2019

Part – II

MATHEMATICS

(General)

Paper – II

Full Marks – 90

Time : 3 Hours

The figures in the right-hand margin indicate marks. Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

GROUP – A

(Differential Calculus)

[Marks – 45]

1. Answer any **one** question : 15×1

(a) (i) Show that every convergent sequence is bounded. Justify with the help of an example that the converse of this theorem is not always true. 3+2

(ii) State D' Alembert's ratio test for convergence of positive term series. Hence show that the series

$$x+\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}+\cdots\cdots\cdots(x\geq 0)$$ is convergent if $0\leq x<1$ and divergent if $x\geq 1$. 2+3

(iii) Show that the function defined by –

$$f(x)=\begin{cases}\dfrac{x}{1+e^{1/x}}, & x\neq 0\\ 0, & x=0\end{cases}$$

is continuous at x = 0 but not derivable at that point. 5

(b) (i) Prove that a monotonic increasing bounded above sequence converges to its supremum. 5

(ii) State Cauchy's General Principle for the convergence of an infinite series.

Examine the convergence of $\sum_{n=1}^{\infty}\frac{n^n}{n!}$

2+3

(iii) In mean value theorem

$f(x + h) = f(x) + hf'(x + \theta h), (0 < \theta < 1)$

Prove that $\lim_{h\to 0^+}\theta=\frac{1}{2}$ if f(x)= sinx.

5

2. Answer any **one** question : 8×1

(a) (i) State and prove Cauchy mean value theorem. 4

(ii) Find the values of a and b in order that

$$\underset{x \to 0}{\text{Lt}} \frac{x(1+a\cos x)-b\sin x}{x^3}=1 \quad 4$$

(b) (i) Find the value of y_n for x=0 when $y = e^{a \cdot \sin^{-1} x}$. 4

(ii) State Euler's theorem on homogeneous function and verify it for the function

$$u=\sin^{-1}\left(\frac{x}{y}\right)+\tan^{-1}\left(\frac{y}{x}\right). \quad 4$$

3 Answer any **four** questions : 4×4

(a) Show that the pedal equation of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with respect to a focus is $\frac{b^2}{p^2}=\frac{2a}{r}-1$. 4

(b) Find the envelope of the parabola $\sqrt{\frac{x}{a}}+\sqrt{\frac{y}{b}}=1$, where $ab=k^2$, a and b being variable parameters. 4

(c) If $f(x,y)=\begin{cases}\dfrac{xy\left(x^2-y^2\right)}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$

Show that $f_{xy}(0, 0) \neq f_{yx}(0, 0)$. 4

(d) Determine the asymptotes of the curve $x^2y^2-x^2y-xy^2+x+y+1=0$. 4

(e) Find the point on the parabola $2y=x^2$, which is nearest to the point (0, 3). 4

(f) Prove that the function cos x possesses an expansion of Maclaurin's infinite series for all values of x and find this series. 4

4. Answer any **three** questions : 2×3

(a) Write the condition that two curve f(x, y)=0 and p(x, y)=0 cut orthogonally. 2

(b) Show that the function $f(x) = x^5$ has neither maximum nor a minimum. 2

(c) Prove that $\sqrt{3}$ is an irrational number. 2

(d) If $y=\dfrac{1}{1+x}$, find $\left(y_5\right)_0$. 2

(e) Show that the function $f(x)=\begin{cases}x\sin\dfrac{1}{x}, & x\neq 0\\ 0, & x=0\end{cases}$

is continuous at $x=0$. 2

GROUP – B

(Integral Calculus)

[Marks – 30]

5. Answer any **one** question : 16×1

A. (a) Evaluate any two : 4×2

(i) $\int \frac{x^2-1}{x^4+x^2+1}dx$ 4

(ii) $\int_0^{\frac{\pi}{4}} \frac{\sin x+\cos x}{9+16\sin 2x}dx$ 4

(iii) $\int \cos^{-1}\sqrt{\frac{x}{a+x}}\,dx$ 4

(b) If $I_n = \int_0^{\frac{\pi}{2}} x^n \sin x \; dx$, n being a positive integer greater than 1, show that –

$$I_n + n(n-1)I_{n-2} = n\left(\frac{\pi}{2}\right)^{n-1}$$

Hence find the value of $\int_0^{\frac{\pi}{2}} x^5 \sin x \, dx$.

5+3

B. (a) Answer any **two** questions : 4×2

(i) Show that, $\int_0^{\frac{\pi}{2}} \log(\sin x)\,dx = \frac{\pi}{2}\log\frac{1}{2}$. 4

(ii) Find, $\lim_{n\to\infty} \frac{n}{(n!)^{\frac{1}{n}}}$ 4

(iii) Show that

$$\int_{-a}^{a} f(x)dx = \int_0^a \{f(x) + f(-x)\}dx.$$

Hence show that if f(x) be even

then $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)\,dx.$ 4

(b) (i) Show that $\int_{-1}^{1} (1+x)^p\ (1+x)^q dx =$

$2^{p+q+1}\frac{\Gamma(p+1)\Gamma q(p+1)}{\Gamma(p+q+2)}$, p, q > –1. 4

(ii) Show that $\int_0^{\infty} e^{-t^2}dt = \frac{\sqrt{\pi}}{2}$. Hence

find $\int_{-\infty}^{\infty} e^{-t^2}dt$.

6. Answer any **one** question : 9×1

(a) (i) Determine the entire area of the lemniscate $r^2=a^2\cos 2\theta$. 4

(ii) Find the total length of the cycloid $x = a(\theta - \sin\theta), y = a(1 - \cos\theta)$. 5

(b) (i) Find the surface area of the solid generated by revolving the curve $r=4(1-\cos\theta)$ about the initial line. 4

(ii) Find the volume of the solid generated by revolution of the area bounded by the x-axis and the lines $y=2x$, $x+2y=5$ about the x-axis. 5

7. Answer any **one** question : 5×1

(i) Show that the of $\iint(x^2+y^2)\,dx\,dy$ over the region enclosed by the triangle having its vertices (0, 0), (1, 0) and (1, 1) is $\frac{1}{3}$. 5

(ii) Evaluate $\iiint z^2\,dx\,dy\,dz$ extended over the hemisphere $z \geq 0, x^2 + y^2 + z^2 \leq a^2$. 5

Group – C

(Differential Equation)

[Marks – 15]

8. Answer any **two** questions : 6×2

(a) (i) *Solve* $xy - \frac{dy}{dx} = y^3 e^{-x^2}$. 4

(ii) Find the integrating factor of

$(x^3+y^3)dx = xy^2dy.$ 2

(b) (i) Obtain the singular solution of

$y = px + \sqrt{1+p^2}$ where $p=\frac{dy}{dx}$. 3

(ii) Find the particular integral of

$\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 6y = (x-2)e^x$. 3

(c) (i) Solve the following simultaneous equations : 3

$\frac{dx}{dt} - 7x + y = 0, \quad \frac{dy}{dt} - 2x - 5y = 0$

(ii) Solve : $x^2\frac{d^2y}{dx^2} + 5x\frac{dy}{dx} + 3y = e^{-x}$ 3

9. Answer any **one** question : 3×1

(i) Find the differential equation of all parabolas each having latus rectum 4a and whose axes are parallel to the x-axis. 3

(ii) Find the eigen values and the eigen functions of the boundary value problem

$\frac{d^2y}{dx^2} + \lambda y = 0 (\lambda > 0)$ with $y(0)=0$ and $y(\pi)=0$.

3
