 PG/IIIS/MTM-305/14

M.Sc. 3rd Semester Examination, 2014

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(Special Paper : *OM / OR* : *Dynamical Meteorology* - I / *Operational Research Modelling* - I)

PAPER—MTM-305

Full Marks : 50

Time : 2 hours

The figures in the right-hand margin indicate marks

(*Dynamical Meteorology* - I)

Answer **Q. No. 8** and any **four** from the rest

1. What is the purpose of aerological diagram ? Derive the area equivalence of Emagram and discuss its important features. 1 + 8

2. (*a*) Derive the momentum equation of motion of an air parcel in the atmosphere in spherical co-ordinate system. 7

(Turn Over)

(*b*) What is meant by relative humidity ? 2

3. (*a*) Derive the effect of ascent and descent of an air parcel on lapse rate in terms of pressure changes. 6

(*b*) Derive the hypsometric equation in the atmosphere. 3

4. (*a*) Derive the expression of the pressure gradient force in the atmosphere. 4

(*b*) Deduce the expression for the density ρ of an air parcel at pressure p if it is adiabatically expands from a level where pressure and density are p_s and ρ_s respectively. 4

(*c*) Write down the second law of thermo-dynamics. 1

5. (*a*) How is a gradient wind generated in the atmosphere ? Discuss different cases of its occurrences. 7

(*b*) Show that the potential temperature of an air parcel is invariant. 2

6. (*a*) Derive Psychrometric equation to measure the actual vapor pressure in terms of dry bulb and wet bulb temperature. 3

   (*b*) State and prove the Clausius-Clapeyron equation in the atmosphere. 6

7. (*a*) What is the difference between Solar radiation and Terrestrial radiation ? 2

   (*b*) Derive the Beer's law indicating the relationship between incident radiative intensity and outgoing transmitted radiative intensity. Hence deduce the coefficient of transmission. 7

8. Answer any *two* questions : 2 + 2

   (*i*) What is lifting condensation level ?

   (*ii*) Define Dew-point temperature.

   (*iii*) Define pseudoadiabatic change.

[*Internal Assessment* : 10 Marks]

(*Operational Research Modelling* - I)

Answer **Q. No. 1** and any **four** from the rest

1. Answer any *four* questions : 2×4

(*a*) What do you mean by pessimistic, optimistic and most likely times in a network ? Explain.

(*b*) What do you mean by simulation ? What are the draw backs of this process ?

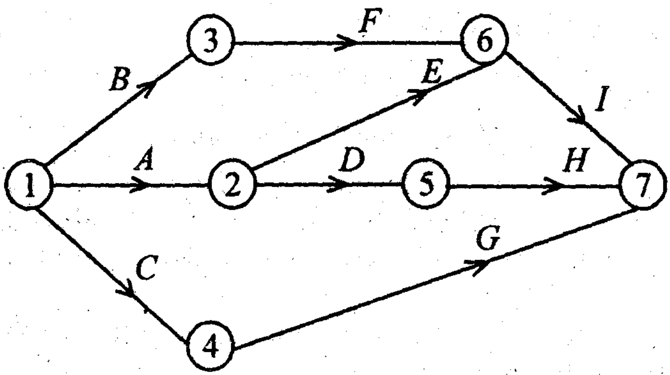
(*c*) What is the basic principle to solve a problem using dynamic programming method ?

(*d*) What do you mean by economic lot-size ?

(*e*) What is the relation between queue length and system length in (M/M/N : ∞/FCFS) queueing system ?

(*f*) What do you mean by the terms lead time and procurement cost in inventory control ?

2. A project is represented by the network shown below and has the following data :



Task	: A	B	C	D	E	F	G	H	I
Least time (week)	: 5	18	26	16	15	6	7	7	3
Greatest time (week)	: 10	22	40	20	25	12	12	9	5
Most likely time (week)	: 8	20	33	18	20	9	10	8	4

Determine the following :

(*i*) expected task time and their variance, (*ii*) the earliest and latest expected times to reach each node,

(*iii*) the critical path, and

(*iv*) the probability to complete the project in 41.5 weeks.

$$\left[\text{Given } \int_{-\infty}^{0.52} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz = 0.70\right]$$ 8

3. Describe dynamic programming method to solve the following problem :

$$\text{Minimize } Z = \sum_{j=1}^{n} f_j(y_j)$$

$$\text{subject to } \sum_{j=1}^{n} a_j y_j \geq b$$

a_j, b are real numbers, $a_j \geq 0$, $y_j \geq 0$, $b > 0$. Use this process find the values of y_1, y_2 and y_3 such that

$$\text{Minimize } Z = y_1^2 + y_2^2 + y_3^2$$

$$\text{subject to } y_1 + y_2 + y_3 \geq 75, y_1, y_2, y_3 \geq 0$$

4 + 4

4. What do you mean by random number ? Explain a method to generate random numbers. Use Monte-Carlo simulation method to find the area of a circle whose radius is a. 4 + 4

5. Suppose in a system all items are new at beginning. Each item has a probability p of failing immediately before the end of the first month of life and probability $q(= 1 - p)$ of failing immediately before the end of the second month. If all items are replaced as they fail. Show that the expected number of failures $f(x)$ at the end of a month is given by

$$f(x) = \frac{N}{1+q}\left[1-(-q)^{x+1}\right]$$

where N be the initial items of the system.
If the cost per item of individual replacement policy is Rs. C_1 and the cost per item of group replacement policy is Rs. C_2. Find the condition under which group replacement policy at the end of the first month is most profitable over individual replacement. 8

6. Derive the differential difference equations for (M/M/C : N/FCFS/∞) queueing system in transient state. 8

7. A small shop produces three machine parts 1, 2, 3 in lots. The shop has only 650 sq. ft. storage space. The appropriate data for three items are presented in the following table :

Items	1	2	3
Demand Rate (units/year)	5000	2000	10000
Procurement cost (Rs.)	100	200	75
Cost per units (Rs.)	10	15	5
Floor space required (sq. ft./unit)	0·7	0·8	0·4

The carring charge on each item is 20% of average inventory valuation per annum. If shortage are not allowed, determine the optimal lot size for each item. 8

[*Internal Assessment* : 10 Marks]