PG/IIS/MTM-203/15

M.Sc. 2nd Semester Examination, 2015

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(*Abstract Algebra and Linear Algebra*)

PAPER – MTM - 203

Full Marks : 50

Time : 2 hours

The figures in the right-hand margin indicate marks

UNIT – I

(*Abstract Algebra*)

[*Marks* : 25]

Answer **Q.No. 1** and any **two** from the rest

1. Answer any *two* questions : 2×2

(*a*) Let S_7 be the group of permutations on 7 symbols. Does S_7 contain an element of order 10 ?

(Turn Over)

(*b*) Let I be an ideal of a ring R. Show that the quotient ring R/I is a commutative ring if and only if $ab-ba \in I$ for all $a, b \in R$.

(*c*) Show that the group $\mathbb{Z} \times \mathbb{Z}$ is not cyclic.

2. (*a*) State the classequation for a finite group. Define simple group. Show that a group of order 63 isnot simple. 2 + 1 +2

(*b*) Show that

$$G = \left\{ \begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix} : a, b, c \in \mathbb{Z}_3 \right\}$$

is a finite non-commutative solvable group w.r.t. usual matrix multiplication. 3

3. (*a*) State the Sylow's first, second and third theorems and prove any one of them. 5

(*b*) If G is abelian group having subgroups

$H_1, H_2, \ldots H_n$ Such that $|H_i \cap H_j| = 1$, for all $i \neq j$, then show that $K = H_1 H_2 \ldots\ldots H_n$ is a subgroup of G and $K \simeq H_1 \times H_2 \times \ldots\ldots \times H_n$. 3

4. (*a*) Let R be a commutative ring with unity. Show that an ideal M of R is maximal if and only if $\frac{R}{M}$ is a field. 5

(*b*) Find all prime ideals and maximal ideals in the ring Z_8. 3

[*Internal Assessment* : 5 Marks]

UNIT – II

(*Linear Algebra*)

[*Marks* : 25]

Answer **Q.No. 5** and any **two** from the rest

The symbols have their usual meanings

5. Answer any *two* questions : 2 × 2

(*a*) Let $V = M_2(R)$. Define $T : V(R) \to R$ by

$$T\left(\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right) = a + b + c + d - 1$$

Is T a linear transformation ? Justify your answer.

(*b*) Define complete lattice with an example. Also give an example which does not form the complete lattice.

(*c*) What is Jordan block ? Give an example.

6. (*a*) Let a, b, c be elements of a field F and

$$A = \begin{bmatrix} 0 & 0 & c \\ 1 & 0 & b \\ 0 & 1 & a \end{bmatrix}$$

Prove that the characteristic polynomial of A is same as that of its minimal polynomial. 3

(*b*) Give the definition of lattice with respect to poset and also give the definition of lattice with respect to algebra. Show that the two definitions are equivalent. 5

7. (*a*) Let T : $R^4 \to R^2$ be a linear transformation defined by

$$T\left(\left(x_1, x_2, x_3, x_4\right)^t\right) = \begin{pmatrix} x_1 - x_2 + 2x_3 \\ x_1 + x_3 + x_4 \end{pmatrix}$$

Find : 1 + 1 + 2

(*i*) Nullity T

(*ii*) Rank T

(*iii*) Verify that, Nullity T + Rank $T = 4$.

(*b*) Find the Jordan canonical form of the given matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

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8. (*a*) Prove that a linear mapping $T : V \to W$ is invertible if and only if T is one-to-one and onto. Here V and W are vector spaces over a field F. 3

(*b*) Prove that the minimal polynomial of a linear operator T divides its characteristic polynomial. 2

(*c*) Let $(\alpha_1, \alpha_2, \alpha_3)$ be an ordered basis of a real vector space V and a linear mapping $T : V \to V$ is defined by $T(\alpha_1) = \alpha_1 + \alpha_2 + \alpha_3$, $T(\alpha_2) = \alpha_1 + \alpha_2$, $T(\alpha_3) = \alpha_1$. Show that T is non-singular. Find the matrix of T^{-1} relative to the ordered basis $(\alpha_1, \alpha_2, \alpha_3)$. 3

[*Internal Assessment* : 5 Marks]