2018

STATISTICS

[Honours]

PAPER – I

Full Marks : 90

Time : 4 hours

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

GROUP – A

(*Descriptive Statistics*)

1. Answer any *two* questions : 10×2

   (*a*) What is rank correlation ? Derive the formula for Spearman's rank correlation coefficient for both non-tied and tied case. 2+8 (*b*) (*i*) What is scatter diagram ? Draw scatter diagram for the following cases :

I. $r = 1$
II. $r = -1$
III $r = 0$

where r is the correlation coefficient. 2+3

(*ii*) Given two regression lines for the two variables obtain the expression of the acute angle (θ) between the two regression lines. 5

(*c*) Write short notes on the following :

Ordinal data, Discrete variable, Time series data, Step diagram and Box Plot.
2+2+2+2+2

(*d*) What is primary data ? Discuss about different methods for collecting primary data. What are their relative merits and demerits ? 2+4+4

2. Answer any *five* questions : 5×5

(*a*) Distinguish between questionnaire and schedule. 5

(*b*) What do you mean by skewness of a distribution ? Give Bowley's measure of skewness. Show that it lies between −1 and 1. 5

(*c*) Derive the expression for the variance of residual in case of linear regression. 5

(*d*) Define correlation index of order *p*. Show that $r_p^2 \geq r_{p-1}^2$ where r_p is the correlation index of order *p*. 5

(*e*) Distinguish between bar diagram and histogram. 5

(*f*) Show that root mean square deviation is minimum when measured from the mean. 5

(g) Prove that $b_2 \geq b_1 + 1$ where b_1 and b_2 are the moment measures of skewness and Kurtosis respectively. 5

(*h*) What is odds ratio ? Discuss about its properties. 2 + 3 (*i*) Discuss about different relative measures of dispersion. 5

(*j*) Define chi-square measure of association. What are its defects ? How do you remove these defects ? (1 + 2 + 2)

GROUP – B

(*Matrix Algebra*)

3. Answer any *one* question : 10 × 1

(*a*) (*i*) Show that the number of vectors in a basis of a vector space is unique. 5

(*ii*) Find a basis of the vector space E_4 containing the vectors $(1, 1, 1, 0)'$ and $(1, 1, 0, 0)'$. 5

(*b*) (*i*) Suppose A is a square matrix of order p. show that

$$A(\text{adj } A) = (\text{adj} A)A = |A|\, I_p$$

where I_p is the unit matrix of order p. 5

(*ii*) Evaluate the determinant of the following matrix of order $p(>2)$ 5

$$\begin{pmatrix} 1 & r' & r' & \cdots & r' \\ r' & 1 & r & \cdots & r \\ r' & r & 1 & \cdots & r \\ \vdots & & & & \\ r' & r & r & \cdots & 1 \end{pmatrix}$$

4. Answer any *two* questions : 5×2

(*a*) Suppose A and B be two matrices such that AB is defined. Show that,

$$\text{rank}(AB) \le \min\{\text{rank}(A), \text{rank}(B)\}$$

(*b*) What is idempotent matrix? If A is an idempotent matrix then show that $(I - A)$ is also an idempotent matrix.

(*c*) Suppose $A = ((a_{ij}))$ is a matrix of order $p \times p$ with $a_{ij} = (\beta_i - \beta_j)^2 \ \forall_{i,j}$. Show that $|A| = 0$ if $p > 3$ and $|A| \neq 0$ if $p \le 3$.

(*d*) Define vector subspace. Let us consider a vector space $V=\{\underset{\sim}{x}:\underset{\sim}{x}=(x_1,x_2,x_3)';x_i \in R, \forall i=1,2,3\}$. Define V_1 as $V_1=\{\underset{\sim}{x}:\underset{\sim}{x}=(x_1,x_2,x_3)'$, $x_i \in R, \forall i=1,2,3;\quad x_1^2+x_2{}^2=x_3{}^2\}$. Show that V_1 is not a vector subspace of V.

GROUP – C

(*Mathematical Analysis*)

5. Answer any *one* question : 10 × 1

(*a*) (*i*) Prove that a monotone increasing sequence which is bounded above is convergent. 5

(*ii*) Show that the sequence $\left\{\left(1+\frac{1}{n}\right)^n\right\}$ is convergent 5

(*b*) (*i*) Show that the function $f(x)=\frac{1}{x},\ x\in[1,\infty)$ is uniformly continuous. 5

(*ii*) Test the convergence of the series

$$1+\frac{1}{2}x+\frac{1\cdot 3}{2\cdot 4}x^2+\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}x^3+\ldots,\ x>0 \quad 5$$

6. Answer any *three* questions : 5×3

(*a*) Find c such that

$$\lim_{x\to 0}\frac{c\sin x-\sin 2x}{\tan^3 x} \text{ is finite.}$$

Also find the limit.

(*b*) Show that,

$$\int_0^\infty \frac{x}{1+x^2}\,dx$$

does not exist.

(*c*) Show that,

$$\int_0^{\pi/2}\sqrt{\cot x}\,dx=\frac{\pi}{\sqrt{2}}$$

(*d*) Examine the convergence of the integral

$$\int_0^\infty \frac{\sin x}{1+x^2}\,dx.$$

(*e*) Obtain the Taylor-Maclaurin's expansion of the function $f(x)=e^x, x \in R$.

(*f*) Show that,

$$B(m,n)=\int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}\,dx,\ \ m>0,\ \ n>0 .$$