UG/I/STAT/H/II/18

2018

STATISTICS

[Honours]

PAPER – II

Full Marks : 90

Time : 4 hours

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

GROUP—A

1. Answer any *five* questions : 5 × 5

(*a*) Define the cumulative distribution function (c.d.f.) of a random variable X. Show that it is monotone non-decreasing. 2 + 3

(*Turn Over*)

(*b*) Show that for n events $A_1, A_2, \dots, A_n$,

$$P\left(\bigcap_{i=1}^{n} A_i\right) \geq \sum_{i=1}^{n} P(A_i) - (n-1). \qquad 5$$

(*c*) The joint density function of x and y is given by

$$f(x,y) = \begin{cases} 2e^{-x-2y} & 0 < x, y < \infty \\ 0 & \text{otherwise} \end{cases}$$

compute $P(x < y)$. 5

(*d*) Show that moment generating function for a Cauchy distribution does not exist. 5

(*e*) A and B throw alternately with a pair of ordinary dice. A wins if he throws 6 before B throws 7, and B wins if he throws 7 before A throws 6. If A begins, show that his chance of wining is $\frac{30}{61}$. 5

(*f*) Show that the odd order central moments of a symmetric distribution are zeros. 5

(*g*) If the probability density function (pdf) of a random variable X is given by

$$f(x)=\begin{cases}\dfrac{1}{b-a} & \text{if} \quad a<x<b \\ 0 & \text{otherwise}\end{cases}$$

then find $P(c \le X < b)$ when $a \le c < b$ and $P(a \le X < c \mid X \le c)$. 5

(*h*) Show that mean deviation about mean of normal distribution is $\sqrt{\frac{2}{\pi}}a$ where a be the s.d. of the distribution. 5

(*i*) Let X and Y be independent, each following a geometric distribution with parameter p. Show that the conditional distribution of X given $X+Y=n$ is uniform over $\{0, 1, 2, 3, \ldots\ldots, n\}$. 5

2. Answer any *two* questions : 10 × 2

(*a*) (*i*) Let (X, Y) be a bivariate discrete random variables with joint probability mass function

$$f(x,y)=\begin{cases}\dfrac{1}{\binom{m+1}{2}} & \text{if } y=1,2,\ldots,x \\ & \text{and } x=1,2,\ldots,m \\ 0 & \text{otherwise}\end{cases}$$

for a given positive integer $m>1$. Find

(*I*) $E(x)$ from the marginal distribution of x.

(*II*) Find the correlation coefficient between x and y. 2 + 4

(*ii*) Show that for two random variables X and Y

$\text{Var}(X)=E(\text{Var}(X\mid Y))+\text{Var}(E(X\mid Y))$. 4

(*b*) Establish an appropriate inequality concerning the mean, the median and the mode of a log-normal distribution with parameters μ and *a*. 10

(*c*) (*i*) Prove that the probability P that at least one of the r events $A_1, A_2, \ldots, A_r$ will occur obeys the inequality

$$P \geq \sum_{i=1}^{r} P(A_i) - \sum_{\substack{i=1 \\ i<j}}^{r} \sum_{j=1}^{r} P(A_i \cap A_j)$$ 7

(*ii*) Show that conditional probability satisfies Kolmogorov's axioms on the definition of probability. 3

(*d*) (*i*) Show that two events cannot be simultaneously mutually exclusive and mutually independent. 2

(*ii*) Let X be a random variable denoting the number of tosses of a coin till the first head appears. Obtain the moment generating function of X and hence its mean and variance. 4 + 2 + 2

GROUP–B

3. Answer any *two* questions : 5 × 2

(*a*) A polynomial $g(\cdot)$ is such that $g(0) = 1$, $g(1) + g(2) = 10$, $g(3) + g(4) = 65 = g(5)$. Find the form of $g(x)$. 5

(*b*) Obtain the convergence criterion of iteration method to obtain numerical solution of a equation in one unknown. 5

(*c*) Derive Newton's formula for backward interpolation. 5

(*d*) Show that *n*th order finite difference of a polynomial of degree *n* is constant. 5

4. Answer any *one* question : 10 × 1

(*a*) Derive Lagrange's interpolation formula. Hence show that the Lagrange's interpolation formula is an weighted average of entries. Discuss the important uses of this formula. 5 + 3 + 2 (*b*) (*i*) Obtain the Trapezoidal rule of numerical integration.

(*ii*) When and how will you use the method of false position ? 5 + 5

GROUP–C

5. Answer any *three* questions : 5 × 3

(*a*) Discuss the ratio to trend method in estimating seasonal component of time series. 5

(*b*) What are the different tests for consistency that a formula for price index number should satisfy ? 5

(*c*) Distinguish between fixed-base index numbers and chain-base index numbers. 5

(*d*) Mention the important publications of the Reserve Bank of India. 5

(*e*) Write a short note on simple exponential smoothing. 5

(*f*) Show that Gini's coefficient of mean difference can be expressed in terms of standard deviation. 5

6. Answer any *one* question : 10 × 1

(*a*) State the different components of a time series with appropriate examples of each. How will you fit a linear trend equation to time series data ? 5 + 5

(*b*) (*i*) Why is Fisher's index number known as an ideal index number ?

(*ii*) Show that Marshall Edgeworth index number his between Laspeyre's and Paasche's index numbers. 6 + 4

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