_1 = n-km = 0$, then the last wholesaler's cycle is the k-th cycle and the inventory level during j-th retailer's cycle of k-th wholesaler's cycle is as follows (cf. Figure 3.10):

$$I^k_{W_l,j}(t) = e^{\gamma[jt_1-t]}\sum_{u=j+1}^{km+1} Q_{R,u}e^{(u-\overline{j+1})\gamma t_1}, \text{ for } (k-1)m+1 \le j \le km \tag{3.58}$$

If $m_1 = n-km \neq 0$, then the inventory level of the last m_1 number of retailer's cycle for $(k+1)$-th wholesaler's cycle can be calculated as follows (cf. Figure 3.11):

$$I^{k+1}_{W_l,j}(t) = e^{\gamma[jt_1-t]}\sum_{u=j+1}^{km+m_1+1} Q_{R,u}e^{(u-\overline{j+1})\gamma t_1}, \text{ for } km+1 \le j \le km+m_1 \tag{3.59}$$

Now, the order quantity of the wholesaler in r-th cycle (excluding the last wholesaler's cycle) is

$$\begin{aligned} Q^r_W &= Q_{R,(r-1)m+1} + I^r_{W,(r-1)m+1}((r-1)mt_1) \\ &= Q_{R,(r-1)m+1} + e^{\gamma t_1}\sum_{u=(r-1)m+2}^{rm} Q_{R,u}e^{[u-\overline{(r-1)m+2}]\gamma t_1} \end{aligned} \tag{3.60}$$

The order quantity of the wholesaler in the last cycle is

$$\begin{aligned}
\text{If } m_1 = 0: \ Q^k_{W_l} &= Q_{R,(k-1)m+1} + I^k_{W_l,(k-1)m+1}((k-1)mt_1) \\
&= Q_{R,(k-1)m+1} + e^{\gamma t_1} \sum_{u=(k-1)m+2}^{km+1} Q_{R,u} e^{[u-\overline{(k-1)m+2}]\gamma t_1} \quad (3.61) \\
\text{If } m_1 \neq 0: \ Q^{k+1}_{W_l} &= Q_{R,km+1} + I^{k+1}_{W_l,km+1}(kmt_1) \\
&= Q_{R,km+1} + e^{\gamma t_1} \sum_{u=km+2}^{km+m_1+1} Q_{R,u} e^{[u-\overline{km+2}]\gamma t_1} \quad (3.62)
\end{aligned}$$

Hence, total order quantity of the wholesaler over the whole period $[0, T_2]$ is

$$Q_W = \begin{cases} \sum_{r=1}^{k-1} Q^r_W + Q^k_{W_l}, & \text{if } m_1 = 0 \\ \sum_{r=1}^{k} Q^r_W + Q^{k+1}_{W_l}, & \text{if } m_1 \neq 0 \end{cases} \quad (3.63)$$

3.3.2.3 Retailer's Profit

The retailer has different types of cost: ordering cost (OC_R), purchase cost (PC_R), holding cost (HC_R), deterioration cost (DC_R), shortage cost (SC_R) and lost sale cost (LSC_R). Retailer's total ordering cost for $(n+1)$ cycles is

$$OC_R = (n+1)A_R \quad (3.64)$$

where, A_R is the ordering cost per order of the retailer.

The purchase cost of the retailer is

$$PC_R = p_R Q_R = p_R \sum_{i=1}^{n+1} Q_{R,i} \quad (3.65)$$

Cumulative quantity of inventory hold during $[(i-1)t_1, s_i]$, i.e., for i-th cycle is

$$\begin{aligned}
H_{R,i} &= \int_{(i-1)t_1}^{s_i} I_{R,i}(t)dt \\
&= \left(-\frac{a}{\alpha} + \frac{b}{\alpha^2} + \frac{2c}{\alpha^3}\right)s - \frac{1}{2}\left(\frac{b}{\alpha} + \frac{2c}{\alpha^2}\right)\{s_i^2 - (i-1)^2t_1^2\} + \frac{c}{3\alpha}\{s_i^3 - (i-1)^3t_1^3\} \\
&\quad -\left[\frac{a+bs_i-cs_i^2}{\alpha^2} - \frac{b-2cs_i}{\alpha^3} - \frac{2c}{\alpha^4}\right](1-e^{\alpha s}) \quad (3.66)
\end{aligned}$$

Therefore, the total holding cost of the retailer is

$$HC_R = h_R \sum_{i=1}^{n} H_{R,i} \tag{3.67}$$

where, h_R is the holding cost per unit per unit time of the retailer.

Number of deteriorating units during the interval $[(i-1)t_1, s_i]$, i.e., for i-th cycle is

$$D_{R,i} = \int_{(i-1)t_1}^{s_i} \alpha I_{R,i}(t)dt = \alpha H_{R,i} \tag{3.68}$$

Therefore, the total deterioration cost of the retailer is

$$DC_R = d_R D_R = d_R \sum_{i=1}^{n} D_{R,i} \tag{3.69}$$

where, d_R is the deterioration cost per unit per unit time and D_R is the total deterioration of the retailer.

Total quantity of shortages during $[s_i, it_1]$, i.e., for i-th cycle is

$$\begin{aligned} S_{R,i} &= -\int_{s_i}^{it_1} I_{S,i}(t)dt \\ &= \frac{c}{6\beta}(i^3t_1^3 - s_i^3) - \frac{cs_i^2}{2\beta}(it_1 - s_i) - \frac{1}{\beta}\left\{b - \frac{c(1+i\beta t_1)}{\beta}\right\}\frac{(it_1 - s_i)^2}{2} \\ &\quad + \frac{1}{\beta^2}\left\{a\beta + b(1+i\beta t_1) - \frac{c}{\beta}(1+i\beta t_1)^2\right\}\Big[(it_1 - s_i)log(1+\beta t_s) \\ &\quad + (it_1 - s_i) - \frac{1}{\beta}\{1+\beta(it_1 - s_i)\}log\{1+\beta(it_1 - s_i)\}\Big] \end{aligned} \tag{3.70}$$

Therefore, the total shortage cost of the retailer is

$$SC_R = c_s \sum_{i=1}^{n} S_{R,i} \tag{3.71}$$

where, c_s is the shortage cost per unit per unit time for backlogged demand of the retailer.

Total quantity of lost sale during $[s_i, it_1]$, i.e., for i-th cycle is

$$\begin{aligned} L_{R,i} &= \int_{s_i}^{it_1} \{D(t) - B_i(t)D(t)\}dt \\ &= a(it_1 - s_i) + \frac{b}{2}(i^2t_1^2 - s_i^2) - \frac{c}{3}(i^3t_1^3 - s_i^3) \\ &\quad - \frac{c}{2\beta}(i^2t_1^2 - s_i^2) + \frac{1}{\beta}\left\{b - \frac{c(1 + i\beta t_1)}{\beta}\right\}(it_1 - s_i) \\ &\quad - \frac{1}{\beta^2}\left\{a\beta + b(1 + i\beta t_1) - \frac{c}{\beta}(1 + i\beta t_1)^2\right\}log(1 + \beta t_s) \end{aligned} \tag{3.72}$$

Therefore, the total lost sale cost of the retailer is

$$LSC_R = c_l \sum_{i=1}^{n} L_{R,i} \tag{3.73}$$

where, c_l is the per unit lost sale cost of the retailer.

Hence, the total cost of the retailer during the whole period $[0, T_2]$ is

$$TC_R = OC_R + PC_R + HC_R + DC_R + SC_R + LSC_R \tag{3.74}$$

With the above costs, retailer spends some promotional cost to increase the demand. The promotional cost per unit time (PrC) is as follows:

$$PrC = g(\rho - 1)^2 M^{\xi} \tag{3.75}$$

where, g and ξ are the parameters so chosen to best fit the promotional cost.

Total selling price of the retailer is

$$TS_R = s_R(Q_R - D_R) \tag{3.76}$$

Using (3.74), (3.75) and (3.76) the total profit of the retailer (TP_R) is

$$TP_R = TS_R - TC_R - PrC.T_2 \tag{3.77}$$

3.3.2.4 Wholesaler's Profit

The wholesaler has the following costs: ordering cost (OC_W), purchase cost (PC_W), holding cost (HC_W) and deterioration cost (DC_W). If $m|n$ (i.e., if m divides n), then there are $k(=[\frac{n}{m}])$ full cycles in wholesaler's inventory period; otherwise there are $m_1(=n-km)$ retailer periods in $(k+1)$-th cycle with k full cycles. So the ordering cost of the wholesaler can be calculated as follows:

$$OC_W = \begin{cases} kA_W & \text{if } m_1 = 0 \\ (k+1)A_W & \text{otherwise} \end{cases} \tag{3.78}$$

where, A_W is the ordering cost per order of the wholesaler.

The purchase cost of the wholesaler is

$$PC_W = p_W Q_W \tag{3.79}$$

where, p_W is the unit purchase price of the wholesaler.

Cumulative quantity of inventory hold during j-th retailer cycle $[(j-1)t_1, jt_1]$ in r-th cycle of the wholesaler (excluding the last wholesaler's cycle) is

$$\begin{aligned} H^r_{W,j} &= \int_{(j-1)t_1}^{jt_1} I^r_{W,j}(t)dt = \frac{(e^{\gamma t_1}-1)}{\gamma}\sum_{u=j+1}^{rm} Q_{R,u}e^{(u-\overline{j+1})\gamma t_1}, \\ & \quad \text{for } (r-1)m+1 \le j \le rm-1 \end{aligned} \tag{3.80}$$

$$\text{and } H^r_{W,j} = 0, \text{ for } j = rm \tag{3.81}$$

For the last wholesaler's cycle, the above quantity will be

$$\begin{aligned} \text{If } m_1 = 0: \ H^k_{W_l,j} &= \frac{(e^{\gamma t_1}-1)}{\gamma}\sum_{u=j+1}^{km+1} Q_{R,u}e^{(u-\overline{j+1})\gamma t_1}, \\ & \quad \text{for } (k-1)m+1 \le j \le km \end{aligned} \tag{3.82}$$

$$\begin{aligned} \text{If } m_1 \neq 0: \ H^{k+1}_{W_l,j} &= \frac{(e^{\gamma t_1}-1)}{\gamma}\sum_{u=j+1}^{km+m_1+1} Q_{R,u}e^{(u-\overline{j+1})\gamma t_1}, \\ & \quad \text{for } km+1 \le j \le km+m_1 \end{aligned} \tag{3.83}$$

So the total cumulative quantity of inventory over the r-th cycle (excluding the last wholesaler's cycle) is

$$H_W^r = \sum_{j=(r-1)m+1}^{rm} H_{W,j}^r \tag{3.84}$$

and cumulative quantity of inventory over the last cycle is

$$\text{If } m_1 = 0: \ H_{W_l}^k = \sum_{j=(k-1)m+1}^{km} H_{W_l,j}^k \tag{3.85}$$

$$\text{If } m_1 \neq 0: \ H_{W_l}^{k+1} = \sum_{j=km+1}^{km+m_1} H_{W_l,j}^{k+1} \tag{3.86}$$

Hence, total quantity of inventory hold over the whole period $[0, T_2]$ is

$$H_W = \begin{cases} \sum_{r=1}^{k-1} H_W^r + H_{W_l}^k, & \text{if } m_1 = 0 \\ \sum_{r=1}^{k} H_W^r + H_{W_l}^{k+1}, & \text{if } m_1 \neq 0 \end{cases} \tag{3.87}$$

Therefore, the holding cost for the wholesaler is

$$HC_W = h_W H_W \tag{3.88}$$

where, h_W is the holding cost per unit per unit time of the wholesaler.

Total deteriorated amount of the wholesaler is

$$D_W = \gamma H_W \tag{3.89}$$

Therefore, the total deterioration cost for the wholesaler is

$$DC_W = d_W D_W \tag{3.90}$$

where, d_W is the deterioration cost per unit per unit time of the wholesaler.

Hence, the total cost of the wholesaler during the whole period $[0, T_2]$ is

$$TC_W = OC_W + PC_W + HC_W + DC_W \tag{3.91}$$

Total selling price of the wholesaler during the whole period $[0, T_2]$ is

$$TS_W = s_W(Q_W - D_W) \tag{3.92}$$

where, s_W is the unit selling price of the wholesaler (i.e., $s_W = p_R$).

Hence, the total profit of the wholesaler over the whole period is

$$TP_W = TS_W - TC_W \tag{3.93}$$

According to the above discussion, two cases may arise:

- the wholesaler does not share any part of the promotional cost. In this case, inventory decisions are made by retailer only. i.e., only retailer's profit is maximized to find marketing decision.
- the wholesaler shares a part, F, of the promotional cost; i.e., the wholesaler pays $F.g(\rho-1)^2M^{\xi}.T_2$ and the retailer pays the remaining part $(1-F).g(\rho-1)^2M^{\xi}.T_2$ of the promotional cost. In this case, inventory decisions are made by retailer and wholesaler jointly. i.e., joint profit of the retailer and the wholesaler is maximized to find marketing decision.

These phenomena are termed as *non-coordination scenario* and *coordination scenario* respectively.

In Non-Coordination Scenario (NCS), there is no coordination between the wholesaler and the retailer and hence the retailer is the sole decision maker. So the retailer's total profit TP_R, which is a function of λ, ρ, m_k and n, is maximized in this scenario. So the mathematical form of the problem in this scenario is as follows:

$$\left.\begin{array}{ll} \text{Determine} & \lambda, \rho, m_k, n \\ \text{to maximize} & TP_R(\lambda, \rho, m_k, n) \\ \text{subject to} & 0 \le \lambda \le 1;\ \rho \ge 1;\ m_k \ge 1;\ n \text{ is an integer.} \end{array}\right\} \tag{3.94}$$

Depending upon the decision of the retailer, the wholesaler tries to improve his/her profit. So in this scenario, the problem of the wholesaler mathematically takes the following form:

$$\left.\begin{array}{l}\text{Determine } \ m \\ \text{to maximize } \ TP_W(\lambda,\rho,m_k,m,n) \\ \text{where, } \ \lambda,\rho,m_k,n \ \text{ are determined by the retailer} \\ \text{and } m \text{ is an integer.}\end{array}\right\} \tag{3.95}$$

In Coordination Scenario (CS), the wholesaler offers to pay a fraction (F) of the promotional cost. Here both the wholesaler and the retailer are decision makers and hence their joint profit TP is maximized to find marketing decisions. In this case retailer's profit is

$$TP_R^F = TS_R - TC_R - (1-F).g(\rho-1)^2M^{\xi}.T_2 \tag{3.96}$$

The wholesaler's profit is

$$TP_W^F = TS_W - TC_W - F.g(\rho-1)^2M^{\xi}.T_2 \tag{3.97}$$

So their joint profit TP is a function of λ, ρ, m_k, m and n and is of the form:

$$TP = TP_R^F + TP_W^F \tag{3.98}$$

So in this scenario the problem is as follows:

$$\left.\begin{array}{ll}\text{Determine} & \lambda,\rho,m_k,m,n \\ \text{to maximize} & TP(\lambda,\rho,m_k,m,n) \\ \text{subject to} & 0\le\lambda\le 1;\ \rho\ge 1;\ m_k\ge 1;\ m \text{ and } n \text{ are integers.}\end{array}\right\} \tag{3.99}$$

3.3.2.5 Fuzzy Supply Chain Model

As discussed in the introduction section that in real life most of the inventory parameters are fuzzy in nature. When some of the inventory parameters of the above model are fuzzy in nature model reduces to a fuzzy supply chain model. Normally ordering cost, holding cost are imprecise in nature. In this model let us consider ordering costs A_R, A_W; holding costs h_R, h_W; shortage cost c_s and the constant g of the promotional cost as fuzzy numbers $\tilde{A}_R$, $\tilde{A}_W$, $\tilde{h}_R$, $\tilde{h}_W$, $\tilde{c}_s$, $\tilde{g}$ respectively, then profits in both the scenario become imprecise in nature. Symbol $\tilde{}$ is used on the top of some notations to indicate fuzzy numbers.

In Non-Coordination Scenario (NCS) of fuzzy supply chain model, the individual profits and total profit of the retailer and the wholesaler are reduced to fuzzy numbers and represented by

$$\begin{aligned} \widetilde{TP}_R &= TS_R - \widetilde{TC}_R - \widetilde{PrC}.T_2 \\ \widetilde{TP}_W &= TS_W - \widetilde{TC}_W \\ \widetilde{TP} &= \widetilde{TP}_R + \widetilde{TP}_W \end{aligned}$$

where,
$$\begin{aligned} \widetilde{TC}_R &= \widetilde{OC}_R + PC_R + \widetilde{HC}_R + DC_R + \widetilde{SC}_R + LSC_R \\ \widetilde{PrC} &= \tilde{g}(\rho-1)^2 M^{\xi} \\ \widetilde{TC}_W &= \widetilde{OC}_W + PC_W + \widetilde{HC}_W + DC_W \end{aligned}$$

where, $\widetilde{OC}_R = (n+1)\tilde{A}_R,\ \ \widetilde{HC}_R = \tilde{h}_R \sum_{i=1}^{n} H_{R,i},\ \ \widetilde{SC}_R = \tilde{c}_s \sum_{i=1}^{n} S_{R,i}$

$$\widetilde{OC}_W = \begin{cases} k\tilde{A}_W & \text{if } m_1 = 0 \\ (k+1)\tilde{A}_W & \text{otherwise} \end{cases} \quad \text{and} \quad \widetilde{HC}_W = \tilde{h}_W H_W$$

Considering the fuzzy numbers $\tilde{A}_R$, $\tilde{A}_W$, $\tilde{h}_R$, $\tilde{h}_W$, $\tilde{c}_s$, $\tilde{g}$ as TFNs (A_{R1}, A_{R2}, A_{R3}), $(A_{W1},\ A_{W2}, A_{W3})$, (h_{R1}, h_{R2}, h_{R3}), (h_{W1}, h_{W2}, h_{W3}), (c_{s1}, c_{s2}, c_{s3}), (g_1, g_2, g_3) respectively, the fuzzy numbers $\widetilde{TP}_R$, $\widetilde{TP}_W$, $\widetilde{TP}$ becomes $(TP_{R1}, TP_{R2}, TP_{R3})$, $(TP_{W1}, TP_{W2}, TP_{W3})$, $(TP_1,\ TP_2, TP_3)$ respectively, where

For $i = 1, 2, 3$
$$\begin{aligned} TP_{Ri} &= TS_R - TC_{R(4-i)} - PrC_{4-i}.T_2 \\ TP_{Wi} &= TS_W - TC_{W(4-i)} \\ TP_i &= TP_{Ri} + TP_{Wi} \end{aligned}$$

where,
$$\begin{aligned} TC_{Ri} &= OC_{Ri} + PC_R + HC_{Ri} + DC_R + SC_{Ri} + LSC_R \\ PrC_i &= g_i(\rho-1)^2 M^{\xi} \\ TC_{Wi} &= OC_{Wi} + PC_W + HC_{Wi} + DC_W \end{aligned}$$

where, $OC_{Ri} = (n+1)A_{Ri}$, $HC_{Ri} = h_{Ri}\sum_{i=1}^{n} H_{R,i}$, $SC_{Ri} = c_{si}\sum_{i=1}^{n} S_{R,i}$

$$OC_{Wi} = \begin{cases} kA_{Wi} & \text{if } m_1 = 0 \\ (k+1)A_{Wi} & \text{otherwise} \end{cases} \quad \text{and} \quad HC_{Wi} = h_{Wi}H_W$$

So in this case the problem reduces to

$$\left.\begin{array}{ll} \text{Maximize} & \widetilde{TP}_R = (TP_{R1}, TP_{R2}, TP_{R3}) \\ \text{subject to} & 0 \le \lambda \le 1;\ \rho \ge 1;\ m_k \ge 1;\ n \text{ is an integer.} \end{array}\right\} \tag{3.100}$$

Depending upon the decision of the retailer, the wholesaler tries to improve his/her profit. So in this scenario, the problem of the wholesaler mathematically takes the following form:

$$\left.\begin{array}{l} \text{Maximize } \widetilde{TP}_W = (TP_{W1}, TP_{W2}, TP_{W3}) \\ \text{where, } \lambda, \rho, m_k, n \text{ are determined by the retailer} \\ \text{and } m \text{ is an integer.} \end{array}\right\} \tag{3.101}$$

The problem is solved using proposed PSO (cf. § 2.2.2.1) where comparisons of the objectives are made by the following approach. Let $\widetilde{TP}_{Ra}$, $\widetilde{TP}_{Rb}$ be the two objectives corresponding to two solutions X_a, X_b respectively. Then the approach of comparison of solutions is given below:

- In this approach credibility measure of fuzzy event is used to compare the solutions. According to this approach X_a dominates X_b if $\mathrm{Cr}(\widetilde{TP}_{Ra} > \widetilde{TP}_{Rb}) > 0.5$. In this approach no crisp equivalent of fuzzy numbers are used to find marketing decisions.

In coordination scenario (CS) of fuzzy supply chain model, the individual profits and the total profit becomes fuzzy numbers and are represented by

$$\begin{aligned} \widetilde{TP}_R^F &= TS_R - \widetilde{TC}_R - (1-F).\widetilde{PrC}.T_2 \\ \widetilde{TP}_W^F &= TS_W - \widetilde{TC}_W - F.\widetilde{PrC}.T_2 \\ \widetilde{TP} &= \widetilde{TP}_R^F + \widetilde{TP}_W^F \end{aligned}$$

So in this case the problem reduces to

$$\left.\begin{array}{ll} \text{Maximize} & \widetilde{TP}_W = (TP_{W1}, TP_{W2}, TP_{W3}) \\ \text{subject to} & 0 \le \lambda \le 1;\ \rho \ge 1;\ m_k \ge 1;\ m \text{ and } n \text{ are integers.} \end{array}\right\} \tag{3.102}$$

The problem is solved using proposed PSO (cf. § 2.2.2.1) where comparisons of the objectives are made by using Credibility Measure approach, which is discussed above.

3.3.3 Numerical Illustration

To illustrate the model, here, a wholesaler-retailer-customer supply chain of apples is considered. Apple is a seasonal fruit of autumn and at that time due to availability, its price decreases and demand increases. Normally apples are available throughout the year with high price and so off-season demand is low. At the beginning of the season it is found that demand of apples increases with time (normally in parabolic curve) to a maximum level and then gradually decreases to off-season level. Seasonal time of apple is about two and half months. Retailer introduces some promotional cost (advertisement, free gift etc.) to increase the maximum demand level of apple of his/her retail shop. Wholesaler share a part of this cost to improve his/her profit also. This supply chain is an example of proposed model of this paper. For this supply chain different parametric values are as follows:

Example 3.3. Base demand of apple is estimated as A = 100 kilograms/week; demand reaches its maximum level after one month, i.e., $T_1 = 4$ weeks; length of season is about two and half months, i.e., $T_2 = 10$ weeks; maximum demand is about 600 kilograms/week, i.e., $M = 600$ kilograms/week; ordering cost of retailer $A_R = \$10$; ordering cost of wholesaler $A_W = \$50$; purchase cost of wholesaler $p_W = \$1.2/$ kilogram; selling price of wholesaler s_W = purchase cost of retailer $p_R = \$2/$ kilogram. Other costs and parameters are $g = 1.2$, $\xi = 0.7$, $\alpha = 0.04$, $\beta = 0.06$, $\delta = 1.65$, $\gamma = 0.03$, $s_R = m_k p_R$, $h_R = \$0.2$, $h_W = \$0.04$, $d_R = \$0.15$, $d_W = \$0.15$, $c_s = \$0.3$, $c_l = \$0.15$.

For fuzzy supply chain model fuzzy parametric values are: $(A_{R1}, A_{R2}, A_{R3}) = (9, 10, 11)$, $(A_{W1}, A_{W2}, A_{W3}) = (48, 50, 52)$, $(h_{R1}, h_{R2}, h_{R3}) = (0.15, 0.20, 0.25)$, $(h_{W1}, h_{W2}, h_{W3}) = (0.03, 0.04, 0.05)$, $(c_{s1}, c_{s2}, c_{s3}) = (0.25, 0.30, 0.35)$, $(g_1, g_2, g_3) = (1.1, 1.2, 1.3)$. All other parametric values are same as crisp model.

For the above set of assumed parametric values, for crisp model, in non-coordination scenario (NCS), TP_R is optimized to find optimum decision for the retailer and optimum λ, ρ, m_k are determined for different values of n. These

TABLE 3.10: Parametric Study on n for Retailer's Profit

Parameter	Decision Variables			Different Expressions			Retailer's Profit
n	λ	ρ	m_k	TS_R	TC_R	$PrC \times T_2$	TP_R
1	0.62	1.30	3.82	1468.75	598.59	97.13	773.02
2	0.62	1.36	3.07	1796.77	737.86	136.45	922.46
3	0.63	1.38	2.88	1910.73	792.99	150.65	967.10
4	0.63	1.39	2.79	1969.42	826.12	158.10	985.21
5	0.63	1.39	2.74	2005.42	850.24	162.75	992.43
6	0.63	1.40	2.70	2030.56	870.41	166.05	**994.10**
7	0.63	1.40	2.68	2046.44	885.78	168.13	992.53
8	0.63	1.40	2.66	2059.80	901.51	169.37	988.93
9	0.63	1.40	2.65	2070.91	916.00	170.99	983.92
10	0.63	1.40	2.64	2080.06	929.65	172.44	977.96

TABLE 3.11: Parametric Study on m for Wholesaler's Profit

Retailer's Decision					Wholesaler's Decision		Total Profit
Decision Variables				TP_R	Parameter	TP_W	TP
λ	ρ	m_k	n		m		
0.628256	1.396440	2.704072	6	994.10	1	3.45	997.54
					2	125.39	1119.48
					3	**145.46**	1139.55
					4	125.66	1119.75
					5	87.19	1081.28
					6	94.17	1088.26

TABLE 3.12: Values of Different Expressions for the Wholesaler

m	TS_W	TC_W	OC_W	PC_W	HC_W	DC_W	TP_W
1	761.09	757.64	300.00	457.09	0.49	0.06	3.45
2	761.09	635.70	150.00	469.64	14.43	1.62	125.39
3	761.09	615.62	100.00	483.02	29.30	3.30	145.46
4	761.09	635.42	100.00	491.88	39.14	4.40	125.66
5	761.09	673.90	100.00	509.08	58.26	6.55	87.19
6	761.09	666.92	50.00	528.32	79.63	8.96	94.17

values and the values of the terms of equation (3.77) are presented in Table 3.10. From Table 3.10, it is found that TP_R is maximum when $n = 6$, $\lambda = 0.63$, $\rho = 1.40$, $m_k = 2.70$. For these values of λ, ρ, m_k, n; TP_W is optimized to find optimum m for the wholesaler and the values are presented in Table 3.11. The values of expressions of wholesaler are shown in Table 3.12 for different values of m. For the optimum values of m and n, the near-optimum results in NCS using PSO technique are presented in Table 3.13.

TABLE 3.13: Near-Optimum Results in NCS using PSO technique

	m	n	TP_R	TP_W	TP	λ	ρ	m_k	F
NCS	3	6	994.09	145.46	1139.55	0.63	1.40	2.70	-

TABLE 3.14: Values of Total Profit (TP) for different m and n using PSO technique

m/ n	1	2	3	4	5	**6**	7	8
1	883.99	1154.99	1216.43	1218.80	1196.90	1162.67	1121.29	1075.41
2	-	1042.11	1179.53	1226.70	1231.44	**1248.60**	1220.15	1226.09
3	-	-	1090.66	1163.87	1218.77	1237.17	1219.07	1234.82
4	-	-	-	1110.88	1146.94	1198.89	1224.43	1232.55
5	-	-	-	-	1119.41	1132.26	1177.95	1205.66

TABLE 3.15: Values of TP_R, TP_W due to different F in CS using PSO technique

m	n	λ	ρ	m_k	F	TP_R	TP_W	TP
2	6	0.64	1.52	1.78	0.67	**992.67**	255.93	1248.60
					0.68	995.71	252.89	1248.60
					0.75	1015.21	233.39	1248.60
					0.85	1043.57	205.03	1248.60
					0.95	1073.18	175.42	1248.60
					1.00	1087.79	160.81	1248.60

TABLE 3.16: Near-Optimum Results in CS using PSO technique

	m	n	TP_R	TP_W	TP	λ	ρ	m_k	F
CS	2	6	1043.57	205.03	1248.60	0.64	1.52	1.78	0.85

Again in coordination scenario (CS), near-optimum results are obtained using PSO technique by optimizing total profit (TP) for different values of m and n and these values are presented in Table 3.14. From this table, it is found that TP is maximum for $m = 2$ and $n = 6$. Now the total profit (TP) is optimized in CS for the above values of m and n and due to sharing of different portion (F) of promotional cost by the wholesaler and the profits of both the parties are tabulated in Table 3.15. From this table, it is found that the appropriate range of F is $(0.68, 1.00)$, i.e., if the wholesaler shares minimum 68%, then the profits of both the parties increase in the CS. In NCS, $TP_R = 994.09$ and $TP_W = 145.46$. If $F = 0.67$, then $TP_R = 992.67 < 994.09$. For the above values of m and n and taking $F = 0.85$, the near-optimum results in CS are presented in Table 3.16.

For fuzzy supply chain model, the experiment is made using PSO technique following credibility measure approach and the results are presented in Table 3.17.

TABLE 3.17: Results of fuzzy supply chain model following Credibility Measure approach

Output Variable	Experiment	
	NCS	CS
$\widetilde{TP}_R$	(964.81, 994.10, 1023.39)	(1014.52, 1043.79, 1073.07)
$\widetilde{TP}_W$	(133.85, 145.17, 156.48)	(170.64, 204.81, 238.97)
$\widetilde{TP}$	(1098.66, 1139.27, 1179.87)	(1185.16, 1248.60, 1312.04)
m	3	2
n	6	6
λ	0.63	0.64
ρ	1.40	1.52
m_k	2.71	1.78
F	-	0.85

Optimization is made by taking fuzzy objectives directly using credibility measure approach of fuzzy events. In coordination scenario the value of F is taken as 0.85.

3.3.4 Discussion

From the above results of the experiment, the following observations are made:

- From Table 3.10 where the optimum number of cycles for retailer is determined, it is found that optimum value of the promotional effort (ρ) is 1.40, i.e., greater than 1. So it is concluded that always promotional effort has a positive effect in a supply chain. This agrees with common expectation.

  Moreover, from the same table, it is observed that the fraction of the duration of positive inventory (λ) is almost constant and the mark-up (m_k) decreases with the number of cycles.

  The effect of these parameters on the profit expression (Eqn. (3.77)) is shown in Table 3.10. Here it is seen that with number of cycles, both revenue and costs are increasing but the rates of increase of these values are different. With the increase of cycle numbers, the rate of increase of costs is more than that of profit and for this reason, after $n = 6$, the profit decreases.

- The effects of ordering, purchase, holding and deterioration costs for different values of m are shown in Table 3.12. It is seen that with the increase of m, the ordering cost decreases as the number of order for wholesaler decreases for increased m, whereas other costs such as purchase, holding and deterioration costs increase with the increase of m due to the increased order quantities.

- From Table 3.13 and 3.16, it is also found that the profits for both the parties (i.e., wholesaler and retailer) increase in the CS than the NCS for a compromise value of F, i.e, if the wholesaler bears a compromise portion of promotional cost, then it is beneficial for both parties. So theoretical expected result agrees with numerical findings.

3.3.5 Managerial Implementation

For a two-level supply chain model connecting a wholesaler and a retailer for a deteriorating seasonal product with dynamic demand, the managerial decision is as follows:

(i) The managements of the both parties should cooperative with each other to boost the demand as it fetches maximum profits for them individually.

(ii) It has been exhibited here that gradual increase of promotional effort does not ensure the gradual increasing profit. For a given set of input data, there is an upper limit of promotional effort for increase of profit (maximum). Management from both side should keep this in mind and the optimum value of promotional effort should be found out in CS system.

(iii) Sharing of the promotional cost between the both parties can not be done arbitrarily. For a given set of data, there are fixed sets of values of proportions for sharing to achieve the maximum profit for individuals. Therefore, in this case also, both parties should play their required roles in consultation for optimum results in CS system.

(iv) In NCS system, the above commitments are observed by the retailer only.

3.4 Model 3.3: A Supply Chain with Fuzzy Production Rate and Demand Under Inflation and Promotional Cost Sharing Using ABC

3.4.1 Assumptions and Notations

The following assumptions and notations are used for mathematical formulation of the model. In the notations the symbol $\sim$ is used over some notations to indicate fuzzy quantities.

(i) The production inventory system involves only one item in a supplier-manufacturer SC.

(ii) $[A_L, A_R]$ represents the α-cut of fuzzy number $\widetilde{A}$ and is denoted by $\widetilde{A}[\alpha]$.

(iii) A_C represents the center of $[A_L, A_R]$, i.e., $A_C = (A_L + A_R)/2$.

(iv) A_W represents the half-width of $[A_L, A_R]$, i.e., $A_W = (A_R - A_L)/2$.

(v) The production rate $\widetilde{K}$ is finite and fuzzy in nature. It is assumed that $\widetilde{K}$ is a triangular fuzzy number (K_1, K_2, K_3) and $\widetilde{K}[\alpha] = [K_L(\alpha), K_R(\alpha)]$.

(vi) The planning horizon $\widetilde{H}$ is finite and fuzzy in nature. It is also assumed that $\widetilde{H}$ is a triangular fuzzy number (H_1, H_2, H_3) and $\widetilde{H}[\alpha] = [H_L(\alpha), H_R(\alpha)]$.

(vii) N manufacturer cycles are completed during the planning horizon.

(viii) $\widetilde{t}_i$ represents the length of i-th manufacturer-cycle.

(ix) $\widetilde{T}_i$ represents the starting time of $(i+1)$-th manufacturer-cycle.

(x) τ_{1i} represents the duration of the production in the i-th manufacturer-cycle. As demand increases in each cycle, τ_{1i} also increase in each cycle. So, τ_{1i} is assumed as follows: $\tau_{1i} = \tau_1 + (i-1)\lambda$, where τ_1 and λ are decision variables.

(xi) τ_2 represents the duration of price discount given by the manufacturer in each cycle, which is a decision variable.

(xii) I represents inflation rate.

(xiii) d represents discount rate.

(xiv) $R = d - I$

(xv) c_r represents the manufacturer's unit purchase cost of the raw material at time $t = 0$.

(xvi) $c_r e^{-R\widetilde{T}_{i-1}}$ is the present value of the manufacturer's unit purchase cost of the raw material in i-th cycle.

(xvii) c_h represents the manufacturer's holding cost of finished good per unit item for 1st cycle, which is a mark-up m_h of c_r, i.e., $c_h = m_h c_r$.

(xviii) $c_h e^{-R\widetilde{T}_{i-1}}$ is the present value of the manufacturer's holding cost of finished good per unit item in i-th cycle.

(xix) c_{hr} represents the manufacturer's holding cost of raw material per unit item for 1st cycle, which is a mark-up m_{hr} of c_r, i.e., $c_{hr} = m_{hr} c_r$.

(xx) $c_{hr} e^{-R\widetilde{T}_{i-1}}$ is the present value of the manufacturer's holding cost of raw material per unit item in i-th cycle.

(xxi) In the absence of inflation, per unit production cost of the item in the i-th cycle c_{pi} is assumed to be decrease in each manufacturer-cycle due to learning effect. The function is taken as: $c_{pi} = c_r + L_i$, where c_r is the manufacturer's purchase cost of the raw material per unit item and L_i is per unit labor charge for production. The labour charge function $L_i = l_0 + \frac{l_1}{i^{\delta_1}}$, where l_0, l_1 and δ_1 are so chosen to best fit the labour charge function. In the presence of inflation, present value of production cost of the item in the i-th cycle is $c_{pi} e^{-R\widetilde{T}_{i-1}}$.

(xxii) In the absence of inflation, the set-up cost c_{si}, of i-th cycle, is assumed to be decrease in each manufacturer-cycle due to learning effect. The function is taken as: $c_{si} = c_0 + \frac{c_1}{i^{\delta_2}}$, where c_0, c_1 and δ_2 are so chosen to best fit the set-up cost function. In the presence of inflation, present value of production cost of the item in the i-th cycle is $c_{si} e^{-R\widetilde{T}_{i-1}}$.

(xxiii) The selling price per unit item in the i-th cycle is a mark-up m of the production cost. Normal value of $m = m_2$. To increase the demand of the item at the beginning of each production cycle the manufacturer offers a price discount for a duration of time τ_2. So during this price discount period, $m = m_1$ and in remaining period of the cycle $m = m_2$, where $m_1 < m_2$ and m_1 is a decision variable. So present value of selling price of the item in i-th cycle is as follows:

$$s_{pi} = mc_{pi}e^{-R\widetilde{T}_{i-1}}$$

(xxiv) $\widetilde{D}_i(t)$ represents demand of the item at time t in the i-th manufacturer-cycle, which depends on selling price and increases in each cycle during price discount period and is of the following form.

$$\widetilde{D}_i(t) = \begin{cases} \dfrac{\widetilde{A}_1 - \widetilde{A}_2.(\frac{m_1}{m_2})^{(i-1)\beta\tau_2}.(\frac{m_1}{m_2})^{\beta(t-\widetilde{T}_{i-1})}}{(m_1 c_{pi} e^{-R\widetilde{T}_{i-1}})^{\gamma}}, & \text{for } \widetilde{T}_{i-1} \leq t \leq \widetilde{T}_{i-1} + \tau_2 \\ \dfrac{\widetilde{A}_1 - \widetilde{A}_2.(\frac{m_1}{m_2})^{i\beta\tau_2}}{(m_2 c_{pi} e^{-R\widetilde{T}_{i-1}})^{\gamma}}, & \text{for } \widetilde{T}_{i-1} + \tau_2 \leq t \leq \widetilde{T}_i \end{cases}$$

where, $\widetilde{A}_1$ and $\widetilde{A}_2$ are estimated as fuzzy parameters from an expert's opinion. β and γ are two parameters so chosen to best fit the demand function. Here, γ is called the price elasticity of the demand function. It is also assumed that $\widetilde{A}_1$, $\widetilde{A}_2$ are triangular fuzzy numbers (A_{11}, A_{12}, A_{13}) and (A_{21}, A_{22}, A_{23}) respectively. Also let $\widetilde{A}_1[\alpha] = [A_{1L}(\alpha), A_{1R}(\alpha)]$, $\widetilde{A}_2[\alpha] = [A_{2L}(\alpha), A_{2R}(\alpha)]$.

(xxv) $\widetilde{q}_i(t)$ represents the inventory level of the manufacturer at time t in the i-th manufacturer-cycle.

(xxvi) M manufacturer-cycles are completed in one supplier-cycle.

(xxvii) c_w represents the supplier's purchase cost of the raw material per unit at time $t = 0$.

(xxviii) $c_w e^{-R\widetilde{T}_{(j-1)M}}$ is the present value of the supplier's purchase cost of the raw material per unit in j-th supplier cycle.

(xxix) c_{hw} represents the supplier's holding cost of raw material per unit item per unit time for 1st cycle, which is a mark-up m_{hw} of c_w, i.e., $c_{hw} = m_{hw}c_w$.

(xxx) $c_{hw}e^{-R\widetilde{T}_{(j-1)M}}$ is the present value of the supplier's holding cost of raw material per unit item in j-th supplier cycle.

(xxxi) c_{sw} represents the ordering cost of the supplier at time $t = 0$.

(xxxii) $c_{sw}e^{-R\widetilde{T}_{(j-1)M}}$ is the present value of the ordering cost of the supplier in j-th supplier cycle.

(xxxiii) $\widetilde{Z}_M$ represents the present value of the profit of the manufacturer from the whole planning horizon.

(xxxiv) $\widetilde{Z}_S$ represents the present value of the profit of the supplier from the whole planning horizon.

(xxxv) $\widetilde{Z}_T$ represents the present value of the total profit of the manufacturer and the supplier from the whole planning horizon.

(xxxvi) F represents the fraction of the promotional cost shared by the supplier.

3.4.2 Mathematical Formulation of the Model

3.4.2.1 Manufacturer's Profit

To develop the model, it is assumed that the planning horizon $\widetilde{H}$ is finite and imprecise in nature. During this planning horizon, N manufacturer-cycles are completed, where the length of the i-th cycle is $\widetilde{t}_i$. Then clearly, $\sum_{i=1}^{N} \widetilde{t}_i \leq \widetilde{H}$. The starting time of i-th manufacturer-cycle is $\widetilde{T}_{i-1}$. At the beginning of the i-th cycle, the manufacturer produces items during a period τ_{1i} at the rate $\widetilde{K}$. To boost the demand, some price discount is offered during the period $[\widetilde{T}_{i-1}, \widetilde{T}_{i-1} + \tau_2]$ in i-th manufacturer-cycle. The inventory levels of the manufacturer and the supplier are presented in Figure 3.12. According to these assumptions, two cases may arise in each cycle: (i) Case-1 ($\tau_{1i} \leq \tau_2$) and (ii) Case-2 ($\tau_{1i} > \tau_2$). The demand function, mentioned in the previous section, can be written as follows:

$$\widetilde{D}_i(t) = \begin{cases} \widetilde{B}_{1i} - \widetilde{B}_{2i}.r^{(i-1)\beta\tau_2}.r^{\beta(t-\widetilde{T}_{i-1})}, & \text{for } \widetilde{T}_{i-1} \leq t \leq \widetilde{T}_{i-1} + \tau_2 \\ \widetilde{C}_{1i} - \widetilde{C}_{2i}.r^{i\beta\tau_2}, & \text{for } \widetilde{T}_{i-1} + \tau_2 \leq t \leq \widetilde{T}_i \end{cases} \tag{3.103}$$

where, $\widetilde{B}_{1i} = \frac{\widetilde{A}_1}{(m_1 c_{pi} e^{-R\widetilde{T}_{i-1}})^\gamma}$, $\widetilde{B}_{2i} = \frac{\widetilde{A}_2}{(m_1 c_{pi} e^{-R\widetilde{T}_{i-1}})^\gamma}$, $\widetilde{C}_{1i} = \frac{\widetilde{A}_1}{(m_2 c_{pi} e^{-R\widetilde{T}_{i-1}})^\gamma}$, $\widetilde{C}_{2i} = \frac{\widetilde{A}_2}{(m_2 c_{pi} e^{-R\widetilde{T}_{i-1}})^\gamma}$ and $r = \frac{m_1}{m_2}$, where, $m_1 < m_2$. Let α-cuts of $\widetilde{B}_{1i}$, $\widetilde{B}_{2i}$, $\widetilde{C}_{1i}$, $\widetilde{C}_{2i}$ be $\widetilde{B}_{1i}[\alpha] = [B_{1iL}, B_{1iR}]$, $\widetilde{B}_{2i}[\alpha] = [B_{2iL}, B_{2iR}]$, $\widetilde{C}_{1i}[\alpha] = [C_{1iL}, C_{1iR}]$, $\widetilde{C}_{2i}[\alpha] = [C_{2iL}, C_{2iR}]$ respectively.

Case-1 ($\tau_{1i} \leq \tau_2$): Instantaneous state of the inventory level, $q_i(t)$ of the item in i-th manufacturer-cycle at any time t ($\widetilde{T}_{i-1} \leq t \leq \widetilde{T}_i$) is given by

$$\frac{d\widetilde{q}_i(t)}{dt} = \begin{cases} \widetilde{K} - \{\widetilde{B}_{1i} - \widetilde{B}_{2i}.r^{(i-1)\beta\tau_2}.r^{\beta(t-\widetilde{T}_{i-1})}\}, & \text{for } \widetilde{T}_{i-1} \leq t \leq \widetilde{T}_{i-1} + \tau_{1i} \\ -\{\widetilde{B}_{1i} - \widetilde{B}_{2i}.r^{(i-1)\beta\tau_2}.r^{\beta(t-\widetilde{T}_{i-1})}\}, & \text{for } \widetilde{T}_{i-1} + \tau_{1i} \leq t \leq \widetilde{T}_{i-1} + \tau_2 \\ -\{\widetilde{C}_{1i} - \widetilde{C}_{2i}.r^{i\beta\tau_2}\}, & \text{for } \widetilde{T}_{i-1} + \tau_2 \leq t \leq \widetilde{T}_i \end{cases} \tag{3.104}$$

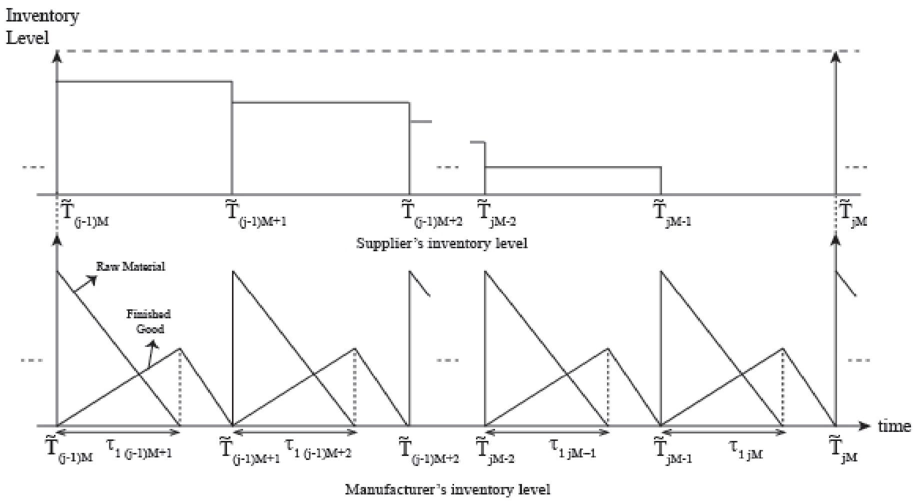

FIGURE 3.12: Inventory levels of the manufacturer and the supplier at the j-th supplier-cycle

with boundary conditions $\widetilde{q}_i(\widetilde{T}_{i-1}) = 0$, $\widetilde{q}_i(\widetilde{T}_{i-1} + \tau_{1i}) = \widetilde{Q}_{1i}$, $\widetilde{q}_i(\widetilde{T}_{i-1} + \tau_2) = \widetilde{Q}_{2i}$. Following Buckley and Feuring [19] (cf. § 2.1.2.6), solving (3.104), α-cut $[q_{iL}(\alpha,t), q_{iR}(\alpha,t)]$ of $\widetilde{q}_i(t)$ is obtained as follows:

$$q_{iL}(\alpha,t) = \begin{cases} (K_L - B_{1iR})(t - T_{(i-1)R}) \\ \qquad + B_{2iL}\dfrac{r^{(i-1)\beta\tau_2}}{\beta log(1/r)}\Big[1 - r^{\beta\{t-T_{(i-1)R}\}}\Big], & \text{for } \widetilde{T}_{i-1} \le t \le \widetilde{T}_{i-1} + \tau_{1i} \\ Q_{1iL} - B_{1iR}\{t - T_{(i-1)L} - \tau_{1i}\} \\ \qquad + B_{2iL}\dfrac{r^{(i-1)\beta\tau_2}}{\beta log(1/r)}\Big[r^{\beta\tau_{1i}} - r^{\beta\{t-T_{(i-1)R}\}}\Big], & \text{for } \widetilde{T}_{i-1} + \tau_{1i} \le t \le \widetilde{T}_{i-1} + \tau_2 \\ Q_{2iL} - (C_{1iR} - C_{2iL}.r^{i\beta\tau_2})\{t - T_{(i-1)L} - \tau_2\}, & \text{for } \widetilde{T}_{i-1} + \tau_2 \le t \le \widetilde{T}_i \end{cases} \tag{3.105}$$

$$q_{iR}(\alpha,t) = \begin{cases} (K_R - B_{1iL})(t - T_{(i-1)L}) \\ \qquad + B_{2iR}\dfrac{r^{(i-1)\beta\tau_2}}{\beta log(1/r)}\Big[1 - r^{\beta\{t-T_{(i-1)L}\}}\Big], & \text{for } \widetilde{T}_{i-1} \le t \le \widetilde{T}_{i-1} + \tau_{1i} \\ Q_{1iR} - B_{1iL}\{t - T_{(i-1)R} - \tau_{1i}\} \\ \qquad + B_{2iR}\dfrac{r^{(i-1)\beta\tau_2}}{\beta log(1/r)}\Big[r^{\beta\tau_{1i}} - r^{\beta\{t-T_{(i-1)L}\}}\Big], & \text{for } \widetilde{T}_{i-1} + \tau_{1i} \le t \le \widetilde{T}_{i-1} + \tau_2 \\ Q_{2iR} - (C_{1iL} - C_{2iR}.r^{i\beta\tau_2})\{t - T_{(i-1)R} - \tau_2\}, & \text{for } \widetilde{T}_{i-1} + \tau_2 \le t \le \widetilde{T}_i \end{cases} \tag{3.106}$$

where,

$$
\begin{aligned}
Q_{1iL} &= (K_L - B_{1iR})\tau_{1i} + B_{2iL}\frac{r^{(i-1)\beta\tau_2}}{\beta log(1/r)}(1 - r^{\beta\tau_{1i}}) \\
Q_{1iR} &= (K_R - B_{1iL})\tau_{1i} + B_{2iR}\frac{r^{(i-1)\beta\tau_2}}{\beta log(1/r)}(1 - r^{\beta\tau_{1i}}) \\
Q_{2iL} &= Q_{1iL} - B_{1iR}(\tau_2 - \tau_{1i}) + B_{2iL}\frac{r^{(i-1)\beta\tau_2}}{\beta log(1/r)}(r^{\beta\tau_{1i}} - r^{\beta\tau_2}) \\
Q_{2iR} &= Q_{1iR} - B_{1iL}(\tau_2 - \tau_{1i}) + B_{2iR}\frac{r^{(i-1)\beta\tau_2}}{\beta log(1/r)}(r^{\beta\tau_{1i}} - r^{\beta\tau_2})
\end{aligned}
$$

The length of i-th manufacturer-cycle $\widetilde{t}_i$ is obtained using the condition $\widetilde{q}_i(\widetilde{T}_i) = 0$ as follows:

$$
\widetilde{t}_i = \tau_2 + \frac{\widetilde{Q}_{2i}}{\widetilde{C}_{1i} - \widetilde{C}_{2i}.r^{i\beta\tau_2}} \tag{3.107}
$$

The α-cut $[t_{iL}(\alpha), t_{iR}(\alpha)]$ of $\widetilde{t}_i$ is as follows:

$$
\begin{aligned}
t_{iL} &= \tau_2 + \frac{Q_{2iL}}{C_{1iR} - C_{2iL}.r^{i\beta\tau_2}} \\
t_{iR} &= \tau_2 + \frac{Q_{2iR}}{C_{1iL} - C_{2iR}.r^{i\beta\tau_2}}
\end{aligned}
$$

The starting time if $(i+1)$-th manufacturer-cycle $\widetilde{T}_i$ can be calculated as follows:

$$
\widetilde{T}_i = \sum_{j=1}^{i} \widetilde{t}_j \tag{3.108}
$$

Let α-cut of $\widetilde{T}_i$ be $\widetilde{T}_i[\alpha] = [T_{iL}(\alpha), T_{iR}(\alpha)]$, where $T_{iL}(\alpha) = \sum_{j=1}^{i} t_{jL}$ and $T_{iR}(\alpha) = \sum_{j=1}^{i} t_{jR}$.

Holding cost of finished good: Present value of the holding cost for finished good of the manufacturer during the i-th cycle is as follows:

$$
\widetilde{HC}_{Mi} = c_h e^{-R\widetilde{T}_{i-1}}[\widetilde{I}_{1i} + \widetilde{I}_{2i} + \widetilde{I}_{3i}] \tag{3.109}
$$

where,

$$\widetilde{I}_{1i} = \int_{\widetilde{T}_{i-1}}^{\widetilde{T}_{i-1}+\tau_{1i}} \left[(\widetilde{K} - \widetilde{B}_{1i})(t - \widetilde{T}_{i-1}) + \widetilde{B}_{2i}\frac{r^{(i-1)\beta\tau_2}}{\beta log(1/r)}\left\{1 - r^{\beta(t-\widetilde{T}_{i-1})}\right\}\right]dt \tag{3.110}$$

$$\widetilde{I}_{2i} = \int_{\widetilde{T}_{i-1}+\tau_{1i}}^{\widetilde{T}_{i-1}+\tau_2} \left[\widetilde{Q}_{1i} - \widetilde{B}_{1i}(t - \widetilde{T}_{i-1} - \tau_{1i}) + \widetilde{B}_{2i}\frac{r^{(i-1)\beta\tau_2}}{\beta log(1/r)}\left\{r^{\beta\tau_{1i}} - r^{\beta(t-\widetilde{T}_{i-1})}\right\}\right]dt \tag{3.111}$$

$$\widetilde{I}_{3i} = \int_{\widetilde{T}_{i-1}+\tau_2}^{\widetilde{T}_i} \left[\widetilde{Q}_{2i} - (\widetilde{C}_{1i} - \widetilde{C}_{2i}.r^{i\beta\tau_2})(t - \widetilde{T}_{i-1} - \tau_2)\right]dt \tag{3.112}$$

Let α-cuts of $\widetilde{I}_{1i}$, $\widetilde{I}_{2i}$ and $\widetilde{I}_{3i}$ be $\widetilde{I}_{1i}[\alpha] = [I_{1iL}(\alpha), I_{1iR}(\alpha)]$, $\widetilde{I}_{2i}[\alpha] = [I_{2iL}(\alpha), I_{2iR}(\alpha)]$ and $\widetilde{I}_{3i}[\alpha] = [I_{3iL}(\alpha), I_{3iR}(\alpha)]$ respectively. Then according to Wu [194] (cf. § 2.1.2.7), the following expressions are obtained.

$$\begin{aligned}
I_{1iL}(\alpha) &= (K_L - B_{1iR})\frac{\tau_{1i}^2}{2} + B_{2iL}\frac{r^{(i-1)\beta\tau_2}}{\beta log(1/r)}\left[\tau_{1i} - \frac{1 - r^{\beta\tau_{1i}}}{\beta log(1/r)}\right] \\
I_{1iR}(\alpha) &= (K_R - B_{1iL})\frac{\tau_{1i}^2}{2} + B_{2iR}\frac{r^{(i-1)\beta\tau_2}}{\beta log(1/r)}\left[\tau_{1i} - \frac{1 - r^{\beta\tau_{1i}}}{\beta log(1/r)}\right] \\
I_{2iL}(\alpha) &= Q_{1iL}(\tau_2 - \tau_{1i}) - B_{1iR}\frac{(\tau_2 - \tau_{1i})^2}{2} \\
&\quad + B_{2iL}\frac{r^{(i-1)\beta\tau_2}}{\beta log(1/r)}\left[r^{\beta\tau_{1i}}(\tau_2 - \tau_{1i}) - \frac{r^{\beta\tau_{1i}} - r^{\beta\tau_2}}{\beta log(1/r)}\right] \\
I_{2iR}(\alpha) &= Q_{1iR}(\tau_2 - \tau_{1i}) - B_{1iL}\frac{(\tau_2 - \tau_{1i})^2}{2} \\
&\quad + B_{2iR}\frac{r^{(i-1)\beta\tau_2}}{\beta log(1/r)}\left[r^{\beta\tau_{1i}}(\tau_2 - \tau_{1i}) - \frac{r^{\beta\tau_{1i}} - r^{\beta\tau_2}}{\beta log(1/r)}\right] \\
I_{3iL}(\alpha) &= Q_{2iL}(t_{iL} - \tau_2) - (C_{1iR} - C_{2iL}.r^{i\beta\tau_2})\frac{(t_{iR} - \tau_2)^2}{2} \\
I_{3iR}(\alpha) &= Q_{2iR}(t_{iR} - \tau_2) - (C_{1iL} - C_{2iR}.r^{i\beta\tau_2})\frac{(t_{iL} - \tau_2)^2}{2}
\end{aligned}$$

Let α-cut of $\widetilde{HC}_{Mi}$ be $\widetilde{HC}_{Mi}[\alpha] = [HC_{MiL}(\alpha), HC_{MiR}(\alpha)]$, where

$$\begin{aligned}
HC_{MiL}(\alpha) &= c_h e^{-RT_{(i-1)R}}[I_{1iL}(\alpha) + I_{2iL}(\alpha) + I_{3iL}(\alpha)] \\
HC_{MiR}(\alpha) &= c_h e^{-RT_{(i-1)L}}[I_{1iR}(\alpha) + I_{2iR}(\alpha) + I_{3iR}(\alpha)]
\end{aligned}$$

Holding cost of raw material: Present value of the holding cost for raw material of the manufacturer during the i-th cycle is as follows:

$$\widetilde{HCR}_{Mi} = \frac{1}{2}c_{hr}e^{-R\widetilde{T}_{(i-1)}}\widetilde{K}\tau_{1i}^2 \tag{3.113}$$

Let α-cut of $\widetilde{HCR}_{Mi}$ be $\widetilde{HCR}_{Mi}[\alpha] = [HCR_{MiL}(\alpha), HCR_{MiR}(\alpha)]$, where

$$\begin{aligned} HCR_{MiL}(\alpha) &= \frac{1}{2}c_{hr}e^{-RT_{(i-1)R}}K_L(\alpha)\tau_{1i}^2 \\ HCR_{MiR}(\alpha) &= \frac{1}{2}c_{hr}e^{-RT_{(i-1)L}}K_R(\alpha)\tau_{1i}^2 \end{aligned}$$

Sell revenue: Present value of the sell revenue of the manufacturer in the i-th cycle is as follows:

$$\widetilde{SR}_{Mi} = c_{pi}e^{-R\widetilde{T}_{i-1}}[m_1\widetilde{I}_{4i} + m_2\widetilde{I}_{5i}] \tag{3.114}$$

where,

$$\widetilde{I}_{4i} = \int_{\widetilde{T}_{i-1}}^{\widetilde{T}_{i-1}+\tau_2}\left[\widetilde{B}_{1i} - \widetilde{B}_{2i}.r^{(i-1)\beta\tau_2}.r^{\beta(t-\widetilde{T}_{i-1})}\right]dt \tag{3.115}$$

$$\widetilde{I}_{5i} = \int_{\widetilde{T}_{i-1}+\tau_2}^{\widetilde{T}_i}\left[\widetilde{C}_{1i} - \widetilde{C}_{2i}.r^{i\beta\tau_2}\right]dt \tag{3.116}$$

Let α-cuts of $\widetilde{I}_{4i}$ and $\widetilde{I}_{5i}$ be $\widetilde{I}_{4i}[\alpha] = [I_{4iL}(\alpha), I_{4iR}(\alpha)]$ and $\widetilde{I}_{5i}[\alpha] = [I_{5iL}(\alpha), I_{5iR}(\alpha)]$ respectively. Then according to Wu [194] (cf. § 2.1.2.7), the following expressions are obtained.

$$\begin{aligned} I_{4iL}(\alpha) &= B_{1iL}.\tau_2 - B_{2iR}\frac{r^{(i-1)\beta\tau_2}}{\beta log(1/r)}(1 - r^{\beta\tau_2}) \\ I_{4iR}(\alpha) &= B_{1iR}.\tau_2 - B_{2iL}\frac{r^{(i-1)\beta\tau_2}}{\beta log(1/r)}(1 - r^{\beta\tau_2}) \\ I_{5iL}(\alpha) &= (C_{1iL} - C_{2iR}.r^{i\beta\tau_2})(t_{iL} - \tau_2) \\ I_{5iR}(\alpha) &= (C_{1iR} - C_{2iL}.r^{i\beta\tau_2})(t_{iR} - \tau_2) \end{aligned}$$

Let α-cut of $\widetilde{SR}_{Mi}$ be $\widetilde{SR}_{Mi}[\alpha] = [SR_{MiL}(\alpha), SR_{MiR}(\alpha)]$, where

$$\begin{aligned} SR_{MiL}(\alpha) &= c_{pi}e^{-RT_{(i-1)R}}[m_1I_{4iL}(\alpha) + m_2I_{5iL}(\alpha)] \\ SR_{MiR}(\alpha) &= c_{pi}e^{-RT_{(i-1)L}}[m_1I_{4iR}(\alpha) + m_2I_{5iR}(\alpha)] \end{aligned}$$

Production cost: Present value of the production cost of the manufacturer in the i-th cycle is as follows:

$$\widetilde{PC}_{Mi} = c_{pi}e^{-R\widetilde{T}_{i-1}}\widetilde{K}\tau_{1i} \tag{3.117}$$

Let α-cut of $\widetilde{PC}_{Mi}$ be $\widetilde{PC}_{Mi}[\alpha] = [PC_{MiL}(\alpha), PC_{MiR}(\alpha)]$, where

$$\begin{aligned} PC_{MiL}(\alpha) &= c_{pi}e^{-RT_{(i-1)R}}K_L(\alpha)\tau_{1i} \\ PC_{MiR}(\alpha) &= c_{pi}e^{-RT_{(i-1)L}}K_R(\alpha)\tau_{1i} \end{aligned}$$

Set-up cost: Present value of the set-up cost of the manufacturer in the i-th cycle is as follows:

$$\widetilde{SC}_{Mi} = c_{si}e^{-R\widetilde{T}_{i-1}} \tag{3.118}$$

Let α-cut of $\widetilde{SC}_{Mi}$ be $\widetilde{SC}_{Mi}[\alpha] = [SC_{MiL}(\alpha), SC_{MiR}(\alpha)]$, where

$$\begin{aligned} SC_{MiL}(\alpha) &= c_{si}e^{-RT_{(i-1)R}} \\ SC_{MiR}(\alpha) &= c_{si}e^{-RT_{(i-1)L}} \end{aligned}$$

Case-2 ($\tau_{1i} > \tau_2$): Instantaneous state of the inventory level, $q_i(t)$ of the item in i-th manufacturer-cycle at any time t ($\widetilde{T}_{i-1} \le t \le \widetilde{T}_i$) is given by

$$\frac{d\widetilde{q}_i(t)}{dt} = \begin{cases} \widetilde{K} - \{\widetilde{B}_{1i} - \widetilde{B}_{2i}.r^{(i-1)\beta\tau_2}.r^{\beta(t-\widetilde{T}_{i-1})}\}, & \text{for } \widetilde{T}_{i-1} \le t \le \widetilde{T}_{i-1} + \tau_2 \\ \widetilde{K} - \{\widetilde{C}_{1i} - \widetilde{C}_{2i}.r^{i\beta\tau_2}\}, & \text{for } \widetilde{T}_{i-1} + \tau_2 \le t \le \widetilde{T}_{i-1} + \tau_{1i} \\ -\{\widetilde{C}_{1i} - \widetilde{C}_{2i}.r^{i\beta\tau_2}\}, & \text{for } \widetilde{T}_{i-1} + \tau_{1i} \le t \le \widetilde{T}_i \end{cases} \tag{3.119}$$

with boundary conditions $\widetilde{q}_i(\widetilde{T}_{i-1}) = 0$, $\widetilde{q}_i(\widetilde{T}_{i-1} + \tau_2) = \widetilde{Q}_{2i}$, $\widetilde{q}_i(\widetilde{T}_{i-1} + \tau_{1i}) = \widetilde{Q}_{1i}$. Similarly, following Buckley and Feuring [19] (cf. § 2.1.2.6) (as in Case-1), solving (3.119), α-cut $[q_{iL}(\alpha,t), q_{iR}(\alpha,t)]$ of $\widetilde{q}_i(t)$ is obtained as follows:

$$q_{iL}(\alpha,t) = \begin{cases} (K_L - B_{1iR})(t - T_{(i-1)R}) \\ \quad + B_{2iL}\dfrac{r^{(i-1)\beta\tau_2}}{\beta log(1/r)}\Big[1 - r^{\beta\{t-T_{(i-1)R}\}}\Big], & \text{for } \widetilde{T}_{i-1} \le t \le \widetilde{T}_{i-1} + \tau_2 \\ Q_{2iL} + (K_L - C_{1iR} + C_{2iL}.r^{i\beta\tau_2})\{t - T_{(i-1)R} - \tau_2\}, & \\ & \text{for } \widetilde{T}_{i-1} + \tau_2 \le t \le \widetilde{T}_{i-1} + \tau_{1i} \\ Q_{1iL} - (C_{1iR} - C_{2iL}.r^{i\beta\tau_2})\{t - T_{(i-1)L} - \tau_{1i}\}, & \text{for } \widetilde{T}_{i-1} + \tau_{1i} \le t \le \widetilde{T}_i \end{cases} \tag{3.120}$$

$$q_{iR}(\alpha,t)=\begin{cases}(K_R-B_{1iL})(t-T_{(i-1)L}) \\ \qquad +B_{2iR}\dfrac{r^{(i-1)\beta\tau_2}}{\beta log(1/r)}\Big[1-r^{\beta\{t-T_{(i-1)L}\}}\Big], & \text{for } \widetilde{T}_{i-1}\le t\le \widetilde{T}_{i-1}+\tau_2 \\ Q_{2iR}+(K_R-C_{1iL}+C_{2iR}.r^{i\beta\tau_2})\{t-T_{(i-1)L}-\tau_2\}, & \\ & \text{for } \widetilde{T}_{i-1}+\tau_2\le t\le \widetilde{T}_{i-1}+\tau_{1i} \\ Q_{1iR}-(C_{1iL}-C_{2iR}.r^{i\beta\tau_2})\{t-T_{(i-1)R}-\tau_{1i}\}, & \text{for } \widetilde{T}_{i-1}+\tau_{1i}\le t\le \widetilde{T}_i\end{cases} \tag{3.121}$$

where,

$$\begin{aligned}Q_{2iL} &= (K_L-B_{1iR})\tau_2+B_{2iL}\frac{r^{(i-1)\beta\tau_2}}{\beta log(1/r)}(1-r^{\beta\tau_2}) \\ Q_{2iR} &= (K_R-B_{1iL})\tau_2+B_{2iR}\frac{r^{(i-1)\beta\tau_2}}{\beta log(1/r)}(1-r^{\beta\tau_2}) \\ Q_{1iL} &= Q_{2iL}+(K_L-C_{1iR}+C_{2iL}.r^{i\beta\tau_2})(\tau_{1i}-\tau_2) \\ Q_{1iR} &= Q_{2iR}+(K_R-C_{1iL}+C_{2iR}.r^{i\beta\tau_2})(\tau_{1i}-\tau_2)\end{aligned}$$

The length of i-th manufacturer-cycle $\widetilde{t}_i$ is obtained using the condition $\widetilde{q}_i(\widetilde{T}_i)=0$ as follows:

$$\widetilde{t}_i = \tau_{1i}+\frac{\widetilde{Q}_{1i}}{\widetilde{C}_{1i}-\widetilde{C}_{2i}.r^{i\beta\tau_2}} \tag{3.122}$$

The α-cut $[t_{iL}(\alpha),t_{iR}(\alpha)]$ of $\widetilde{t}_i$ is as follows:

$$\begin{aligned}t_{iL} &= \tau_{1i}+\frac{Q_{1iL}}{C_{1iR}-C_{2iL}.r^{i\beta\tau_2}} \\ t_{iR} &= \tau_{1i}+\frac{Q_{1iR}}{C_{1iL}-C_{2iR}.r^{i\beta\tau_2}}\end{aligned}$$

The starting time of $(i+1)$-th manufacturer-cycle $\widetilde{T}_i$ can be calculated as in Case-1.

Holding cost of finished good: Present value of the holding cost for finished good of the manufacturer during the i-th cycle is as follows:

$$\widetilde{HC}_{Mi} = c_h e^{-R\widetilde{T}_{i-1}}[\widetilde{I}_{1i}+\widetilde{I}_{2i}+\widetilde{I}_{3i}] \tag{3.123}$$

where,

$$\widetilde{I}_{1i} = \int_{\widetilde{T}_{i-1}}^{\widetilde{T}_{i-1}+\tau_2} \Bigg[(\widetilde{K} - \widetilde{B}_{1i})(t - \widetilde{T}_{i-1}) + \widetilde{B}_{2i}\frac{r^{(i-1)\beta\tau_2}}{\beta log(1/r)}\Big\{1 - r^{\beta(t-\widetilde{T}_{i-1})}\Big\}\Bigg]dt \quad (3.124)$$

$$\widetilde{I}_{2i} = \int_{\widetilde{T}_{i-1}+\tau_2}^{\widetilde{T}_{i-1}+\tau_{1i}} \Bigg[\widetilde{Q}_{2i} + (\widetilde{K} - \widetilde{C}_{1i} + \widetilde{C}_{2i}.r^{i\beta\tau_2})(t - \widetilde{T}_{i-1} - \tau_2)\Bigg]dt \quad (3.125)$$

$$\widetilde{I}_{3i} = \int_{\widetilde{T}_{i-1}+\tau_{1i}}^{\widetilde{T}_i} \Bigg[\widetilde{Q}_{1i} - (\widetilde{C}_{1i} - \widetilde{C}_{2i}.r^{i\beta\tau_2})(t - \widetilde{T}_{i-1} - \tau_{1i})\Bigg]dt \quad (3.126)$$

Let α-cuts of $\widetilde{I}_{1i}$, $\widetilde{I}_{2i}$ and $\widetilde{I}_{3i}$ be $\widetilde{I}_{1i}[\alpha] = [I_{1iL}(\alpha), I_{1iR}(\alpha)]$, $\widetilde{I}_{2i}[\alpha] = [I_{2iL}(\alpha), I_{2iR}(\alpha)]$ and $\widetilde{I}_{3i}[\alpha] = [I_{3iL}(\alpha), I_{3iR}(\alpha)]$ respectively. Then according to Wu [194] (cf. § 2.1.2.7), the following expressions are obtained.

$$\begin{aligned}
I_{1iL}(\alpha) &= (K_L - B_{1iR})\frac{\tau_2^2}{2} + B_{2iL}\frac{r^{(i-1)\beta\tau_2}}{\beta log(1/r)}\Big[\tau_2 - \frac{1 - r^{\beta\tau_2}}{\beta log(1/r)}\Big] \\
I_{1iR}(\alpha) &= (K_R - B_{1iL})\frac{\tau_2^2}{2} + B_{2iR}\frac{r^{(i-1)\beta\tau_2}}{\beta log(1/r)}\Big[\tau_2 - \frac{1 - r^{\beta\tau_2}}{\beta log(1/r)}\Big] \\
I_{2iL}(\alpha) &= Q_{2iL}(\tau_{1i} - \tau_2) - (K_L - C_{1iR} + C_{2iL}.r^{i\beta\tau_2})\frac{(\tau_{1i} - \tau_2)^2}{2} \\
I_{2iR}(\alpha) &= Q_{2iR}(\tau_{1i} - \tau_2) - (K_R - C_{1iL} + C_{2iR}.r^{i\beta\tau_2})\frac{(\tau_{1i} - \tau_2)^2}{2} \\
I_{3iL}(\alpha) &= Q_{1iL}(t_{iL} - \tau_{1i}) - (C_{1iR} - C_{2iL}.r^{i\beta\tau_2})\frac{(t_{iR} - \tau_{1i})^2}{2} \\
I_{3iR}(\alpha) &= Q_{1iR}(t_{iR} - \tau_{1i}) - (C_{1iL} - C_{2iR}.r^{i\beta\tau_2})\frac{(t_{iL} - \tau_{1i})^2}{2}
\end{aligned}$$

Let α-cut of $\widetilde{HC}_{Mi}$ be $\widetilde{HC}_{Mi}[\alpha] = [HC_{MiL}(\alpha), HC_{MiR}(\alpha)]$, where

$$\begin{aligned}
HC_{MiL}(\alpha) &= c_h e^{-RT_{(i-1)R}}[I_{1iL}(\alpha) + I_{2iL}(\alpha) + I_{3iL}(\alpha)] \\
HC_{MiR}(\alpha) &= c_h e^{-RT_{(i-1)L}}[I_{1iR}(\alpha) + I_{2iR}(\alpha) + I_{3iR}(\alpha)]
\end{aligned}$$

The holding cost for raw material, sell revenue, production cost and set-up cost in this case are same as in Case-1.
Hence, present value of the total profit of the manufacturer from the whole planning horizon is as follows:

$$\begin{aligned}
\widetilde{Z}_M &= \sum_{i=1}^{N}\Big[\widetilde{SR}_{Mi} - \widetilde{PC}_{Mi} - \widetilde{HC}_{Mi} - \widetilde{HCR}_{Mi} - \widetilde{SC}_{Mi}\Big] \\
&= \widetilde{SR}_M - \widetilde{PC}_M - \widetilde{HC}_M - \widetilde{HCR}_M - \widetilde{SC}_M \quad (3.127)
\end{aligned}$$

Let α-cut of $\widetilde{Z}_M$ be $\widetilde{Z}_M[\alpha] = [Z_{ML}(\alpha), Z_{MR}(\alpha)]$, where

$$Z_{ML}(\alpha) = SR_{ML} - PC_{MR} - HC_{MR} - HCR_{MR} - SC_{MR} \quad (3.128)$$

$$Z_{MR}(\alpha) = SR_{MR} - PC_{ML} - HC_{ML} - HCR_{ML} - SC_{ML} \quad (3.129)$$

3.4.2.2 Supplier's Profit

The supplier purchases the raw materials and supplies to the manufacturer. Here it is assumed that M manufacturer-cycles are completed in one supplier-cycle. If $M|N$ (M divides N), then there are p full cycles in supplier's inventory period; otherwise, there are $M_1(= N - pM)$ manufacturer-cycles in $(p+1)$-th supplier-cycle with p full cycles, where $p = [\frac{N}{M}]$ and $[X]$ represents integral part of X.

Amount of raw material required for i-th manufacturer-cycle, $\widetilde{R}_i = \widetilde{K}\tau_{1i}$. The order quantity of raw material for the supplier's j-th cycle (except last cycle of the supplier) is as follows:

$$\widetilde{RW}_j = \widetilde{R}_{(j-1)M+1} + \widetilde{R}_{(j-1)M+2} + \ldots + \widetilde{R}_{jM} \quad (3.130)$$

The order quantity of raw material for the supplier's last cycle is as follows:

$$\text{If } M|N,\ \widetilde{RW}_{l,p} = \widetilde{R}_{(p-1)M+1} + \widetilde{R}_{(p-1)M+2} + \ldots + \widetilde{R}_{pM} \quad (3.131)$$

$$\text{If } M \nmid N,\ \widetilde{RW}_{l,p+1} = \widetilde{R}_{pM+1} + \widetilde{R}_{pM+2} + \ldots + \widetilde{R}_{pM+M_1} \quad (3.132)$$

Hence, the total order quantity of the raw material for the supplier is as follows:

$$\widetilde{RW} = \begin{cases} \sum_{j=1}^{p-1} \widetilde{RW}_j + \widetilde{RW}_{l,p}, \text{ for } M|N \\ \sum_{j=1}^{p} \widetilde{RW}_j + \widetilde{RW}_{l,p+1}, \text{ for } M \nmid N \end{cases} \quad (3.133)$$

The holding amount of the supplier in j-th supplier-cycle (except the last cycle of the supplier) is as follows:

$$\widetilde{HA}_j = \sum_{i=(j-1)M+1}^{jM} \widetilde{R}_i(\widetilde{T}_{i-1} - \widetilde{T}_w); \text{ where, } w = (j-1)M \quad (3.134)$$

The holding amount of the supplier in the last supplier-cycle is as follows:

$$\text{If } M|N,\ \widetilde{HA}_{l,p} = \sum_{i=(p-1)M+1}^{pM} \widetilde{R}_i(\widetilde{T}_{i-1} - \widetilde{T}_w);\ \text{where, } w = (p-1)M \tag{3.135}$$

$$\text{If } M \nmid N,\ \widetilde{HA}_{l,p+1} = \sum_{i=pM+1}^{pM+M_1} \widetilde{R}_i(\widetilde{T}_{i-1} - \widetilde{T}_w);\ \text{where, } w = pM \tag{3.136}$$

Sell revenue: Present value of the sell revenue of raw material of the supplier in the i-th manufacturer-cycle is as follows:

$$\widetilde{SR}_{Si} = c_r e^{-R\widetilde{T}_{i-1}} \widetilde{R}_i \tag{3.137}$$

Hence, present value of the total sell revenue for raw material of the supplier is as follows:

$$\widetilde{SR}_S = \sum_{i=1}^{N} \widetilde{SR}_{Si} \tag{3.138}$$

Purchase cost: Present value of the purchase cost of raw material of the supplier in the j-th supplier-cycle (except the last cycle of the supplier) is as follows:

$$\widetilde{PC}_{Sj} = c_w e^{-R\widetilde{T}_{(j-1)M}} \widetilde{RW}_j \tag{3.139}$$

Present value of the purchase cost of raw material of the supplier in the last supplier-cycle is as follows:

$$\text{If } M|N,\ \widetilde{PC}_{Sl,p} = c_w e^{-R\widetilde{T}_{(p-1)M}} \widetilde{RW}_{l,p} \tag{3.140}$$

$$\text{If } M \nmid N,\ \widetilde{PC}_{Sl,p+1} = c_w e^{-R\widetilde{T}_{pM}} \widetilde{RW}_{l,p+1} \tag{3.141}$$

Hence, present value of the total purchase cost of the supplier is as follows:

$$\widetilde{PC}_S = \begin{cases} \sum_{j=1}^{p-1} \widetilde{PC}_{Sj} + \widetilde{PC}_{Sl,p}, & \text{for } M|N \\ \sum_{j=1}^{p} \widetilde{PC}_{Sj} + \widetilde{PC}_{Sl,p+1}, & \text{for } M \nmid N \end{cases} \tag{3.142}$$

Holding cost of raw material: Present value of the total holding cost of raw material of the supplier is as follows:

$$\widetilde{HCR}_S = \begin{cases} c_{hw}\left(\sum_{j=1}^{p-1} e^{-R\widetilde{T}_{(j-1)M}}\widetilde{HA}_j + e^{-R\widetilde{T}_{(p-1)M}}\widetilde{HA}_{l,p}\right), \text{ for } M|N \\ c_{hw}\left(\sum_{j=1}^{p} e^{-R\widetilde{T}_{(j-1)M}}\widetilde{HA}_j + e^{-R\widetilde{T}_{pM}}\widetilde{HA}_{l,p+1}\right), \text{ for } M \nmid N \end{cases} \tag{3.143}$$

Ordering cost: Present value of the total ordering cost of the supplier is as follows:

$$\widetilde{OC}_S = \begin{cases} \sum_{j=1}^{p} c_{sw}e^{-R\widetilde{T}_{(j-1)M}}, \text{ for } M|N \\ \sum_{j=1}^{p+1} c_{sw}e^{-R\widetilde{T}_{(j-1)M}}, \text{ for } M \nmid N \end{cases} \tag{3.144}$$

Hence, present value of the total profit of the supplier from the whole planning horizon is as follows:

$$\widetilde{Z}_S = \widetilde{SR}_S - \widetilde{PC}_S - \widetilde{HCR}_S - \widetilde{OC}_S \tag{3.145}$$

Let α-cut of $\widetilde{Z}_S$ be $\widetilde{Z}_S[\alpha] = [Z_{SL}(\alpha), Z_{SR}(\alpha)]$, where

$$Z_{SL}(\alpha) = SR_{SL} - PC_{SR} - HCR_{SR} - OC_{SR} \tag{3.146}$$

$$Z_{SR}(\alpha) = SR_{SR} - PC_{SL} - HCR_{SL} - OC_{SL} \tag{3.147}$$

Promotional cost: Present value of the promotional cost for the manufacturer's i-th cycle is as follows:

$$\begin{aligned}\widetilde{PrC}_i &= (m_2 - m_1)c_{pi}e^{-R\widetilde{T}_{i-1}}\int_{\widetilde{T}_{i-1}}^{\widetilde{T}_{i-1}+\tau_2}\widetilde{D}_i(t)dt \\ &= (m_2 - m_1)c_{pi}e^{-R\widetilde{T}_{i-1}}\widetilde{I}_{4i}\end{aligned} \tag{3.148}$$

Hence, present value of the total promotional cost is $\widetilde{PrC} = \sum_{i=1}^{N}\widetilde{PrC}_i$. Let α-cut of $\widetilde{PrC}_i$ be $\widetilde{PrC}_i[\alpha] = [PrC_{iL}(\alpha), PrC_{iR}(\alpha)]$, where

$$PrC_{iL}(\alpha) = (m_2 - m_1)c_{pi}e^{-RT_{(i-1)R}}I_{4iL}$$

$$PrC_{iR}(\alpha) = (m_2 - m_1)c_{pi}e^{-RT_{(i-1)L}}I_{4iR}$$

When the supplier does not share any part of this promotional cost, then the inventory decisions are made by the manufacturer only, i.e., only the manufacturer's profit $\widetilde{Z}_M$ is maximized to identify a marketing decision. This phenomenon is known as *non-coordination scenario* (NCS). So the mathematical form in this scenario is given by:

$$\left.\begin{array}{l}\text{Determine } \tau_1, \lambda, m_1, \tau_2, N \\ \text{to maximize } [Z_{ML}, Z_{MR}] \\ \text{subject to } \sum_{i=1}^{N} \widetilde{t}_i \leq \widetilde{H}\end{array}\right\} \tag{3.149}$$

As the fuzzy constraints are not well defined, using fuzzy chance constraints [103, 104], the above problem can be written in optimistic and pessimistic sense in the following form.

$$\left.\begin{array}{l}\text{Maximize } [Z_{ML}, Z_{MR}] \\ \text{subject to } Pos\Big(\sum_{i=1}^{N} \widetilde{t}_i \leq \widetilde{H}\Big) \geq \eta_1\end{array}\right\} \tag{3.150}$$

$$\left.\begin{array}{l}\text{Maximize } [Z_{ML}, Z_{MR}] \\ \text{subject to } Nes\Big(\sum_{i=1}^{N} \widetilde{t}_i \leq \widetilde{H}\Big) \geq \eta_2\end{array}\right\} \tag{3.151}$$

where, η_1 and η_2 are predefined levels of possibility and necessity respectively, which are entirely determined by the decision makers (DM).

Depending upon the manufacturer's decision, the supplier tries to improve his/her profit. So the problem of the supplier mathematically takes the following form:

$$\left.\begin{array}{l}\text{Determine } M \\ \text{to maximize } [Z_{SL}, Z_{SR}] \\ \text{subject to } \sum_{i=1}^{N} \widetilde{t}_i \leq \widetilde{H}\end{array}\right\} \tag{3.152}$$

If the supplier offers to pay a fraction (F) of the promotional cost, then this phenomenon is known as *coordination scenario* (CS). Here, both the manufacturer and the supplier are DM and hence their joint profit ($\widetilde{Z}_T = \widetilde{Z}_M + \widetilde{Z}_S$) is maximized to find the marketing decision. In this case, the profits of the manufacturer and the supplier are as follows:

$$\widetilde{Z}_M^F = \widetilde{SR}_M - \widetilde{PC}_M - \widetilde{HC}_M - \widetilde{HCR}_M - \widetilde{SC}_M + \widetilde{PrC} - (1-F)\widetilde{PrC} \quad (3.153)$$

$$\widetilde{Z}_S^F = \widetilde{SR}_S - \widetilde{PC}_S - \widetilde{HCR}_S - \widetilde{OC}_S - F\widetilde{PrC} \quad (3.154)$$

So their joint profit $\widetilde{Z}_T$ $(= \widetilde{Z}_M^F + \widetilde{Z}_S^F)$ is to be maximized. The mathematical form in this scenario is given by:

$$\left.\begin{array}{l} \text{Determine } \tau_1, \lambda, m_1, \tau_2, N, M \\ \text{to maximize } [Z_{TL}, Z_{TR}] \\ \text{subject to } \sum_{i=1}^{N} \widetilde{t}_i \le \widetilde{H} \end{array}\right\} \quad (3.155)$$

As the fuzzy constraints are not well defined, using fuzzy chance constraints [103], the above problem can be written in an optimistic and pessimistic sense in the following form.

$$\text{Maximize } [Z_{TL}, Z_{TR}] \quad (3.156)$$

subject to constraints are given in (3.150) and (3.151) for optimistic and pessimistic sense respectively.

Lemma 3.1. For any types of fuzzy numbers, $Pos(\sum_{i=1}^{N} \widetilde{t}_i \le \widetilde{H}) \ge \eta_1$, iff $\sum_{i=1}^{N} t_{iL}(\eta_1) \le H_R(\eta_1)$.

Proof. cf. [99]. □

Lemma 3.2. For any types of fuzzy numbers, $Nes(\sum_{i=1}^{N} \widetilde{t}_i \le \widetilde{H}) \ge \eta_2$, iff $\sum_{i=1}^{N} t_{iR}(1-\eta_2) \le H_L(1-\eta_2)$.

Proof. cf. [99]. □

Using Lemma 3.1 and Lemma 3.2, the above problems are reduced to the following problems for an optimistic decision maker (ODM) and a pessimistic decision maker (PDM) respectively.

Model 3.3.1 (ODM):

$$\left.\begin{array}{l}\text{For NCS: Determine } \tau_1, \lambda, m_1, \tau_2, N \text{ to maximize } [Z_{ML}, Z_{MR}] \\ \text{For CS: Determine } \tau_1, \lambda, m_1, \tau_2, N, M \text{ to maximize } [Z_{TL}, Z_{TR}] \\ \text{subject to } \sum_{i=1}^{N} t_{iL}(\eta_1) \leq H_R(\eta_1)\end{array}\right\} \quad (3.157)$$

Model 3.3.2 (PDM):

$$\left.\begin{array}{l}\text{For NCS: Determine } \tau_1, \lambda, m_1, \tau_2, N \text{ to maximize } [Z_{ML}, Z_{MR}] \\ \text{For CS: Determine } \tau_1, \lambda, m_1, \tau_2, N, M \text{ to maximize } [Z_{TL}, Z_{TR}] \\ \text{subject to } \sum_{i=1}^{N} t_{iR}(1-\eta_2) \leq H_L(1-\eta_2)\end{array}\right\} \quad (3.158)$$

3.4.3 Numerical Illustration and Discussion

To illustrate the model following hypothetical example is used:

Example 3.4. The following parametric values are used in appropriate units: $\widetilde{K} = (K_1, K_2, K_3) = (124, 126, 127)$, $\widetilde{A}_1 = (A_{11}, A_{12}, A_{13}) = (397, 400, 403)$, $\widetilde{A}_2 = (A_{21}, A_{22}, A_{23}) = (58, 60, 62)$, $\widetilde{H} = (H_1, H_2, H_3) = (16.5, 17, 17.5)$, $m_2 = 2.3$, $d = 0.09$, $I = 0.07$, $\beta = 2.5$, $\gamma = 2.2$, $c_r = 1.5$, $L_0 = 0.35$, $L_1 = 0.4$, $\delta_1 = 0.7$, $c_0 = 30$, $c_1 = 18$, $\delta_2 = 0.6$, $m_h = 0.08$, $m_{hr} = 0.0533$, $c_w = 0.7$, $m_{hw} = 0.0429$, $c_{sw} = 50$, $\alpha = 0.5$, $\eta_1 = 0.9$, $\eta_2 = 0.9$.

Model 3.3.1: For the above parametric values the Model 3.3.1 (Eqn. (3.157)) is solved for non-coordination scenario using proposed MCABC due to different values of N and results are presented in Table 3.18. From Table 3.18, it is observed that the manufacturer's profit initially increases with N, reaches a maximum limit and then decreases as N increases. For the above Example 3.4, the maximum profit of the manufacturer is obtained for $N = 4$ and the corresponding values of the decision variables τ_1, λ, m_1, τ_2 are given in the Table 3.18.

Taking $N = 4$, the results are obtained for the same model for 5 different runs of the algorithm using different seeds of the random number generator and the results are presented in Table 3.19. From Table 3.19 it is clear that obtained results are non-dominated with respect to mid values and widths of the α-cuts of the profits. So a DM will choice a solution according to his/her requirement.

Table 3.18: Parametric Study of N to maximize Manufacturer's Profit in NCS for Model 3.3.1

N	τ_1	λ	m_1	τ_2	$[Z_{ML}, Z_{MR}]$	Z_{MC}	Z_{MW}
1	1.635870	0.066011	1.724796	3.014075	[268.275909, 325.794159]	297.035034	28.759125
2	0.968247	0.481006	1.807258	3.075621	[384.521729, 463.939178]	424.230453	39.708725
3	0.685424	0.269990	1.900273	3.433820	[418.268677, 511.371033]	464.819855	46.551178
4	0.444196	0.233530	1.864071	2.619529	[426.520996, 532.887817]	479.704407	53.183411
5	0.374780	0.152535	1.899697	2.550462	[422.369873, 536.941895]	479.655884	57.286011
6	0.320549	0.112788	1.820056	1.819064	[407.616821, 532.827576]	470.222198	62.605377

Table 3.19: Compare the Manufacturer's Profit for $N = 4$ with different runs in NCS for Model 3.3.1

N	Run	τ_1	λ	m_1	τ_2	$[Z_{ML}, Z_{MR}]$	Z_{MC}	Z_{MW}
4	1	0.444196	0.233530	1.864071	2.619529	[426.520996, 532.887817]	479.704407	53.183411
4	2	0.388709	0.264885	1.895802	2.808316	[426.631012, 531.518188]	479.074600	52.443588
4	3	0.508968	0.182206	1.930757	3.102330	[426.526276, 529.546021]	478.036148	51.509872
4	4	0.426350	0.213700	1.905722	2.370341	[427.401642, 527.475891]	477.438766	50.037125
4	5	0.448132	0.240536	1.875520	2.896881	[426.120514, 534.058472]	480.089493	53.968979

Table 3.20: Parametric Study of M to maximize Supplier's Profit in NCS for Model 3.3.1

N	M	τ_1	λ	m_1	τ_2	$[Z_{SL}, Z_{SR}]$	Z_{SC}	Z_{SW}
4	1	0.444196	0.233530	1.864071	2.619529	[87.317589, 107.243599]	97.280594	9.963005
4	2	0.444196	0.233530	1.864071	2.619529	[131.624146, 150.067032]	140.845589	9.221443
4	3	0.444196	0.233530	1.864071	2.619529	[120.765884, 139.282333]	130.024109	9.258224
4	4	0.444196	0.233530	1.864071	2.619529	[83.091049, 99.492584]	91.291817	8.200768

In NCS, as the manufacturer is the leader and the supplier is the follower, supplier will try to maximize his/her profit depending upon the decision of the manufacturer. So the profit of the supplier is obtained due to different values of M for $N = 4$. From Table 3.20, it is found that the supplier's profit is maximum for $M = 2$.

In coordination scenario, the supplier likes to pay a fraction F of the promotional cost to improve the channel performance. So, in this scenario, the joint profit $\widetilde{Z}_T$ is maximized. A parametric study of N and M are obtained for maximizing the joint profit of the manufacturer and the supplier. The results are presented in Table 3.21. From this table, it is found that the joint profit of both the parties is maximum, when $N = 6$ and $M = 2$.

Taking the above values of N and M, i.e., for $N = 6$ and $M = 2$, the joint profit is maximized for different values of F and in different runs. In all the runs channel profit as well as individual profits are calculated and presented in Table

TABLE 3.21: Parametric Study of N & M to maximize Joint Profit in CS for Model 3.3.1

N	M	τ_1	λ	m_1	$tau2$	$[Z_{TL}, Z_{TR}]$	Z_{TC}	Z_{TW}
1	1	2.134890	0.161472	1.260253	3.012188	[395.750153,474.269562]	435.009857	39.259705
2	1	1.324556	0.794996	1.361757	2.940942	[532.317810,666.442627]	599.380219	67.062408
2	2	1.033177	0.750686	1.608317	3.178465	[487.123505,592.450684]	539.787094	52.663589
3	1	1.107810	0.543120	1.385446	3.392156	[567.155029,789.882629]	678.518829	111.363800
3	2	0.829873	0.621440	1.455760	3.284660	[575.881226,766.807617]	671.344421	95.463196
3	3	0.746652	0.563368	1.450600	2.601966	[503.896423,667.063721]	585.480072	81.583649
4	1	0.904255	0.381949	1.339244	2.769432	[544.440247,850.973267]	697.706757	153.266510
4	2	0.792710	0.281702	1.548447	3.194561	[587.618530,804.875183]	696.246857	108.628326
4	3	0.586568	0.376480	1.493080	2.627738	[573.773804,784.557739]	679.165771	105.391968
4	4	0.545268	0.334028	1.648120	3.118318	[518.918823,694.842590]	606.880707	87.961884
5	1	0.756052	0.296582	1.308280	2.397151	[496.780212,898.115234]	697.447723	200.667511
5	2	0.559030	0.225691	1.570960	2.728707	[587.534180,829.069153]	708.301666	120.767487
5	3	0.656831	0.272640	1.348654	2.198505	[551.300293,874.151489]	712.725891	161.425598
5	4	0.475112	0.210760	1.649572	2.614281	[559.735413,758.913818]	659.324615	99.589203
5	5	0.485280	0.198843	1.654654	2.548760	[503.400879,693.374695]	598.387787	94.986908
6	1	0.573411	0.166962	1.403634	1.907482	[476.551147,799.732178]	638.141663	161.590515
6	2	0.558340	0.212000	1.317682	1.862773	[529.298584,914.546021]	721.922302	192.623718
6	3	0.488285	0.168798	1.564466	2.440883	[576.980347,849.680908]	713.330627	136.350281
6	4	0.481591	0.178160	1.501000	2.173954	[566.460938,851.366211]	708.913574	142.452637
6	5	0.422211	0.162675	1.540960	1.934062	[528.053955,764.884460]	646.469208	118.415253
6	6	0.330616	0.147887	1.643594	1.720940	[492.587341,672.769958]	582.678650	90.091309
7	1	0.448063	0.129440	1.505064	1.959720	[442.883057,762.079041]	602.481049	159.597992
7	2	0.448063	0.129440	1.505063	1.959710	[541.406250,857.344238]	699.375244	157.968994
7	3	0.448528	0.129842	1.503586	1.959990	[550.849731,865.574219]	708.211975	157.362244
7	4	0.448353	0.129440	1.504673	1.958995	[554.977051,865.263489]	710.120270	155.143219
7	5	0.449590	0.129864	1.501499	1.954568	[537.205444,849.461731]	693.333588	156.128143
7	6	0.450541	0.129440	1.500800	1.949239	[484.405518,794.193970]	639.299744	154.894226
7	7	0.380469	0.131180	1.538183	1.860602	[453.275879,720.247314]	586.761597	133.485718

3.22. It is clear from the table that profits of both the parties improve in CS, if $F \in (0.11, 0.18)$. Optimum results of NCS and CS (for $F = 0.15$) are presented in Table 3.23.

Model 3.3.2: For Model 3.3.2, problem (3.158) is solved using MCABC, following the same approach as followed for Model 3.3.1 and results are presented in Table 3.24. In this case also, the same sort of results are obtained (as Model 3.3.1). From Table 3.24, it is observed that the manufacturer's profit is maximum for $N = 4$ in NCS. For this value of N, the parametric study of M is presented in Table 3.25. In CS, the parametric study of N and M are done and the results are

TABLE 3.22: Individual & Joint Profit for different values of F in CS for Model 3.3.1

N	M	F	Run	$[Z_{ML}, Z_{MR}]$	Z_{MC}	Z_{MW}	$[Z_{SL}, Z_{SR}]$	Z_{SC}	Z_{SW}	$[Z_{TL}, Z_{TR}]$	Z_{TC}	Z_{TW}
6	2	0.100	1	[312.89,634.52]	473.70	160.82	[210.63,285.80]	248.22	37.59	[529.30,914.55]	721.92	192.62
6	2	0.100	2	[367.74,623.78]	495.76	128.02	[187.31,246.55]	216.93	29.62	[559.18,866.20]	712.69	153.51
6	2	0.100	3	[367.02,630.39]	498.70	131.69	[191.66,252.84]	222.25	30.59	[562.96,878.94]	720.95	157.99
6	2	0.100	4	[363.62,625.91]	494.77	131.14	[188.61,248.24]	218.43	29.81	[556.31,870.08]	713.20	156.88
6	2	0.110	1	[324.55,646.76]	485.65	161.11	[198.39,274.14]	236.27	37.87	[529.30,914.55]	721.92	192.62
6	2	0.110	2	[377.04,633.49]	505.27	128.22	[177.59,237.25]	207.42	29.83	[559.18,866.20]	712.69	153.51
6	2	0.110	3	[376.54,640.35]	508.45	131.90	[181.70,243.31]	212.50	30.81	[562.96,878.94]	720.95	157.99
6	2	0.110	4	[373.00,635.69]	504.35	131.35	[178.83,238.87]	208.85	30.02	[556.31,870.08]	713.20	156.88
6	2	0.120	1	[336.21,659.00]	497.60	161.40	[186.15,262.48]	224.32	38.16	[529.30,914.55]	721.92	192.62
6	2	0.120	2	[386.34,643.20]	514.77	128.43	[167.88,227.95]	197.91	30.04	[559.18,866.20]	712.69	153.51
6	2	0.120	3	[386.07,650.30]	518.19	132.12	[171.74,233.78]	202.76	31.02	[562.96,878.94]	720.95	157.99
6	2	0.120	4	[382.37,645.47]	513.92	131.55	[169.05,229.49]	199.27	30.22	[556.31,870.08]	713.20	156.88
6	2	0.130	1	[347.87,671.24]	509.56	161.69	[173.91,250.82]	212.37	38.45	[529.30,914.55]	721.92	192.62
6	2	0.130	2	[395.64,652.92]	524.28	128.64	[158.17,218.65]	188.41	30.24	[559.18,866.20]	712.69	153.51
6	2	0.130	3	[395.60,660.26]	527.93	132.33	[161.78,224.25]	193.02	31.23	[562.96,878.94]	720.95	157.99
6	2	0.130	4	[391.74,655.25]	523.50	131.76	[159.27,220.12]	189.70	30.43	[556.31,870.08]	713.20	156.88
6	2	0.140	1	[359.53,683.48]	521.51	161.97	[161.67,239.16]	200.42	38.74	[529.30,914.55]	721.92	192.62
6	2	0.140	2	[404.94,662.63]	533.79	128.84	[148.45,209.35]	178.90	30.45	[559.18,866.20]	712.69	153.51
6	2	0.140	3	[405.13,670.22]	537.68	132.54	[151.83,214.72]	183.28	31.45	[562.96,878.94]	720.95	157.99
6	2	0.140	4	[401.12,665.04]	533.08	131.96	[149.49,210.75]	180.12	30.63	[556.31,870.08]	713.20	156.88
6	2	0.150	1	[371.19,695.72]	533.46	162.26	[149.44,227.50]	188.47	39.03	[529.30,914.55]	721.92	192.62
6	2	0.150	2	[414.24,672.34]	543.29	129.05	[138.74,200.05]	169.39	30.65	[559.18,866.20]	712.69	153.51
6	2	0.150	3	[414.66,680.18]	547.42	132.76	[141.87,205.20]	173.53	31.66	[562.96,878.94]	720.95	157.99
6	2	0.150	4	[410.49,674.82]	542.65	132.16	[139.71,201.38]	170.54	30.83	[556.31,870.08]	713.20	156.88
6	2	0.160	1	[382.85,707.96]	545.41	162.55	[137.20,215.83]	176.52	39.32	[529.30,914.55]	721.92	192.62
6	2	0.160	2	[423.54,682.06]	552.80	129.26	[129.03,190.75]	159.89	30.86	[559.18,866.20]	712.69	153.51
6	2	0.160	3	[424.19,690.13]	557.16	132.97	[131.91,195.67]	163.79	31.88	[562.96,878.94]	720.95	157.99
6	2	0.160	4	[419.86,684.60]	552.23	132.37	[129.93,192.00]	160.97	31.04	[556.31,870.08]	713.20	156.88
6	2	0.170	1	[394.52,720.20]	557.36	162.84	[124.96,204.17]	164.56	39.61	[529.30,914.55]	721.92	192.62
6	2	0.170	2	[432.85,691.77]	562.31	129.46	[119.31,181.45]	150.38	31.07	[559.18,866.20]	712.69	153.51
6	2	0.170	3	[433.72,700.09]	566.90	133.19	[121.95,186.14]	154.05	32.09	[562.96,878.94]	720.95	157.99
6	2	0.170	4	[429.24,694.38]	561.81	132.57	[120.15,182.63]	151.39	31.24	[556.31,870.08]	713.20	156.88
6	2	0.180	1	[406.18,732.44]	569.31	163.13	[112.72,192.51]	152.61	39.90	[529.30,914.55]	721.92	192.62
6	2	0.180	2	[442.15,701.48]	571.82	129.67	[109.60,172.15]	140.87	31.27	[559.18,866.20]	712.69	153.51
6	2	0.180	3	[443.25,710.05]	576.65	133.40	[111.99,176.61]	144.30	32.31	[562.96,878.94]	720.95	157.99
6	2	0.180	4	[438.61,704.16]	571.38	132.77	[110.37,173.26]	141.81	31.44	[556.31,870.08]	713.20	156.88
6	2	0.190	1	[417.84,744.68]	581.26	163.42	[100.48,180.85]	140.66	40.19	[529.30,914.55]	721.92	192.62
6	2	0.190	2	[451.45,711.20]	581.32	129.88	[99.88,162.85]	131.37	31.48	[559.18,866.20]	712.69	153.51
6	2	0.190	3	[452.77,720.01]	586.39	133.62	[102.04,167.08]	134.56	32.52	[562.96,878.94]	720.95	157.99
6	2	0.190	4	[447.98,713.94]	580.96	132.98	[100.59,163.88]	132.24	31.65	[556.31,870.08]	713.20	156.88

TABLE 3.23: Individual & Joint Profit in NCS and CS for Model 3.3.1

	N	M	F	$[Z_{ML}, Z_{MR}]$	Z_{MC}	Z_{MW}	$[Z_{SL}, Z_{SR}]$	Z_{SC}	Z_{SW}	$[Z_{TL}, Z_{TR}]$	Z_{TC}	Z_{TW}
NCS	4	2	-	[426.52,532.89]	479.70	53.18	[131.62,150.07]	140.85	9.22	[558.15,682.95]	620.55	62.40
CS	6	2	0.150	[371.19,695.72]	533.46	162.26	[149.44,227.50]	188.47	39.03	[529.30,914.55]	721.92	192.62

presented in Table 3.26. From Table 3.27, it is observed that the profits of both the parties are improved in CS, if $F \in (0.12, 0.18)$. Optimum results of NCS and CS (for $F = 0.15$) are presented in Table 3.28.

Table 3.24: Parametric Study of N to maximize Manufacturer's Profit in NCS for Model 3.3.2

N	τ_1	λ	m_1	τ_2	$[Z_{ML}, Z_{MR}]$	Z_{MC}	Z_{MW}
1	1.616015	0.008619	1.670448	3.121371	[261.043762, 315.998688]	288.521225	27.477463
2	0.861548	0.580769	1.728580	2.596009	[360.456757, 435.665649]	398.061203	37.604446
3	0.498080	0.339494	1.923177	2.839843	[387.717987, 469.326508]	428.522247	40.804260
4	0.425721	0.186058	1.840142	2.031072	[387.646820, 480.490356]	434.068588	46.421768
5	0.287998	0.160200	1.879254	2.096062	[379.626709, 479.973907]	429.800308	50.173599
6	0.264240	0.103701	1.910317	1.951767	[366.014099, 469.982941]	417.998520	51.984421

Table 3.25: Parametric Study of M to maximize Supplier's Profit in NCS for Model 3.3.2

N	M	τ_1	λ	m_1	τ_2	$[Z_{SL}, Z_{SR}]$	Z_{SC}	Z_{SW}
4	1	0.425721	0.186058	1.840142	2.031072	[59.211803, 76.202530]	67.707167	8.495363
4	2	0.425721	0.186058	1.840142	2.031072	[110.962357, 126.728859]	118.845608	7.883251
4	3	0.425721	0.186058	1.840142	2.031072	[100.139969, 115.924706]	108.032337	7.892368
4	4	0.425721	0.186058	1.840142	2.031072	[76.992043, 91.086372]	84.039207	7.047165

Table 3.26: Parametric Study of N & M to maximize Joint Profit in CS for Model 3.3.2

N	M	τ_1	λ	m_1	$tau2$	$[Z_{TL}, Z_{TR}]$	Z_{TC}	Z_{TW}
4	1	0.812311	0.293040	1.441000	3.056950	[473.891602,708.928101]	591.409851	117.518250
4	2	0.632088	0.242530	1.640958	3.163715	[528.403687,692.822693]	610.613190	82.209503
4	3	0.609823	0.257410	1.578280	2.717598	[508.565430,675.284790]	591.925110	83.359680
4	4	0.631480	0.240996	1.640800	3.140959	[464.959137,625.128296]	545.043716	80.084579
5	1	0.468417	0.179518	1.624842	2.441100	[446.041473,627.302368]	536.671921	90.630447
5	2	0.557123	0.182240	1.547200	2.513613	[503.374969,713.700256]	608.537613	105.162643
5	3	0.574855	0.219360	1.464744	2.514971	[498.737854,744.102051]	621.419952	122.682098
5	4	0.488261	0.185095	1.553406	2.217347	[487.160339,675.657349]	581.408844	94.248505
5	5	0.417592	0.168240	1.727256	2.560136	[466.464233,618.289673]	542.376953	75.912720
6	1	0.481308	0.130298	1.511915	2.004640	[390.411285,623.624329]	507.017807	116.606522
6	2	0.485928	0.129440	1.499957	1.957885	[482.437683,713.345337]	597.891510	115.453827
6	3	0.485928	0.129577	1.500078	1.960239	[487.790314,716.312500]	602.051407	114.261093
6	4	0.454574	0.155192	1.478370	2.047207	[482.891846,731.002808]	606.947327	124.055481
6	5	0.438970	0.135472	1.508422	1.857131	[452.383911,667.890625]	560.137268	107.753357
6	6	0.346050	0.131040	1.647520	1.917322	[446.101288,616.096130]	531.098709	84.997421

TABLE 3.27: Individual & Joint Profit for different values of F in CS for Model 3.3.2

N	M	F	$[Z_{ML}, Z_{MR}]$	Z_{MC}	Z_{MW}	$[Z_{SL}, Z_{SR}]$	Z_{SC}	Z_{SW}	$[Z_{TL}, Z_{TR}]$	Z_{TC}	Z_{TW}
5	3	0.110	[330.25,537.86]	434.05	103.81	[164.98,209.75]	187.37	22.39	[498.74,744.10]	621.42	122.68
5	3	0.120	[338.83,546.76]	442.80	103.96	[156.07,201.17]	178.62	22.55	[498.74,744.10]	621.42	122.68
5	3	0.130	[347.42,555.67]	451.55	104.12	[147.17,192.58]	169.87	22.71	[498.74,744.10]	621.42	122.68
5	3	0.140	[356.01,564.58]	460.29	104.28	[138.26,183.99]	161.13	22.87	[498.74,744.10]	621.42	122.68
5	3	0.150	[364.59,573.48]	469.04	104.44	[129.36,175.41]	152.38	23.03	[498.74,744.10]	621.42	122.68
5	3	0.160	[373.18,582.39]	477.78	104.60	[120.45,166.82]	143.64	23.19	[498.74,744.10]	621.42	122.68
5	3	0.170	[381.77,591.29]	486.53	104.76	[111.55,158.23]	134.89	23.34	[498.74,744.10]	621.42	122.68
5	3	0.180	[390.35,600.20]	495.28	104.92	[102.64,149.65]	126.14	23.50	[498.74,744.10]	621.42	122.68
5	3	0.190	[398.94,609.10]	504.02	105.08	[93.73,141.06]	117.40	23.66	[498.74,744.10]	621.42	122.68

TABLE 3.28: Individual & Joint Profit in NCS and CS for Model 3.3.2

	N	M	F	$[Z_{ML}, Z_{MR}]$	Z_{MC}	Z_{MW}	$[Z_{SL}, Z_{SR}]$	Z_{SC}	Z_{SW}	$[Z_{TL}, Z_{TR}]$	Z_{TC}	Z_{TW}
NCS	4	2	-	[387.65,480.49]	434.07	46.42	[110.96,126.73]	118.85	7.88	[498.61,607.22]	552.91	54.31
CS	5	3	0.150	[364.59,573.48]	469.04	104.44	[129.36,175.41]	152.38	23.03	[498.74,744.10]	621.42	122.68

3.5 Conclusion

In the Model 3.1, a coordinated SC sharing of the promotional cost among the wholesaler and the retailer with stock and promotional effort influenced demand is formulated and solved. With the above mentioned demand of a deteriorating item, sharing of promotional cost between SC partners is determined for the benefit of the individual profits as well as channel profit. Here, a conventional PSO algorithm is implemented, tested and tacitly used to solve the above problem in crisp and imprecise environments. Its performance is compared with the LINGO 14.0 software and ABC algorithm. From the different tables of the obtained results of the models in different scenarios following marketing decisions can be outlined:

- It is found in all the experiments that the promotional effort (ρ) of the item is grater than 1. So the promotional effort has a positive effect in a SC of deteriorating item.
- It is also found that the profits for both the parties (i.e., the wholesaler and the retailer) increase in the CS than the NCS for a compromise value of $F \in (F_{min}, F_{max})$, i.e., if the wholesaler bears a compromise portion of promotional cost then it is beneficial for both the parties. So, the theoretical expected result agrees with the numerical findings.
- In all the studies it is observed that $t_r > 0$, i.e., two warehouse strategy is beneficial for the retailer with limited outlet capacity.

- In all the studies it is observed that $t_s > 0$, i.e., backlogging is beneficial for the proposed SC model.

- Efficiency of the PSO of solving such real life complex decision making problem with respect to accuracy and computational time is well established by this study.

- Moreover, for the first time, Taguchi method is used for the parameter setting of a heuristic algorithm to solve any SC/inventory model.

In the Model 3.2, a coordinated SC sharing the promotional cost among the wholesaler and retailer is formulated and solved with time, price and promotional cost dependent demand. For the first time, with the above demand, sharing of promotional cost between SC partners is determined for maximum channel profit. Here, the algorithm of the proposed PSO is presented and used for the above problem. An approach is proposed where fuzzy objective is directly optimized without transforming it into equivalent crisp objective. The present model can be modified with other types of demand function, variable deterioration etc.

Advantages of the proposed model are as follows:

(i) It gives a ready-made answer to the wholesaler and the retailer for their shares of investment (for promotions) for the maximum benefit of both parties.

(ii) Nowadays, due to several factors, system parameters are taken fuzzy. Normally, these are converted to crisp values by different methods such as graded mean value, expectation etc. By this process, the original problem is approximated and we get an approximate optimum result. By the present method, the fuzzy objective function is not approximated, rather directly calculated as fuzzy numbers and their comparison is made using the credibility theory. This gives better results than the other processes. This can be used for fuzzy optimization problems in other areas such as transportation, portfolio management etc.

In the Model 3.3, ABC algorithm is modified to create its new variant MCABC and it is established that its performance is acceptable level for solving continuous optimization problems. The algorithm is capable of solving continuous optimization problems in crisp and imprecise environments. The proposed MCABC algorithm is better compared to any existing heuristic algorithm of continuous optimization problem in the literature with respect to accuracy and consistency. This algorithm is used to find the marketing decision of a real life supplier-manufacturer SC model with fuzzy demand and fuzzy production rate under promotional cost sharing and inflation. In this model, the promotional cost is used to provide price discount to the customers to increase the demand. It is established that if the supplier shares some portion of this promotional cost, then profits of both the parties (the supplier and the manufacturer) increase. Also for the first time, a SC model is studied under inflation when demand is price dependent.