NEW

2019

Part-III 3-Tier

MATHEMATICS

PAPER—IV

(General)

Full Marks : 90

Time : 3 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Group—A

(Compulsory)

(Elements of Computer Science)

[Marks : 45]

1. Answer any *one* question : 1×15

(a) (i) Write the differences between machine language and assembly language ? What are the computer hardware and computer software. 3+2

(ii) What are the differences between STOP and END statements. 4

(iii) Prove that the complement of each element in a Boolean algebra B is unique. 2

(iv) Draw a circuit using only NAND gates that realizes the function $f(x, y, z) = x + yz$. 4

(b) (i) What do you mean by input/output devices ? Give some examples of commonly used input and output devices. 1+2

(ii) What is the fundamental (basic) unit to measure memory ? Write the relations between bit, byte, KB, MB, GB, TB. What is the source program and object program. 1+2+2

(iii) Why NOR gate is called a universal data ? 4

(iv) Simplify the following Boolean expression and draw the corresponding logic circuits.

$$a'b + abc' + abc$$ 2+1

2. Answer any *two* questions : 2×8

(a) (i) Express the following expression in FORTRAN equivalent :

$$A = \frac{1-\cos\theta}{1+\cos\theta} + \frac{e^{\beta^2}\sin\theta}{\sqrt{2\alpha x+\gamma}} \cdot \frac{ab^2}{c}$$ 2

(ii) Find the value of 3/2**2*2 + 3/8*3 + 3**2/2. 2

(iii) Write a FORTRAN program that reads in a given value of x and y and computers and prints the value o $F(x)$ defined by

$$f(x) = \begin{cases} 1 + \dfrac{x}{\sqrt{1+x^2}}, & \text{if } |x| < 3 \\ 0, & \text{if } 3 \le |x| < a \\ 1 - \dfrac{x}{\sqrt{1+x^2}}, & \text{if } |x| \ge a \end{cases}$$

where $a = (4 + |y|)^{3/2}$. 4

(b) (i) Explain different types of IF statements with example. 4

(ii) Find the following :

$(111011)_2 \times (11001)_2$, $(1101101)_2 - (101101)_2$ 2+2

(c) (i) Write the decimal equivalence of the binary number (11011.0011) 2

(ii) Write a program in FORTRAN to find the product of two matrices of order $m \times n$ and $n \times l$. 6

3. Answer any *three* questions : 3×4

(a) State and prove De Morgan's laws without using truth table. 4

(b) Indicate the printed from of the output for the following statement :

PRINT 9, I, M, A, B

I FORMAT (5X, I2, 2X, I4 // F 6.2, 5X, E15.5)

where I = −4, M = 2, A = −4.1, B = −0.00142 4

(c) Write a FORTRAN statement to read the following with suitable FORMAT specifications.

A = 345.257, B = 0.298×10^{-7}, M = 5123, C = "VIDYASAGAR UNIVERSITY".

(d) Write an algorithm to test whether an integer is even or odd. 4

(e) Draw a flow chart to final the area of a triangle when three sides are given. 4

4. Answer any *one* question : 1×2

(a) What are the rules for naming a FORTRAN variable ? What are those variables ? 2

(b) Differentiate between sources program and object program.

Group—B

(Probability and Statistics)

[Marks : 45]

5. Answer any *one* question : 1×15

(a) (i) State and prove Bayes theorem. 5

(ii) An urn contains 6 red and 4 black balls. Two balls are drawn at random. Find the probability that

I. both the balls drawn are red,

II. One ball is red and other ball is black,

III. both of them are other ball is black. 5

(iii) Find the mean and variance of the Normal distribution. 2+3

(b) (i) For what value of k, $f(x, y)$ represents the probability density function of two continuous random variables X and Y, where

$$f(x,y) = K(4 - 2x + y), 0 < x < 3, 2 < y < 4$$
$$= 0, \text{ elsewhere}$$

and find $P(X < 2 / Y < 3)$. 2+2

(ii) For any two evens A and B, prove that

I. if $A \subseteq B$, then $P(A) \leq P(B)$

II. $P(AB) \leq P(A) \leq P(A + B) \leq P(A) + P(B)$

III. If $P(A) = \frac{3}{4}$ and $P(B) = \frac{5}{8}$, then $\frac{3}{8} \leq P(AB) \leq P(AB) \leq \frac{5}{8}$.

2+2+2

(iii) The probability density function of a random variable X is symmetric about the origin. Prove that X and $-X$ have the same distribution. 5

6. Answer any *three* questions : 8×3

(a) (i) If θ be the acute angle between the two regression lines, then show that

$$\tan\theta = \frac{1-r^2}{r}\frac{S_x S_y}{S_x^2 + S_y^2},$$ where r is the correlation coefficient of the variables X and Y; S_x and S_y are the standard deviations of X and Y respectively. Give the interpretations when $r = 0, \pm 1$. 4+2

(ii) Define skewness. What does skewness indicate ? Explain with diagram. 4

(b) If r_{xy} denotes the correlation coefficient between two variables x and y, then prove that $-1 \le r_{xy} \le 1$. 8

(c) Calculate the mean, median and mode of the following frequency distribution : 8

Height in inches	56-60	61-65	66-70	71-75	76-80
No. of persons :	7	25	43	28	7

(d) (i) Obtain the maximum likelihood estimates of α and β for random sample from the exponential population

$$f(x, \alpha, \beta) = \beta e^{-\beta(x-\alpha)}, \alpha \leq x < \infty.$$ 4

(ii) Show that sample mean and sample variance are uncorrelated if third central moment is zero. 4

(e) A sample of 11 rats from a central population had an average blood viscosity of 3.92 with standard deviation of 0.61. On the basis of this sample find 95% confidence interval of the population mean blood viscosity (assume the population is normal and $P(t > 2.228) = 0.025$).

7. Answer any *two* questions : 2×3

(a) If the mutually independent random variables X_1, X_2, ..., X_n all have the same distribution and their sum $X_1 + X_2 + \ldots + X_n$ is normally distributed then show that each of them is normally distributed.

(b) Two cricketers A and B made the following runs in 5 successive innings. Use a measure of relative variability to compare their consistency.

Cricketer A :	26	5	76	112	97
Cricketer B :	47	51	58	60	36

(c) Give an axiomatic definition of probability and from it deduce the classical definition of probability.

———