M.Sc. 2nd Semester Examination, 2014

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(*General Topology & Fuzzy Sets and Their Applications*)

PAPER—MTM-205

Full Marks : 50

Time : 2 hours

The figures in the right-hand margin indicate marks

Notations have their usual meanings

(*General Topology*)

[NEW SYLLABUS]

[*Marks* : 25]

Answer **Q. No. 1** and any **two** from the rest

1. Answer any *two* questions : 2 × 2

   (*a*) Consider the subspace $Y = (0, 1]$ of the real line $\mathbb{R}$. Let $A = \left(0, \frac{1}{7}\right)$. Find the closure of A in Y.

(Turn Over)

(*b*) Define subbasis for a topology on a set X.

(*c*) Give example of a topological space which is connected but not path connected.

(*d*) State Urysohn's Lemma.

2. (*a*) Define order topology on an ordered set X. Show that the order topology on $\mathbb{Z}_+$ is the discrete topology. 2 + 2

(*b*) Let A be a subset of the topological space X. Show that $x \in \overline{A}$ if and only if every open set U containing x intersects A. 4

3. (*a*) Define homeomorphism between two topological spaces with examples. Show that $\mathbb{R}^n$ and $\mathbb{R}$ are not homeomorphic if $n > 1$. 2 + 2

(*b*) Show that every closed subset of a compact space is compact. 4

4. (*a*) If X is a Hausdorff space, then show that a sequence of points of X converges to at most one point of X. 4

(*b*) Show that $\mathbb{R}^{w}$ in the product topology is connected. 4

5. (*a*) Define limit point compact space. Show that compactness implies limit point compactness. 1 + 3

(*b*) Show that every metrizable space is normal. 4

[*Internal Assessment* : 5 Marks]

(*Fuzzy Sets and Their Applications*)

[*Marks* : 25]

Time : 1 hour

Answer **Q. No. 1** and any **three** from the rest

1. Answer any *two* questions : 2 × 1

(*a*) What do you meant by α-cut of a fuzzy set ?

(*b*) Illustrate a trapezoidal fuzzy number.

(*c*) Write the membership function of a non-convex fuzzy set.

2. (*a*) Find $f(\tilde{A}) = \tilde{B}$.

Given $f(x) = x^2$ and

$\tilde{A} = \{(-5, 0{\cdot}1), (-4, 0{\cdot}3), (-3, 0{\cdot}5), (-2, 0{\cdot}7), (-1, 0{\cdot}9), (0, 1), (1, 0{\cdot}8), (2, 0{\cdot}6), (3, 0{\cdot}5), (4, 0{\cdot}4), (5, 0{\cdot}2)\}$. 2

(*b*) Using Zimmermann's method, determine the crisp LPP equivalent to the following fuzzy LPP :

$$\widetilde{\text{Max}}\ g_0(x) = 4x_1 + 5x_2 + 9x_3 + 11x_4 \geq \tilde{b}_0$$

$$\text{subject to } g_1(x) = x_1 + x_2 + x_3 + x_4 \leq \tilde{b}_1$$

$$g_2(x) = 7x_1 + 9x_2 + 3x_3 + 2x_4 \leq \tilde{b}_2$$

$$g_3(x) = 3x_1 + 5x_2 + 10x_3 + 15x_4 \leq \tilde{b}_3$$

$$x_1, x_2, x_3 \geq 0.$$

where the goal b_0 of the fuzzy objective is 111·57 and its corresponding tolerance p_0 is 10 and the fuzzy resource b_i and their tolerances p_i are as follows : 2 + 2

$b_1 = 15$, $b_2 = 80$, $b_3 = 100$, $p_1 = 5$, $p_2 = 40$, $p_3 = 30$

3. (*a*) Find $\tilde{A} \cup \tilde{B}$ and $\tilde{A} \cap \tilde{B}$.
Where $\tilde{A} = (3, 4, 6, 7)$ and $\tilde{B} = (2, 4, 6, 8)$. 2 + 2

(*b*) What are causes of uncertainly ? 2

4. (*a*) What is the value of $\tilde{A} / \tilde{B}$
where $\tilde{A} = [a_1, a_2]$, $\tilde{B} = [b_1, b_2]$. 1

(*b*) For $\tilde{A} = (-3, 2, 4)$ and $\tilde{B} = (-1, 0, 5)$, find $\mu_{\tilde{A}-\tilde{B}}(x)$ using α-cuts and present it geometrically. 5

5. (*a*) Let $\tilde{A}$ be a fuzzy set in X with membership function $\mu_{\tilde{A}}(x)$. Let A_α be the α-cuts of $\tilde{A}$ and $\chi_{A\alpha}(x)$ be the characteristic function of the crisp set A_α for $\alpha \in (0, 1]$. Then for each $x \in X$, show that

$$\mu_{\tilde{A}}(x) = \sup\left\{\alpha \wedge \chi_{A\alpha}(x) : 0 < \alpha \leq 1\right\} \quad 4$$

(*b*) Evaluate the following expression

$$4\,[3, 4, 6] - 3[10,\ 15] + 17 \quad 2$$

6. Discuss Verdegay's method to solve a fuzzy linear programming problem. 6

[*Internal Assessment* : 5 Marks]

(*Functional Analysis*)

[OLD SYLLABUS]

Answer **Q. No. 1** and any **four** from **Q.No. 2** to **Q. No. 7**

1. Answer any *four* questions : 2 × 4

   (*a*) Define Banach space with an example.

   (*b*) In an inner product space if $\langle x, u \rangle = \langle x, v \rangle$ for all x, then show that $u = v$.

   (*c*) Give an example of a normed linear space which is not complete.

   (*d*) Is the set $\{(x, y) \in \mathbb{R}^2 : x^2 - y^2 = 1\}$ compact in $\mathbb{R}^2$?

(*e*) When are two norms on a linear space V said to be equivalent ?

2. Define a bounded linear operator. Let X and Y be normed linear spaces and $T : X \to Y$ be a linear operator. Prove that T is continuous if and only if T is bounded. 8

3. Define fixed point of an operator f on a metric space (X, d). Prove that every contraction mapping f on a complete metric space (X, d) to itself has a unique fixed point. 1 + 7

4. (*a*) If X is a non-zero normed linear space, then show that there exists a non-zero element in X^*.

(*b*) State open mapping theorem. 6 + 2

5. (*a*) Define an adjoint operator of a bounded linear operator.

(*b*) Prove that

$$||T||^2 = ||T^*T|| = ||TT^*||$$

where $T \in B(H)$, H is a Hilbert space. 2 + 6

6. (*a*) Prove Cauchy-Schwarz inequality

$$|\langle x, y\rangle| \leq ||x|| \, ||y||, \; \forall x, y \in X$$

where X is an inner product space.

(*b*) If $x, y \in H$ (Hilbert space) and $x \perp y$ then prove that $||x+y||^2 = ||x||^2 + ||y||^2$ 5 + 3

7. (*a*) Define positive operator. Prove that T^*T is a positive operator.

(*b*) If T_1 and T_2 are self-adjoint operators then prove that T_1T_2 is self-adjoint if and only if $T_1T_2 = T_2T_1$. 1 + 3 + 4

[*Internal Assessment* : 10 Marks]