UG/II/PHS/H/III/18(New)

2018

PHYSICS

[Honours]

PAPER – III

Full Marks : 90

Time : 4 hours

The figures in the right-hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

[NEW SYLLABUS]

GROUP – A

Answer any two questions : 15 × 2

1. (*a*) Show that

$$A = \begin{pmatrix} 1/\sqrt{2} & -i/\sqrt{2} & 0 \\ i/\sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

is a unitary matrix. Find A^{-1}. 2 + 1

(*b*) Find the eigen values and eigen vectors of the matrix

$$\begin{pmatrix} 1 & -2 \\ -2 & -2 \end{pmatrix}$$ 4

(*c*) The Gaussian probability distribution is given by

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}.$$

Show that

(*i*) the mean of the distribution is μ.

(*ii*) Probability within the range $\pm 2\sigma$ from the mean is 95·44% of the total probability.

$$\left[\text{Given}: \frac{1}{\sqrt{2\pi}}\int_0^2 e^{-\frac{z^2}{2}}dz = 0{\cdot}4772.\right]$$

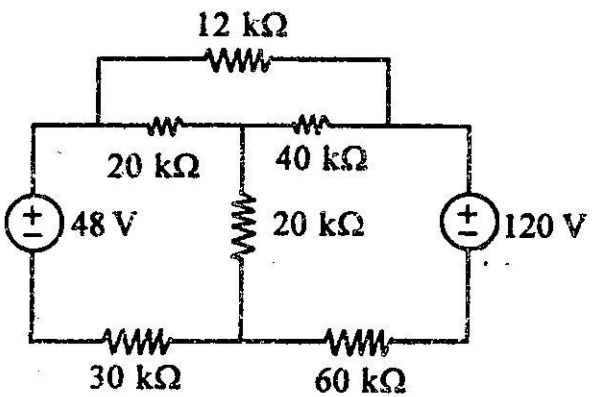
2 + 2

(*d*) Find the fourier series expansion of the rectified half wave given by :

$$V(t) = \begin{cases} V_0 \sin\omega t & 0 \le \omega t \le \pi \\ 0 & \pi \le \omega t \le 2\pi \end{cases}$$ 4

2. (*a*) State and prove the maximum power transfer theorem. 2 + 1

(*b*) Find the current through to the 12 kΩ resistance in the following figure. 3



(*c*) An inductance L, a capacitance C and resistance R are connected in series. What will be the complex impedance offered by this combination to the complex voltage $= V_0 e^{j\omega t}$? 2

(*d*) Find the expression for the current in the circuit if the applied voltage is $V_0 \cos\omega t$. 2

(*e*) Find the resonant frequency and the band width of the circuit. 2 + 3

3. (*a*) The wave equation in a homogeneous linear medium with zero charge density and conductivity σ is given by

$$\nabla^2 \vec{E} - \mu \epsilon \frac{\partial^2 E}{\partial t^2} - \sigma\mu \frac{\partial \vec{E}}{\partial t} = 0$$

obtain the solution for a plane wave with wavefront parallel the $y - z$ plane. 4

(*b*) Show that in a conductor, the magnetic field lags behind the electrical field. 3

(*c*) Give the concept of skin depth. What will be the expression of skin depth for a good conductor. 1 + 2

(*d*) Give the expression of Rayleigh scattering cross-section. What is the condition of Rayleigh scattering ? 1 + 1

(*e*) In a dielectric medium the propagation vector of an electromagnetic wave of angular frequency 213·36 THz is $10{\cdot}668 \times 10^5 m^{-1}$. What is the value of the dielectric constant of the medium ? 3

4. (*a*) Starting from the time dependent Schrödinger equation in one dimension, derive the equation of continuity. 5

(*b*) Find the energy values and the normalized wave functions of a particle confined in a one dimensional box of infinite height. 4

(*c*) State the postulates of special theory of relativity. A frame of reference S' moves along X axis with velocity v with respect to an inertial frame S. Another frame S'' moves also along X axis with velocity v' with respect to S'. Derive the expression of the velocity of S'' with respect to S. 2 + 4

GROUP – B

Answer any **five** questions : 8 × 5

5. (*a*) Two cards are drawn one after another randomly from a well-shuffled packet of 52 cards. What will be the probability that both the cards will be diamonds ? 2

(*b*) In a measurement of diameters of 50 similar balls, you get the following results : 2

Number of balls	2	6	11	14	4	9	4
Diameter (cm)	5·14	5·12	5·09	5·10	5·06	5·11	5·07

Determine the standard deviation of the measured diameter.

(*c*) Find the Fourier transform $F(k)$ for the function $f(x)$ given by :

$$f(x) = \begin{cases} 1 & -a \le x \le a \\ 0 & |x| > a \end{cases}.$$

Plot $F(k)$ vs k between $-\pi \le k \le \pi$. 2 + 2

6. (*a*) Show that the eigen vectors of a symmetric real matrix belonging to different eigen values are orthogonal. 3

(*b*) Classify the singularities and calculate the residue for

$$f(z) = \frac{1}{1+z^2}$$ 2

(*c*) Use the complex variables technique to find the value of the integral

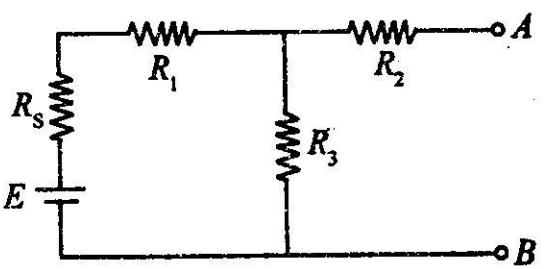
$$\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$$ 3

7. (*a*) If A is non-singular, show that the eigenvalues of A^{-1} are reciprocals of those of A and every eigenvector of A is also an eigenvector of A^{-1}. 2 + 2

(*b*) Solve with the help of matrices, the simultaneous equations.

$$x + y + z = 3$$
$$x + 2y + 3z = 4$$
$$x + 4y + 9z = 6$$ 4

8. (*a*) Prove Thevenin's theorem for the following circuit : 4



(*b*) Show that the equivalent inductance of two

coils of self inductance L_1, L_2 and mutual inductance M connected in parallel is given by

$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 \mp 2M}$$ 4

9. (*a*) Starting from time dependent Schrödinger equation in one dimension derive the equation :

$$\frac{\partial \rho(\vec{r},t)}{\partial t} + \vec{\nabla} \cdot \vec{J}(\vec{r},t) = 0,$$

where $\rho(\vec{r},t)$ is the probability density and $\vec{J}(\vec{r},t)$ is the probability current density. 3

(*b*) What are the units of $\rho(\vec{r},t)$ and $\vec{J}(\vec{r},t)$? 1

(*c*) Interpret the equation you derived. 2

(*d*) Find the value of A to normalize

$$\psi(x) = \frac{A}{\sqrt{a}}\sin\left(\frac{\pi x}{a}\right) + \sqrt{\frac{3}{5a}}\sin\left(\frac{3\pi x}{a}\right) + \frac{1}{\sqrt{5a}}\sin\left(\frac{5\pi x}{a}\right).$$ 2

10. (*a*) (*i*) Give Wien's distribution law.

(*ii*) How does it differ with experimental E_λ *vs.* λ curves ?

(*iii*) State the assumptions made by Plank to develop his blackbody radiation formula. 1 + 2 + 2

(*b*) In the laboratory the life time of a particle moving with speed $2{\cdot}8 \times 10^{10}$ cm/s is found to be $2{\cdot}5 \times 10^{-7}$ sec. Calculate the proper life time of the particle. 3

11. (*a*) State Faraday's law of electromagnetic induction and give the integral form of the law. 1 + 3

(*b*) Write down Maxwell's electromagnetic field equations. Explain the physical significance of each. 2 + 2

12. (*a*) Show that in suitable special cases Lorentz dispersion equation reduce to Sellmeier's formula and Cauchy's formula. 2 + 2

(*b*) Starting from Fresnel's equation (derivation not required) for reflection of an electromagnetic wave having the electric field vector parallel to the plane of incidence express Brewster's angle in terms of refractive indices of the media on the two sides of the interface. Hence calculate the Brewster's angle when light is incident on glass ($n = 1{\cdot}5$.) surface from air. 3 + 1

GROUP –C

Answer any **five** questions : 4 × 5

13. (*a*) Show that the Kronecker delta is a mixed tensor of order two. 2

(*b*) Show that the covariant derivatives of fundamental tensors vanish. 2

14. Write down in detail the difference between a deat-beat and a ballistic galvanometer. 4

15. A charged capacitor of capacitance 2μF when left alone loses 2% of its charge in 1 min. But when connected across a high resistance R it loses 25% of its charge in 1 min. Find the value of R. 4

16. (*a*) Energy eigen value of a particle freely moving in a 1D infinite square well potential of length a is $\frac{9\pi^2\hbar^2}{2ma^2}$ and that of a particle freely

moving in a 3D infinite square well potential having sides of length a is also $\frac{9\pi^2\hbar^2}{2ma^2}$. What are the degrees of degeneracy in these two cases ? 2

(*b*) A particle freely moving in a 1D infinite square well potential of length a. Find the position(s) where the probability of finding the particle at 2nd excited state is maximum. 2

17. (*a*) Determine the polarisation state of an electromagnetic wave given by :

$$\vec{E} = \vec{E}_0 \sin(kz - \omega t) + 2\vec{E}_0 \sin\left(kz - \omega t + \frac{\pi}{2}\right). \quad 2$$

(*b*) At a frequency 400 MHz sea water has permittivity $\epsilon = 81\epsilon_0$, permeability $\mu = \mu_0$ and resistivity $\rho = 0.23\ \Omega m$. Find the ratio

of conduction current to displacement current. 2

18. A plane EM wave falls obliquely on air ($n_1 = 1.0$) glass ($n_2 = 1.5$) interface. Find the angle of incident for which the reflection and transmission coefficients are equal to 0·5, i.e, $R = T = 0{\cdot}5$. 4

19. A space ship of rest length 358 m has speed 0·728 c with respect to certain reference frame. A small meteorite with speed of 0.817c in this frame passes the space ship on an antiparallel track. How long does it take for this meteorite to pass the spaceship, if one observes from the meteorite ? 4

20. (*a*) Establish the reaction :

$$E^2 = p^2c^2 + m_0{}^2c^4,$$

the symbols having usual meaning. 2

(*b*) Find the speed of a particle whose kinetic energy is equal to its rest mass energy. 2
