 PG/IIIS/MTM-304/14

M.Sc. 3rd Semester Examination, 2014

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

Special Paper : OM/OR : (*Dynamical Oceanology*-I/ *Advanced Optimization and Operations Research*)

PAPER — MTM- 304

Full Marks : 50

Time : 2 hours

The figures in the right hand margin indicate marks

(*Dynamical Oceanology*-I)

Answer any **five** questions :

1. Considering sea-water an as two components mixture of pure water and sult, obtain Gibbs relation and hence deduce the Gibbs-Duhem relation. 8

(*Turn Over*)

2. State the principle of conservation of mass'. Obtain the followinmg pair of equations :

$$\frac{D\rho}{Dt} + \rho \operatorname{div} \vec{q} = 0$$

$$\rho \frac{Ds}{Dt} = -\operatorname{div} \vec{I}s$$

by considering sea-water to be a two components mixture of salt and pure water (symbols have their usual meanings). 8

3. What are the assumptions regarding Boussinesq approximation ? Derive the approximate form of the field equation under these assumptions. 2 + 6

4. Show that, under usual notions,

$$T = -\frac{1}{\lambda},\ \mu_s = -U - \frac{\lambda_s}{\lambda} + \frac{\vec{q}^{\,2}}{2}$$

$$\mu_w = -U - \frac{\lambda_w}{\lambda} + \frac{\vec{q}^{\,2}}{2},\ \vec{q} = -\frac{\vec{a}}{\lambda} - \frac{1}{\lambda}(\vec{b} \times \vec{r})$$

are the necessary conditions of thermodynamical equilibrium of a finite volume of sea-water. Hence deduce the hydrostatic pressure equation. (symbols have their usual meanings). 6 + 2

5. Derive Fridman's equation for diffusion of absolute vorticity in a viscous flow in terms of motion relative to the earth. Deduce Ertel's formula for the evaluation of potential vorticity. 6 + 2

6. Assuming the sea-water to be a viscous incompressible heat conducting fluid, derive the energy equation in the form

$$\frac{\partial}{\partial t}(\rho E_m) = -div\,\vec{I}_E,$$

where symbols have their usual meanings. 8

7. Derive the equations for small amplitude wave motion in the ocean. 8

8. Show, by the method of separation of variables, that the problem of free oscillations of an ocean reduces to that of the determination of the eigen-value curves of two distinct eigen-value problem. 8

[*Internal Assessment* – 10 *Marks*]

(*Advanced Optimization and Operation Research*)

Answer **Q.No.1** and any **four** from the rest

1. Answer any *four* questions : 2 × 4

(*a*) What are the limitations of the Lagrangian multiplier technique ?

(*b*) What is the criteria to apply revised simplex method to solve an LPP ?

(*c*) Give an example showing that we may not get the optimal solution of IPP just rounding off the optimal solution of the corresponding LPP.

(*d*) In Golden section method why it is called Golden Section ?

(*e*) State Kuhun-Tucker necessary conditions for optimality test of a function.

(*f*) What do you mean by post-optimality analysis ?

2. Maximize

$$f(x_1, x_2) = x_1^2 + x_2^2 + 2gx_1 + 2fx_2 + c$$

starting from the point $\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$ by using steepest descent method. 8

3. Discuss the revised simplex method to find the optimum solution of the following LPP

Max. $Z = CX$

subject to the constraints

$$AX = b$$
$$X \geq 0.$$ 8

4. The Optimal solution of the LPP

Max. $Z = 3x_1 + 5x_2$

sub. to $x_1 + x_2 \leq 1$

$2x_1 + 3x_2 \leq 1$

and $x_1, x_2 \geq 0$

is contained in the table

C_B	Y_B	X_B	y_1	y_2	y_3	y_4
0	x_3	2/3	1/3	0	1	−1/3
5	x_2	1/3	2/3	1	0	1/3
		5/3	1/3	0	0	5/3

Find the ranges of c_1 and c_2 for which the optimal solution remains optimum when changes one at a time. 8

5. Solve the following goal programming problem graphically

$$\text{Min. } Z = P_1 d_1^- + P_2 d_2^- + P_3 d_3^-$$

subject to constraints

$$20x_1 + 10x_2 \le 60$$
$$10x_1 + 10x_2 \le 40$$
$$40x_1 + 80x_2 + d_1^- - d_1^+ = 1000$$
$$x_1 \quad + d_2^- - d_2^+ = 2$$
$$x_2 \quad + d_3^- - d_3^+ = 2$$

and $x_1, x_2, d_i^-, d_i^+ \ge 0 \quad i = 1, 2, 3.$ 8

6. Using Gomory,s cutting plane method solve the following IPP

Maximize $Z = x_1 + x_2$

subject to $5x_1 + 6x_2 \leq 30$

$5x_1 + 2x_2 \leq 16$

$x_1, x_2 \geq 0$

x_1, x_2 are integers. 8

7. Use Kuhn-Tucher method to determine x_1, x_2, x_3 so as to

Maximize $Z = -x_1^2 - x_2^2 - x_3^2 + 4x_1 + 6x_2$

subject to the constraints

$x_1 + x_2 \leq 2$

$2x_1 + 3x_2 \leq 12$ and

$x_1, x_2 \geq 0.$ 8

[*Internal Assessment* – 10 *Marks*]
