Total Pages—8 **UG/II/STAT/H/III/18(New)**

2018

STATISTICS

[Honours]

PAPER – III

Full Marks : 90

Time : 4 hours

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

Notations and symbols used bear their usual significance

[NEW SYLLABUS]

GROUP – A

1. Answer any *five* question : 5×5

(*a*) Let the radom variable X be such that $E(e^{ax})$ exists where $a > 0$. Show that for any $t > 0$,

$$P(X > t) < \frac{E(e^{ax})}{e^{at}}.$$

(*Turn Over*)

(*b*) A symmetric coin is tossed 1600 times. What is the probability that the head will be shown up more than 1200 times ?

(*c*) Suppose X_n are random variables ($n = 1, 2, ...$) defined by

$$X_n = \begin{cases} 0, & \text{with probability } \frac{1}{n} \\ 1, & \text{with probability } 1 - \frac{1}{n} \end{cases}.$$

Then show that $X_n \xrightarrow[n]{P} 1$.

(*d*) Show that the laws of large Numbers holds for the sequence of independent randaom variables $\{X_n\}$ defined by

$$P\left[X_n = 2^{-n}\right] = P\left[X_n = -2^{-n}\right] = \frac{1}{2}.$$

(*e*) Show that if X and Y are normal variables with zero means and unit variances and correlation coefficient ρ, then

$$E\left[\max(X, Y) = \sqrt{\frac{1-\rho}{\pi}}\right]$$

(*f*) Obtain both in SRSWR and SRSWOR from a population of N units and when sample size is n, the following :

(*I*) the probabilitiy that a particular population unit is included in the sample.

(*II*) the probability that two particular population units are included in the sample.

(*g*) Show that for a random sample of size 100, drawn with replacements, the standard error of sample proportion cannot exceed 0·05.

2. Answer any *one* question : 10×1

(*a*) Find the sampling distribution of the sample regression coefficient of y on x based on n pairs of observation (x_i, y_i), $i = 1, 2, \ldots, n$, where x is non-stochastic and y is normal with constant variance and the true regression of y on x is linear.

(*b*) Let X_n ($n = 1, 2, \ldots$) be iid random variables with common mean μ and common variance

σ^2. Show that if $S_n = \sum_{i=1}^{n} X_i$, $S_n - n\mu$ does not converge stochastically to zero, but $a_n(S_n - n\mu)$ does provided $a_n \cdot \sqrt{n} \to 0$ as $n \to \infty$.

GROUP – B

3. Answer any *four* questions : 5×4

(*a*) Let $X_1, X_2, \ldots, X_n$ be a random sample of size n drawn from a rectangular distribution with p.d.f.

$$f_{\underset{\sim}{\theta}}(x) = \frac{1}{\theta_2 - \theta_1};\ \theta_1 < x < \theta_2 < \infty$$

$$\underset{\sim}{\theta} = (\theta_1, \theta_2) = 0 \qquad \text{o.w.}$$

where $-\infty < \theta_1 < \theta_2 < \infty$. Obtain the sufficient statistics from θ_1 and θ_2.

(*b*) Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from $N(\mu, \sigma^2)$, where $N(\mu, \sigma^2)$ denotes the normal distribution with mean μ and variance σ^2. Find a consistent estimator of μ^2.

(*c*) State and prove the Rao-Blackwell theorem.

(*d*) Let $X_1, X_2,...X_n$ be a random sample from a normal distribution with unknown mean μ and unknown variance σ^2. Obtain a size-α critical region for testing $H_0 : \mu = \mu_0$ againts $H_1 : \mu \neq \mu_0$, where μ_0 is a specified real value.

(*e*) Let $(x_1, y_1),...(x_n, y_n)$ be a random sample of size n from a bivariate normal distribution with parameters $\mu_1, \mu_2, \sigma_1^2, \sigma_2^2$ and ρ. Obtain a level α test for testing $H_0 : \rho = 0$ against $H_1 : \rho \neq 0$. Also obtain the 100 $(1 - \alpha)$ % confidence interval for ρ.

(*f*) Let $x_1, x_2,..., x_n$ be a random sample of size n from Rectangular distribution with p.d.f. $f_\theta(x) = 1$; $\theta - 1 < x < \theta + 1$ $\theta > 0$. Show that the maximum likelihood estimator of θ is not unique.

4. Answer any *one* question : 10×1

(*a*) Suppose that we have a single observation X which is distributed as Bernoulli (p), where p is the unknown parameter. Obtain the maximum likelihood estimator of p if the parametric space happens to be (i) H $= [0, 1]$ and (ii) $H = \left[\frac{1}{3}, \frac{2}{3}\right]$.

(b) Describe the method of minimum χ^2.

GROUP – C

5. Answer any *three* questions : 5×3

(a) If $\rho_{12\cdot3} = 0$, prove that

$$\rho_{13\cdot2} = \rho_{13}\sqrt{\frac{1-\rho_{23}^2}{1-\rho_{12}^2}}.$$

(b) Show that the dispersion matrix of a random vector $\underset{\sim}{X} = (X_1, X_2, ..., X_p)'$ can never be negative definite.

(c) Explain the term 'partial regression coefficient' in the context of multiple regression equation of x_1 on $x_2, x_3, ..., x_K$. Obtain its form in terms of the co-factors of the correlation matrix.

(*d*) Write down the probability mass function of a $(K-1)$-variate multinomial distribution involving the variables $X_1, X_2, ..., X_{K-1}$. Obtain the conditional distribution of X_1 given $X_2 = x_2, ..., X_{K-1} = x_{K-1}$.

(*e*) Explain concentration ellipsoid for two variables and extend the same to the case of p-variables.

6. Answer any *one* question : 10×1

(*a*) Suppose that $X_1, X_2, ..., X_{2p}$ denote scores on $2p$ questions in an aptitude test. Suppose they have a common mean μ and a common variance σ^2, while the correlation coefficient between any pair of them is the same, $\rho > 0$. Let Y_1 be the sum of scores on the odd numbered questions and Y_2 be the sum of scores on the even-numbered questions. Show that the correlation between Y_1 and Y_2 tends to unity as p increases. 10

(*b*) (*i*) Obtain the moment generating function of $\underset{\sim}{X}$, where $\underset{\sim}{X} \sim N_p(\mu, \Sigma)$.

(*ii*) If $\rho_{ij} = \rho (i, j = 1, 2, ..., p;\ i \neq j)$, what will be the value of $\rho_{1\cdot 23 \ldots p}$? 5 + 5
