2019

Part – II

MATHEMATICS
(General)

Paper – III

Full Marks – 90

Time : 3 Hours

The figures in the right-hand margin indicate marks.
Candidates are required to give their answers in their own words as far as practicable.
Illustrate the answers wherever necessary.

Group – A

[Linear Programming Problem]

[Marks : 35]

1. Answer any **one** question : 15×1

(a) (i) Solve the following L.P.P. graphically. 5

Maximize $z = 5x + 7y$

Subject to $2x + 3y \le 6$

$3x + 4y \le 12$

$x, \ y \ge 0$

(ii) Define convex combination and convex set. Prove that hyperplane is a convex set. 5

(iii) Show that all the basic feasible solution of the system –

$2x_1 + 6x_2 + 2x_3 + x_4 = 3$

$6x_1 + 4x_2 + 4x_3 + 6x_4 = 2$

are degenerate. 5

(b) (i) Examine whether the set $S = \{(x, y) : y^2 \geq 4x\}$ is convex or not. Find the extreme points, if any of the following set 5

$$S_1 = \{(x_1, x_2) : x_1^2 + x_2^2 \leq 1, x_1 \geq 0, x_2 \geq 0\}$$

(ii) Prove that a basic feasible solution of the system of equations $Ax=b$, $x \geq 0$ corresponds to an extreme point of the convex set of feasible solutions. 5

(iii) A person has two types of machines and he must have at least 2 first type of machine and 5 second type of machine. The cost of first type machine is Rs.2000 and it required $20m^2$ space whereas the cost of each 2nd type machine is Rs. 1500 and it requires $30m^2$ space. His capital is Rs. 20,000 and the available space is 220 m^2. Profit from each first type machine is Rs. 70 and that from each 2nd type machine is Rs. 110. Formulate on LPP for maximizing the profit earned. 5

2. Answer any **two** question : 8×2

(a) Solve the following linear programming problem by penalty method.

Maximize $z = 2x_1 - 6x_2$

Subject to $3x_1 + 2x_2 \le 6$

$x_1 - x_2 \ge -1$

$-x_1 - 2x_2 \ge 1$

$x_1, \quad x_2 \ge 0$

(b) An office has four workers and four tasks have to be performed. Workers differ in efficiency and tasks differ in their intrinsic difficulty. Time each worker would take to complete each task is given in the effectiveness matrix. How the tasks should be allocated to each worker so as to minimize the total man-hour ?

		Workers			
		I	II	III	IV
	A	5	23	14	8
Tasks	B	10	25	1	23
	C	35	16	15	12
	D	16	23	21	7

(c) Find the optimal solution of thefollowing transportation problem.

Capacities

	D_1	D_2	D_3	D_4	
O_1	5	4	6	14	15
O_2	2	9	9	6	4
O_5	6	11	7	13	8
Demand	9	7	5	6	

3. Answer any **one** question : 4×1

(a) Reduce the feasible solution (1, 3, 2) of the system of linear equations

$$2x_1 + 4x_2 - 2x_3 = 10$$

$$10x_1 + 3x_2 + 7x_3 = 33$$

to a basic feasible solution.

(b) Show that the number of basic variables is a balanced transportation problem is atmost $(m+n-1)$ where m is the number of origins and n destinations.

Group – B

[Numerical Analysis]

[Marks : 20]

4. Answer any **one** question : 8×1

(a) (i) Solve the system of equations by Gauss-Elimination method :

$$2x_1 + 3x_2 + x_3 = 9$$
$$x_1 + 2x_2 + 3x_3 = 6$$
$$3x_1 + x_2 + 2x_3 = 8$$

Correct upto 3-significant figures. 6

(ii) Show that $\left(\frac{\Delta^2}{E}\right)x^3 = 6x$ 2

(b) From the following table, find $\frac{dy}{dx}$ at x=1.1

x	1.0	1.1	1.2	1.3	1.4	1.5	1.6
y	7.989	8.403	8.781	9.129	9.451	9.750	10.031

8

5. Answer any **three** questions : 4×3

(a) Given $\log_{10}654=2.8156$, $\log_{10}658=2.8182$, $\log_{10}659=2.8189$, $\log_{10}661=2.8202$

Find $\log_{10}656$ using Lagrange's interpolation formula.

(b) Explain Gauss-Seidel iteration method for solving system of Linear equations.

(c) Evaluate $\int_2^{10} \frac{1}{1+x}dx$ by using Trapezoidal rule. take h=1.

(d) Evaluate the missing term in the following table :

m	0	1	2	3	4	5
f(x)	0	–	8	15	–	35

(e) Find the value of $\sqrt[3]{2}$ correct upto four decimal places using Newton-Raphson method.

Group – C
[Analytical Dynamics]
[Marks : 35]

6. Answer any **one** question : 15×1

(a) (i) A heavy particle is attached to the lower end of an elastic string, the upper end of which is fixed. The modulus of elasticity of the string is equal to the weight of the particle. The string is drawn vertically down till it is four times its natural length and then let go. Prove that the particle will return to this point in time $\sqrt{\frac{a}{g}}\left(2\sqrt{3}+\frac{4\pi}{3}\right)$, where 'a' is the unstretched length of the string. 8

(ii) A particle moves with a central acceleration $\mu \div (\text{distance})^3$. Find the path. 7

(b) (i) A particle is projected with a velocity u at an inclination α above the horizontal in a medium whose resistance per unit mass is k times the velocity. Show that its direction will again make α below the horizontal after a time

$$\frac{1}{k}\log\left(1+\frac{2ku}{g}\sin\alpha\right).$$ 7

(ii) An engine is pulling a train and works at a constant power, doing H units of work per second. If M be the mass of the whole train and F the registance supposed to be constant, show that the time of generating the velocity V from rest in 8

$$\frac{MH}{F^2}\log\frac{H}{H-FV}-\frac{MV}{F}\text{ seconds}.$$

7. Answer any **two** questions : 8×2

(a) A particle moves with a central acceleration F in a medium of which the resistance is K(velocity)^2. Show that the equation of the path is

$$\frac{d^2u}{d\theta^2}+u=\frac{F}{h_0^2u^2}e^{2ks}$$, where s is the length of the arc described and h_0 is the initial moment of momentum about the centre of force. 8

(b) A planet is describing an ellipse about the sun as focus. Show that its velocity away from the sun is greater when the radius vector to the planet is at right angle to the major axis of the path and that it then is $\frac{2\pi ae}{T\sqrt{1-e^2}}$, where 2a is the major axis, e be the ecentricity and T be the pariodic time. 8

(c) An insect crawls at a constant rate u along the spoke of a cart wheel of radius a, the cart moving with a constant velocity v. Find the acceleration along and perpendicular to the spoke. 8

8. Answer any **one** question : 4×1

a) Prove that in a central orbit, the radius vector drawn from the centre of force to the moving particle traces out area as the particle moves, at a constant rate and twice this rate of description of the area by the radius vector is $r^2 \overset{\circ}{\theta}$.

b) A particle moves with a SHM, its position of rest being at a distance a from the center. Find by the principle of energy, the velocity at the centre.
