 C/18/B.Sc./P-II/3T(N)/MTMG/2

NEW

2018

Part-II 3-Tier

MATHEMATICS

(General)

PAPER—II

Full Marks : 90

Time : 3 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group—A

(*Differential Calculus*)

[Marks : 45]

1. Answer any *one* question : 1×15

(a) (i) Define Cauchy sequence. Prove that every convergent sequence is a Cauchy sequence. 2+3

(ii) Prove that $\sqrt{7}$ is not a rational number. 4

(iii) State Cauchy's root test for the convergence of positive term series. Use it to examine the convergence or divergence of the series

$$2x + \frac{3^2x^2}{2^3} + \frac{4^3x^3}{3^4} + \frac{5^4x^4}{4^5} + \cdots\cdots$$ 2+4

(b) (i) Use Raabe's Test to examine the convergence of the series $\sum\left(\frac{1.3.5.\ (2n-1)}{2.4.6.\ (2n+2)}\right)$. 5

(ii) A mapping $f : R \to R$ is defined as

$$f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ -1 & \text{if } x \text{ is irrational}. \end{cases}$$

Find the range of f. What is $f(\sqrt{7})$? 1+1

(iii) Show that $f(x) = |x - 1|$ is continuous at $x = 1$ but not derivable at that point. 1+2

(iv) Show that the series $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^p}$, $p > 0$ converges for $p > 1$ and diverges for $p \leq 1$. 5

2. Answer any *one* question : 1×8

(a) (i) State and prove Rolle's Mean Value theorem. 4

(ii) If f(x) is an even function and $f'(0)$ exists, show that $f'(0) = 0$. 4

(b) (i) Determine the value of a, b such that

$$\lim_{x \to 0} \frac{ae^{x} - b\cos x + e^{-x}}{x \sin x} = 2.$$ 4

(ii) Establish the inequality $x < \sin^{-1} x < \frac{x}{\sqrt{1-x^2}}$,

$0 < x < 1$. 4

3. Answer any *four* questions : 4×4

(a) Find the radius of curvature of the curve $r^2\cos 2\theta = a^2$ at any point on it. 4

(b) Find the rectilinear asymptotes of
$$y^3 - xy^2 - x^2y + x^3 + x^2 - y^2 - 1 = 0.$$ 4

(c) Prove that the points of the curve $y^2 = 4a\left\{x + a\sin\frac{x}{n}\right\}$ at which the tangents is parallel to the x-axis, lie on a parabola. 4

(d) Suppose f(x, y) is defined as follows :

$$f(x) = \begin{cases} xy, & |x| \geq |y| \\ -xy, & |x| < |y| \end{cases}$$

Show that,
$$f_{xy}(0, 0) \neq f_{yx}(0, 0).$$ 4

(e) If $y = 2\cos x(\sin x - \cos x)$, show that $(y_{10})_0 = 2^{10}$. 4

(f) Let $f(x) = x^2 \sin\left(\frac{1}{x}\right), \quad x \neq 0$

$= \quad 0 \quad , \quad x = 0$

Show that f(x) is continuous and differentiable at x = 0. 4

4. Answer any *three* questions : 3×2

(a) State geometrical interpretation of $\frac{dy}{dx}$. 2

(b) Show that the existence of a maximum or a minimum value of a function f(x) at a certain point need not imply $\frac{df}{dx} = 0$ at that point. 2

(c) For what value of 'a' the function $f(x) = \begin{cases} x^2 - 1, & x < 3 \\ 2ax, & x \geq 3 \end{cases}$ is continuous everywhere ? 2

(d) Find the range of real valued function of a real variable where $f(x) = \frac{|x|}{x} + 2$. 2

(e) Show that $\lim\limits_{(x, y) \to (0, 0)} \frac{2xy}{x^2 + y^2}$ does not exist. 2

Group—B

(Integral Calculus)

[Marks : 30]

5. Answer any *one* question : 1×16

A. (a) Evaluate any *two* : 2×4

(i) $\int \frac{x^2}{x^4 + x^2 - 2} dx$ 4

(ii) $\int \frac{dx}{5 - 13\sin x}$ 4

(iii) $\lim_{n\to\infty} \left[\frac{1^{10} + 2^{10} + \cdots\cdots + n^{10}}{n^{11}}\right]$. 4

(b) Obtain the reduction formula of $\int_0^{\pi/4} \sec^n x\, dx$. Hence evaluate $\int_0^a \left(a^2 + x^2\right)^{5/2} dx$. 4+4

B. (a) Answer any *two* questions : 2×4

(i) Evaluate

$$\lim_{n\to\infty} \left\{\left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)^2\left(1+\frac{3^2}{n^2}\right)^3 \cdots\cdots \left(1+\frac{n^2}{n^2}\right)^n\right\}^{\frac{2}{n^2}}.$$

4

(ii) Prove that

$$\int_0^{3a} f(x)dx = 3\int_0^{a} f(x)dx \text{ if } f(a+x) = f(x).$$ 4

(iii) Evaluate $\int_0^{\pi} \frac{x\tan x}{\sec x + \cos x} dx$. 4

(b) (i) Prove that $B(m, n) = 2\int_0^{\frac{\pi}{2}} \sin_{\theta}^{2m-1} \cos_{\theta}^{2n-1} d\theta$. 4

(ii) Prove that $\int_0^{\infty} e^{-x^4} dx \times \int_0^{\infty} e^{-x^4} . x^2 dx = \frac{\pi}{8\sqrt{2}}$. 4

6. Answer any *one* question : 1×9

(a) (i) Determine the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ between the curve and its latus rectum. 4

(ii) An arc of the sine curve y = sinx from x = 0 to x = π is revolved about the x-axis. Find the surface of the solid thus generated. 5

(b) (i) Find the perimeter of the curve represented by

$$x = \frac{1-t^2}{1+t^2}, \; y = \frac{2t}{1+t^2}.$$ 4

(ii) Find the volume of revolution generated by the region enclosed by $y = \sqrt{x}$ and the lines $y = 1$, $x = 4$ about x-axis. 5

7. Answer any *one* question : 1×5

(i) Compute $\iiint xyz\,dx\,dy\,dz$ over a domain bounded by $x = 0$, $y = 0$, $z = 0$ and $x + y + z = 1$. 5

(ii) Evaluate $\iint\left[2a^2 - 2a(x+y) - (x^2+y^2)\right]dx\,dy$, the region of integration being the circle $x^2 + y^2 + 2a(x + y) = 2a^2$. 5

Group—C

(*Differential Equation*)

[Marks : 15]

8. Answer any *two* questions : 2×6

(a) (i) Obtain the singular solution of $yp = xp^2 - p - 2$ where $p = \dfrac{dy}{dx}$. 3

(ii) Find the particular integral of

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = e^x \sin x.$$ 3

(b) (i) Solve the following simultaneous equations :

$$\frac{dx}{dt} + \frac{dy}{dt} + 2y = 0,$$

$$\frac{dx}{dt} - 3x - 2y = 0.$$ 4

(ii) Find the integrating factor of

$$1 + y^2 + \left(x - e^{-\tan^{-1} y}\right) \frac{dy}{dx} = 0.$$ 2

(c) (i) Evaluate : $\frac{1}{D+2} e^{-2x} \sin 3x$. 3

(ii) Solve : $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \log x$. 3

9. Answer any *one* question : 1×3

(i) Find the differential equation of the function $y = Ae^x + Be^{-x} + x^2$ where A and B are arbitrary constants. 3

(ii) Find the orthogonal trajectories of the family of curves

$x^{2/3} + y^{2/3} = a^{2/3}$; a being a parameter. 3
