

BCA 1st Semester Examination, 2019

DISCRETE MATHEMATICS

PAPER – 1103

Full Marks : 70

Time : 3 hours

Answer **Q.No.1** and any **six** questions from the rest

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

1. Answer any *five* questions : 2 × 5

(*a*) Let $A, B \subseteq R^2$ where $A = \{ (x, y) : y = 3x \}$ and $B = \{(x, y) : x - y = 7\}$. Find $\bar{A} \cup \bar{B}$.

(*b*) Does there exists a 4-regular graph on 6 vertices ? If so construct a graph.

(*c*) Give an example of a relation which is reflexive and symmetric but not transitive.

(*d*) If the function $f: R \to R$ defined by $f(x) = x^2$ then find $f^{-1}(-4)$.

(*e*) Give an example to show that the union of two subgroups is not a subgroup of that group.

(*f*) Is it possible to draw a simple graph with 4 vertices and 7 edges ? Justify.

(*g*) If a be fixed element of a ring R, then prove that the set $S = \{S \in R : xa = 0\}$ is a subring of R.

(*h*) Find a closed form for the generating function for the sequence 1, 1, 0, 1, 1,1,1,........

2. (*a*) Prove that the number of edges in a bipartite graph with n vertices is at most $\frac{n^2}{4}$. 5

(*b*) Solve $a_n = a_{n-1} + 2a_{n-2}$, $n \geq 2$ with the initial conditions $a_0 = 0$, $a_1 = 1$. 5

3. (*a*) Prove that a graph G is a tree if and only if there is a unique path between every pair of vertices in G. 5

(*b*) Examine whether the mapping $f : z \rightarrow z$ defined by $f(x) = 2x + 1$ for all $x \in z$, z is the set of all integers, is injective or surjective or both. 5

4. (*a*) Prove that a connected graph has at least one spanning tree. 5

(*b*) Prove that the set H of all real matrices

$$\left\{ \begin{pmatrix} a & b \\ -b & a \end{pmatrix} : a^2 + b^2 = 1 \right\}$$

forms a commutative group with respect to matrix multiplication. 5

5. (*a*) Prove by mathematical induction that $6^{n+2} \pm 7^{2n+1}$ is divisible by 43 for each positive integer n. 5

(*b*) Find the principal conjuctive normal form of $(\sim p \Rightarrow r) \wedge (q \Leftrightarrow p)$. 5

6. (*a*) Draw the Hasse diagram for the divisibility relation on the set $\{2, 3, 6, 12, 24, 36\}$. 4

(*b*) Prove that the ring of matrices of the form $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ of real numbers is a field under matrix addition and matrix multiplication. 6

7. (*a*) There are 3 copies each of 4 different books. Find the number of ways of arranging them on a shelf. 5

(*b*) Simplify the following Boolean expression using *k*-map.

$$x_1x_2 + \bar{x}_1\bar{x}_2x_3 + x_2\bar{x}_3 + x_1x_2x_3 + x_1\bar{x}_2\bar{x}_3.$$ 5

8. (*a*) If R be a relation in the set of integers z defined by $R = \{ (x, y) : x \in z, y \in z, (x - y)$

is multiple of 3 }. Show that it is an equivalence relation. Find all equivalence classes. 3 + 3

(*b*) Write down the negation of each of the following statements propositional logic :

(*i*) He swims if and only if the water is warm.

(*ii*) Every one likes ice cream. 2 + 2

9. (*a*) Draw the graph with the help of a adjacency matrix : 5

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

(*b*) Define integral domain. Show that the ring of integers (z, + , .) is an integral domain. 5

10. (*a*) If a graph G has more than two vertices of odd degree then show that there can be no Euler path in G. 5

(*b*) Prove that every field is an integral domain. 5

[*Internal Assessment* : 30 Marks]