2012
M.Sc.
1st Semester Examination

DISCRETE STRUCTURE

PAPER—COS-101

Full Marks : 50

Time : 2 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

All notations have their usual meaning.

Module—1

(Set Theory)

(Marks : 25)

Answer Q. No. 1 and any four from the rest.

1. Answer any two questions : 2+2
   
   (a) Define partition of a set. Explain it with an example. 1+1

   (b) Let \( n \) and \( k \) be positive integers with \( n \geq k \). Then

   \[
   \binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}.
   \]

   2
(c) State Pigeonhole principle. How many students must be in a class to guarantee that at least two students received the same score on the final exam, if the exam is graded on a scale from 0 to 10 points. 1+1

2. Let A and B be subsets of a universal set U. Show that 
\[(A \cup B)' = A' \cap B'\] 4

3. State the induction principle. If n be a positive integer then prove that 
\[1^3 + 2^3 + \ldots + n^3 = \left[ \frac{n(n+1)}{2} \right]^2\] 1+3

4. Suppose there are 10 members in a club.
   (i) How many way can we select 5 members to stand in line for a picture? 2+2
   (ii) How many way can we select to make a 5 members committee? 2+2

5. (i) How many bit string of length eight either start with a ‘0’ bit or end with ‘1’ bits.
   (ii) A particular brand of shirt comes in 8 colours, has a male version and female version and comes in three sizes for each sex. How many different types of this shirt are made? 2+2

6. If A, B, C be subsets of a universal set U. Prove that 
\[(A \cap B) \cup (B \cap C) \cup (C \cap A) = (A \cup B) \cap (B \cup C) \cap (C \cup A).\] 4

7. Among the first 100 positive integers, determine the integers which are divisible by 2 and 3, not by 5. 4

[Internal Assessment — 5 marks]
Module—2

(Graph Theory)

(Marks : 25)

Answer any two questions: 2×10

1. (a) Define component of a graph. How many edges are there in a forest with n vertices and k components. 1+4

(b) Define simple graph. What is the maximum number of edges in a simple graph with n vertices. 1+4

2. (a) Define isomorphic between two graphs. Show that the following graphs are isomorphic:

(b) Define walk, trail, path and cycle. Explain it with taking an example. 5

3. (a) Define Eulerian graph. For which value of n, Kn (completely graph of n vertices) is Eulerian and why? 5

(b) Define tree. Show that a circuit free graph with n vertices and (n – 1) edges is a tree. 5
4. (a) Define Eccentricity, center of a graph. Show that every tree has either one or two center.

(b) Demonstrate the Prim's algorithm of the following weighted graph:

![Diagram of a weighted graph with vertices labeled 1, 2, 3, and 4, and edges with weights 1, 2, 3, and 4.]

[Internal Assessment — 05]